Analysis of coupled queues

Eline De Cuypere Koen De Turck and Dieter Fiems

November 8, 2012

イロン イヨン イヨン イヨン

æ

Eline De Cuypere Koen De Turck and Dieter Fiems Analysis of coupled queues



◆□ > ◆□ > ◆臣 > ◆臣 > ○

Э



Eline De Cuypere Koen De Turck and Dieter Fiems

Analysis of coupled queues

Coupled queues

service is only possible if none of the queues are empty.



-≣->



・ロ・・(四・・)を注・・(注・・)注



slow convergence of simulation results

・ロン ・四と ・ヨン ・ヨン



- slow convergence of simulation results
- null-recurrence

・ロ・ ・ 日・ ・ 日・ ・ 日・



- slow convergence of simulation results
- null-recurrence
- state-space explosion

イロン イヨン イヨン イヨン



- slow convergence of simulation results
- null-recurrence
- state-space explosion
- rather complicated matrix structure

< 17 b

< ≣ >

→ Ξ →



- slow convergence of simulation results
- null-recurrence
- state-space explosion
- rather complicated matrix structure

< 17 b

< ≣ >

→ Ξ →

Table of contents

Applications

Kitting process Decoupling inventory Energy harvesting process

Numerical techniques

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

Conclusion

3 × 4 3 ×

Kitting process Decoupling inventory Energy harvesting process

イロン 不同と 不同と 不同と

æ

Applications

- Kitting process
- Decoupling buffer
- Energy harvesting process

Kitting process Decoupling inventory Energy harvesting process

Kitting process



Eline De Cuypere Koen De Turck and Dieter Fiems

Analysis of coupled queues

Э





Kitting process Decoupling inventory Energy harvesting process

Decoupling inventory in a hybrid push-pull system



Raw materials are **pushed** into the semi-finished product inventory while customers **pulls** products by placing orders

 Applications
 Kitting process

 Numerical techniques
 Decoupling inventory

 Conclusion
 Energy harvesting process



・ロト ・回ト ・ヨト ・ヨト

Kitting process Decoupling inventory Energy harvesting process

イロト イヨト イヨト イヨト

Energy harvesting process



Energy harvesting is the process by which

ambient energy is converted into electrical energy and

Kitting process Decoupling inventory Energy harvesting process

イロト イポト イヨト イヨト

Energy harvesting process



Energy harvesting is the process by which

- ambient energy is converted into electrical energy and
- this energy is stored in small autonomous devices called sensor nodes.

Applications Kitting process Numerical techniques Decoupling inventory Conclusion Energy harvesting process



▲口 → ▲圖 → ▲ 国 → ▲ 国 → □

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロト イヨト イヨト イヨト

Numerical techniques

Sparse matrix techniques

- Matrix-analytic methods
- Taylor series expansion approach

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

・ロト ・回ト ・ヨト

• 3 > 1

Sparse matrix techniques

Sparse method

> sparse matrix: matrix with most of its elements equal to zero

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロト イポト イヨト イヨト

Sparse matrix techniques

Sparse method

- sparse matrix: matrix with most of its elements equal to zero
- generator matrix of paired queues where e.g. $C_1 = C_2 = 100$:
 - standard: 40804² elements
 - ► sparse: 3x40804 elements

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロト イポト イヨト イヨト

Sparse matrix techniques

Sparse method

- sparse matrix: matrix with most of its elements equal to zero
- generator matrix of paired queues where e.g. $C_1 = C_2 = 100$:
 - standard: 40804² elements
 - sparse: 3x40804 elements
- less memory consumption and processing time than for standard matrices

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロト イヨト イヨト イヨト

Sparse matrix techniques

Sparse method

- sparse matrix: matrix with most of its elements equal to zero
- generator matrix of paired queues where e.g. $C_1 = C_2 = 100$:
 - standard: 40804² elements
 - sparse: 3x40804 elements
- less memory consumption and processing time than for standard matrices

