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Locally self-similar Gaussian processes: extremes and Pickands constants

Pickands theorem is a powerful tool in the analysis of exact asymptotics of

$$P\left(\sup_{t \in [0, T]} X(t) > u\right), \quad u \rightarrow \infty, \quad (1)$$

where $\{X(t) : t \in [0, T]\}$ is a centered Gaussian process with continuous trajectories. One of its classical applications deals with the class of Gaussian processes with variance function $Var(X(t))$ which attains its maximum at $t^* \in [0, T]$ and in the neighborhood of t^* :

$$\{(X(t^* + t/u) - X(t^*))u^\alpha\} \rightarrow_d \{B_\alpha(t)\} \quad (2)$$

as $u \rightarrow \infty$, where $\{B_\alpha(t) : t \geq 0\}$ is a fractional Brownian motion with Hurst parameter $\alpha \in (0, 1]$ ([5],[1]). Then, under some mild additional assumptions,

$$P\left(\sup_{t \in [0, T]} X(t) > u\right) = \mathcal{H}_\alpha \text{ Const } u^\beta \Psi\left(\frac{u}{\sqrt{Var(X(t^*))}}\right) (1+o(1)), \quad u \rightarrow \infty,$$

where Const and β are known constants, and \mathcal{H}_α is the celebrated Pickands constant, defined by the following limit

$$\mathcal{H}_\alpha = \lim_{S \rightarrow \infty} E \exp\left(\sup_{t \in [0, S]} (\sqrt{2}B_\alpha(t) - t^{2\alpha})/S\right).$$

Condition (2) restricts the applicability of Pickands's theory to cases, for which the analyzed Gaussian process locally behaves as a fractional Brownian motion.

In the talk we analyze problem (1) for a broader class of Gaussian processes. In particular, we focus on the case where (2) is replaced by

$$\{(X(t^* + t/u) - X(t^*))u^\alpha\} \rightarrow_d \{I_\alpha(t)\}, \quad u \rightarrow \infty$$

for some self-similar centered Gaussian process $\{I_\alpha(t) : t \geq 0\}$.

As an application of the obtained results, we derive the exact asymptotics of (1) for locally self-similar Gaussian processes. Additionally, we analyze properties of counterparts of Pickands constants that appear in the obtained asymptotics.

The talk is based on a joint work with Kamil Tabiś (University of Wrocław).

Bibliography

- [1] Dębicki, K., Kisowski, P. (2008) Asymptotics of supremum distribution of $(\alpha(t); A(t))$ -locally stationary Gaussian processes. *Stochastic Processes and their Applications*. **118**, 2022-2037.
- [2] Dębicki, K., Kisowski, P. (2008) A note on upper estimates for Pickands constants. *Statistics and Probability Letters* **78**, 2046-2051.
- [3] Dębicki, K., Tabiś, K. (2011) Extremes of the time-average of stationary Gaussian processes. *Stochastic Processes and their Applications*. **121**, 2049–2063.
- [4] Dębicki, K., Tabiś, K. (2012) Extremes of locally self-similar Gaussian processes. *In preparation*.
- [5] Piterbarg, V.I. *Asymptotic methods in the theory of Gaussian processes and fields*. Translations of Mathematical Monographs 148, AMS, Providence, 1996.