General Minimal Residual Method (GMRES)

- iterative method to solve sparse matrix equations
- fast and sufficiently accurate
- however, size of the state space is strictly limited

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロト イヨト イヨト イヨト

Numerical techniques

- Sparse matrix techniques
- Matrix-analytic methods
- Taylor series expansion approach

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロン イヨン イヨン イヨン

æ

QBD Processes

We define a Markov process with state $S = \{(n, i) : n > 0, 1 \le i \le m\}$

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

QBD Processes

We define a Markov process with state $S = \{(n, i) : n > 0, 1 \le i \le m\}$

and with infinitesimal generator

$$Q = \begin{bmatrix} B_0 A_0 & 0 & 0 & \cdots \\ A_2 A_1 A_0 & 0 & \cdots \\ 0 & A_2 A_1 A_0 & \cdots \\ 0 & 0 & A_2 A_1 \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

イロト イヨト イヨト イヨト

æ

Eline De Cuypere Koen De Turck and Dieter Fiems Analysis of coupled queues

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

QBD Processes

We define a Markov process with state $S = \{(n, i) : n > 0, 1 \le i \le m\}$

and with infinitesimal generator

$$Q = \begin{bmatrix} B_0 \ A_0 \ 0 \ 0 \ \cdots \\ A_2 \ A_1 \ A_0 \ 0 \ \cdots \\ 0 \ A_2 \ A_1 \ A_0 \ \cdots \\ 0 \ 0 \ A_2 \ A_1 \ \cdots \\ \vdots \ \vdots \ \vdots \ \ddots \end{bmatrix}$$

・ロト ・回ト ・ヨト

The one-step transitions are restricted to states in the same level (n, *) or in two adjacent levels (n + 1, *) or (n - 1, *).

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

QBD Processes

We define a Markov process with state $S = \{(n, i) : n > 0, 1 \le i \le m\}$

and with infinitesimal generator

$$Q = \begin{bmatrix} B_0 \ A_0 \ 0 \ 0 \ \cdots \\ A_2 \ A_1 \ A_0 \ 0 \ \cdots \\ 0 \ A_2 \ A_1 \ A_0 \ \cdots \\ 0 \ 0 \ A_2 \ A_1 \ \cdots \\ \vdots \ \vdots \ \vdots \ \ddots \end{bmatrix}$$

イロト イヨト イヨト イヨト

The one-step transitions are restricted to states in the same level (n, *) or in two adjacent levels (n + 1, *) or (n - 1, *).

The following equation determine the stationary distribution: $\pi_{\mathbf{n}}=\pi_{\mathbf{0}}\mathbf{R}^{\mathbf{n}}$

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

QBD Processes

We define a Markov process with state $S = \{(n, i) : n > 0, 1 \le i \le m\}$

and with infinitesimal generator

$$Q = \begin{bmatrix} B_0 \ A_0 \ 0 \ 0 \ \cdots \\ A_2 \ A_1 \ A_0 \ 0 \ \cdots \\ 0 \ A_2 \ A_1 \ A_0 \ \cdots \\ 0 \ 0 \ A_2 \ A_1 \ \cdots \\ \vdots \ \vdots \ \vdots \ \ddots \end{bmatrix}$$

イロン イヨン イヨン イヨン

The one-step transitions are restricted to states in the same level (n, *) or in two adjacent levels (n + 1, *) or (n - 1, *).

The following equation determine the stationary distribution: $\pi_{\bf n}=\pi_{\bf 0}{\bf R}^{\bf n}$

with the *rate matrix* \mathbf{R} , solution of the equation :

$$\mathbf{R} = \mathbf{A}_0 + \mathbf{R}\mathbf{A}_1 + \mathbf{R}^2\mathbf{A}_2$$

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

decoupling inventory model



・ロン ・回 と ・ ヨン ・ ヨン

Э

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

・ロン ・回 と ・ ヨン ・ ヨン

Э

decoupling inventory model



Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロン イヨン イヨン イヨン

Э

decoupling inventory model



Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

decoupling inventory model



◆□> ◆□> ◆目> ◆目> ・目 ・のへぐ

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

decoupling inventory model



Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

・ロト ・回ト ・ヨト

3

decoupling inventory model



Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

-

decoupling inventory model



> 3-dimensional modulating Markov chain with state (n, m, i)

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

decoupling inventory model



- ▶ 3-dimensional modulating Markov chain with state (n, m, i)
 - n = number of backlogged orders

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

decoupling inventory model



3-dimensional modulating Markov chain with state (n, m, i)

- n = number of backlogged orders
- m = number of semi-finished products

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

• • • • • • • • • • •

decoupling inventory model



▶ 3-dimensional modulating Markov chain with state (*n*, *m*, *i*)

- n = number of backlogged orders
- m = number of semi-finished products
- *i* = state of the modulating chain

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

Numerical results



- capacity: C_p = 20
- product arrival rate: $\lambda_p = 1$
- order arrival rate: $\lambda_o = 0.85$
- order processing rate: $\mu = 1$
- no set-up time

Eline De Cuypere Koen De Turck and Dieter Fiems

イロン イヨン イヨン イヨン

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

block matrix structure for 3 finite capacity queues

 $C_i = 2$ for queue $i = \{1, 2, 3\}$

generator matrix with a tridiagonal block structure

1	0	λ_3	0	λ_2	0	0	0	0	0	λ_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ł	0	0	λ_3	0	λ_2	0	0	0	0	0	λ_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	λ_2	0	0	0	0	0	λ_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	λ_3	0	λ_2	0	0	0	0	0	λ_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	λ_{1}	0	λ_2	0	0	0	0	0	λ_1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	Ő	0	0	λ_2	0	0	0	0	0	λ_1	0	0	0	0	0	0	0	0	0	0	0	0
ł	0	0	0	0	0	0	0	λ_3	0	0	0	0	0	0	0	λ_1	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	λ_3	0	0	0	0	0	0	0	λ_1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	Ő	0	0	0	0	0	0	0	0	λ_1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	1.0	0	0			0	1.5			1.0	0	0	1.	0	0	1.0	0	0	1.0	0	0
ł	0	0	0	0	0	0	0	0	0		- 13		A2 0	÷.		0	0	0	14	- <u>.</u> .		0	0	0		0	0
	0	0	0	0	0	0	0	0	0	l X		~3	, č	~~	N.	1.0		0		- 11	Ň	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	v	0	0	•		A2	0	0	0	0	0	A1	0	0	0	0	0	0
ł	0	0	0	0	0	0	0	0	0	0	0	0	0	λ_3	.0	λ_2	.0	0	0	0	0	λ_1	0	0	0	0	0
	μ	0	0	0	0	0	0	0	0	0	0	0	0	0	λ_3	0	λ_2	0	0	0	0	0	λ_1	0	0	0	0
1	0	μ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	λ_2	0	0	0	0	0	λ_1	0	0	0
ł	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	λ_3	0	0	0	0	0	0	0	0	0	λ_1
	0	0	0	μ	0	0	0	0	0	0	0	0	0	0	0	0	0	λ_3	0	0	0	0	0	0	0	λ_1	0
	0	0	0	0	μ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	λ_1
l	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.0	0	0	1.0	λ.	0	10	0	0	1.0	0	0
	ŏ	ő	ő	ŏ	ŏ	ŏ	ŏ.	ŏ	ŏ	ŏ	ŏ	ő	ŏ	ŏ	ŏ	lő.	ő	ŏ	۱ő.	õ	λ.	0	No.	ŏ	ŏ	ő	ő
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	Ó	0	0	0	X2	0	0	0
ł	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	λa	0	30	0	0
	ŏ	ő	ő	ŏ	ŏ	ŏ	ŏ.	ŏ	ŏ	u.	ŏ	ŏ	ŏ	ŏ	ŏ	ŏ.	ő	ŏ	lő.	ŏ	ő	ŏ	ñ	X.	0	x.	ő.
I	ŏ	ő	ő	ŏ	ő	ŏ	ŏ	ŏ	ŏ	0	ü	ŏ	ŏ	ŏ	ő	ŏ	ő	Ű.	ŏ.	ŏ	ő	ŏ	ŏ	0	ō	0	22
ł	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	<u>).</u>	0
L	ň	ň	ň	0	ŏ	ŏ	lŏ.	0	0	ŏ.	ŏ	ň		ŏ	ŏ	lă.	ň	0	lő.	ň	ň	ň	0	ŏ	ň	- 73	¥.
L	ŏ	0	0	0	ő	ŏ	l ő	0	0	lă.	ő	0	6		ŏ	l ő.			Lő.	ő	0	ŏ	0	ő	l ő	ŏ	- Ca
	. 0	0	0	U .	0	0	1.0	0	0	0	0	0	9	μ	0	1.0	0	0	1.0	0	0	0	0	0		9	• /

1

イロト イヨト イヨト イヨト

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロト イヨト イヨト イヨト

Numerical techniques

- Sparse matrix techniques
- Matrix-analytic methods
- Taylor series expansion approach

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロト イポト イヨト イヨト

Taylor series expansion approach

Consider a family of (continuous-time) Markov processes $\{X_{\epsilon}(t)\}$ over a finite state space \mathcal{X} of size M with generator matrix

$$Q_{\epsilon} = Q^{(0)} + \epsilon Q^{(1)},$$

 π_ϵ denotes the corresponding stationary distribution

 $\pi_{\epsilon} Q_{\epsilon} = 0$

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロト イポト イラト イラト

Taylor series expansion approach

Consider a family of (continuous-time) Markov processes $\{X_{\epsilon}(t)\}$ over a finite state space \mathcal{X} of size M with generator matrix

$$Q_\epsilon = Q^{(0)} + \epsilon Q^{(1)},$$

 π_ϵ denotes the corresponding stationary distribution

$$\pi_{\epsilon} Q_{\epsilon} = 0$$

Numerical computation of the steady-state vector has an asymptotic time complexity of $O(M^3)$ Models that suffer from state-space explosion stay generally out of reach of a numerical analysis

Applications	Sparse matrix techniques
Numerical techniques	Matrix-analytic methods
Conclusion	Taylor series expansion approach

Taylor series expansion around $\epsilon = 0$

Vector π_ϵ is required to be analytic in a neighbourhood of $\epsilon = 0$

Taylor series expansion around $\epsilon = 0$

Vector π_{ϵ} is required to be analytic in a neighbourhood of $\epsilon = 0$

- we only consider finite state space
- entries of the generator matrix depend analytically on the parameter
- only one recurrent class for $\epsilon = 0$

"Regular perturbation"

From now on: assume $Q_0 = Q^{(0)}$ is a generator matrix with one recurrent class

Recursive solution of the series expansion

 $\pi^{(0)}Q^{(0)} = 0$

$$\pi^{(n+1)}Q^{(0)} = -\pi^{(n)}Q^{(1)}$$

We have to solve a linear system of equations for each term $\pi^{(i)}$ in the expansion

・ロン ・回と ・ヨン・

Applications	Sparse matrix techniques
Numerical techniques	Matrix-analytic methods
Conclusion	Taylor series expansion approach

Impose the extra condition that Q⁽⁰⁾ for some ordering of the state space be triangular

・ロト ・回ト ・ヨト

< ≣⇒

Impose the extra condition that Q⁽⁰⁾ for some ordering of the state space be triangular

イロン イヨン イヨン イヨン

æ

• Solution by backward substitutions $\rightarrow O(M^2)$

Impose the extra condition that Q⁽⁰⁾ for some ordering of the state space be triangular

イロン イヨン イヨン イヨン

- Solution by backward substitutions $\rightarrow O(M^2)$
- ▶ In practice $Q^{(0)}$ and $Q^{(1)}$ often sparse $\rightarrow O(M)$

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach



- system of K queues, queue i having finite capacity C_i
- Poisson arrivals in each of the queues at rate λ_i
- exponentially distributed service rate μ
- Lexicographical order
- Expansion around $\epsilon = \mu = 0$, all queues are full for $\epsilon = 0$

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach



- system of K queues, queue i having finite capacity C_i
- Poisson arrivals in each of the queues at rate λ_i
- exponentially distributed service rate μ
- Lexicographical order
- Expansion around $\epsilon = \mu = 0$, all queues are full for $\epsilon = 0$

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach



- system of K queues, queue i having finite capacity C_i
- Poisson arrivals in each of the queues at rate λ_i
- exponentially distributed service rate μ
- Lexicographical order
- Expansion around $\epsilon = \mu = 0$, all queues are full for $\epsilon = 0$

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

イロト イヨト イヨト イヨト



- system of K queues, queue i having finite capacity C_i
- Poisson arrivals in each of the queues at rate λ_i
- exponentially distributed service rate μ
- Lexicographical order
- Expansion around $\epsilon = \mu = 0$, all queues are full for $\epsilon = 0$

Applications Sparse matrix techniques Numerical techniques Matrix-analytic methods Conclusion Taylor series expansion approach

1. Balance equation

$$\begin{aligned} \pi(i_1, i_2, \dots, i_{\mathcal{K}}) \left(\mu \prod_{\ell=1}^{\mathcal{K}} \mathbf{1}_{\{i_\ell > 0\}} + \sum_{\ell=1}^{\mathcal{K}} \mathbf{1}_{\{i_\ell < C_\ell\}} \lambda_\ell \right) = \\ \pi(i_1 + 1, i_2 + 1, \dots, i_{\mathcal{K}} + 1) \mu \prod_{\ell=1}^{\mathcal{K}} \mathbf{1}_{\{i_\ell < C_\ell\}} \\ + \sum_{\ell=1}^{\mathcal{K}} \pi(i_1, \dots, i_{\ell-1}, i_\ell - 1, i_{\ell+1}, \dots, i_{\mathcal{K}}) \lambda_\ell \mathbf{1}_{\{i_\ell > 0\}} \end{aligned}$$

・ロ・・ (日・・ (日・・ (日・)

Applications Sparse matrix techniques Numerical techniques Matrix-analytic methods Conclusion Taylor series expansion approach

2. Expansion

$$\pi(\mathbf{i}) = \sum_{n=0}^{\infty} \pi_n(\mathbf{i}) \mu^n$$

3. Substitution

$$\sum_{n=0}^{\infty} \pi_n(i_1, i_2, \dots, i_K) \mu^n \left(\mu \prod_{\ell=1}^K \mathbf{1}_{\{i_\ell > 0\}} + \sum_{\ell=1}^K \mathbf{1}_{\{i_\ell < C_\ell\}} \lambda_\ell \right) = \sum_{n=0}^{\infty} \pi_n(i_1 + 1, i_2 + 1, \dots, i_K + 1) \mu^{n+1} \prod_{\ell=1}^K \mathbf{1}_{\{i_\ell < C_\ell\}} + \sum_{n=0}^{\infty} \sum_{\ell=1}^K \pi_n(i_1, \dots, i_{\ell-1}, i_\ell - 1, i_{\ell+1}, \dots, i_K) \lambda_\ell \mu^n \mathbf{1}_{\{i_\ell > 0\}}$$

・ロト ・回ト ・ヨト ・ヨト

Applications Sparse matrix techniques Numerical techniques Matrix-analytic methods Conclusion Taylor series expansion approach

4. Compare terms in μ^n

$$\pi_{n}(i_{1}, i_{2}, \dots, i_{K}) = \frac{1}{\sum_{\ell=1}^{K} \mathbf{1}_{\{i_{\ell} < C_{\ell}\}} \lambda_{\ell}} \times \left(\mathbf{1}_{\{n>0\}} \pi_{n-1}(i_{1}+1, i_{2}+1, \dots, i_{K}+1) \prod_{\ell=1}^{K} \mathbf{1}_{\{i_{\ell} < C_{\ell}\}} + \sum_{\ell=1}^{K} \pi_{n}(i_{1}, \dots, i_{\ell-1}, i_{\ell}-1, i_{\ell+1}, \dots, i_{K}) \lambda_{\ell} \mathbf{1}_{\{i_{\ell}>0\}} - \mathbf{1}_{\{n>0\}} \pi_{n-1}(i_{1}, i_{2}, \dots, i_{K}) \prod_{\ell=1}^{K} \mathbf{1}_{\{i_{\ell}>0\}}\right)$$

・ロン ・回 と ・ ヨン ・ モン

Э

5. Normalisation condition

$$\pi_0([C_1, C_2, \ldots, C_K]) = 1$$

・ロト ・回ト ・ヨト ・ヨト

æ

$$\pi_n([C_1, C_2, \ldots, C_K]) = -\sum_{i \in C^*} \pi_n(\mathbf{i})$$

with $\mathcal{C}^* = \mathcal{C} \setminus \{[C_1, C_2, \dots, C_K]\}$

Applications	Sparse matrix techniques
Numerical techniques	Matrix-analytic methods
Conclusion	Taylor series expansion approach

What's the speed?

$C \setminus N$	5	10	20	50	100
10	0.340	0.376	0.415	0.534	0.735
20	0.796	1.237	2.144	4.678	8.960
30	3.783	6.842	12.984	32.856	64.660
40	14.640	30.422	53.202	128.375	257.236

Table: Computation time in seconds for 5 queues

- - 4 回 ト - 4 回 ト

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

・ロン ・回 と ・ ヨン ・ ヨン

æ

Numerical results



5 queues, $\lambda = 1$ and C = 10 for all queues

Sparse matrix techniques Matrix-analytic methods Taylor series expansion approach

・ロン ・回 と ・ヨン ・ヨン

æ

Numerical results



5 queues, $\lambda_1=0.8$ and $\lambda_{2-5}=1,~C=10$

Conclusion

Conclusion

Numerical analysis of coupled queues in a Markovian setting

個 ト く ヨ ト く ヨ ト

- Sparse-matrix techniques
- Matrix-analytic methods
- Taylor series expansion approach

Conclusion

Conclusion

Numerical analysis of coupled queues in a Markovian setting

< E → < E →</p>

- Sparse-matrix techniques
- Matrix-analytic methods
- Taylor series expansion approach

Future work

Compare the analysis methodology

Conclusion

Conclusion

- Numerical analysis of coupled queues in a Markovian setting
 - Sparse-matrix techniques
 - Matrix-analytic methods
 - Taylor series expansion approach

Future work

- Compare the analysis methodology
- Investigate the matrix-analytic methods to cope with the tridiagonal block matrix structure of the generator matrix for the multidimensional case.

回 と く ヨ と く ヨ と

Conclusion

Conclusion

- Numerical analysis of coupled queues in a Markovian setting
 - Sparse-matrix techniques
 - Matrix-analytic methods
 - Taylor series expansion approach

Future work

- Compare the analysis methodology
- Investigate the matrix-analytic methods to cope with the tridiagonal block matrix structure of the generator matrix for the multidimensional case.

回 と く ヨ と く ヨ と

Investigate other methods: mean field analysis, stochastic fluid model,...

Questions?



・ロン ・回と ・ヨン・

Э

Eline De Cuypere Koen De Turck and Dieter Fiems Analysis of coupled queues