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Some results on inhomogeneous percolation

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Let $\mathbb{L}$ be the d-dimensional hypercubic lattice and let $\mathbb{L}_0$ be an s-dimensional hypercubic sublattice of $\mathbb{L}$ which contains the origin (and where $2 \leq s < d$).

Percolation at densities (p, σ) can be set up in $\mathbb{L}$ by declaring edges in $\mathbb{L}_0$ open with probability σ, and edges in $\mathbb{L} \setminus \mathbb{L}_0$ open with probability p. The probability that the open cluster C at the origin is open is given by

$$\theta^I(p, \sigma) = P^I_{p,\sigma}(|C| = \infty).$$

In this talk we examine the (p, σ)-parameter space of this model. We prove existence of a critical curve $\sigma^*(p)$ such that the model is subcritical if $\sigma < \sigma^*(p)$ and supercritical if $\sigma > \sigma^*(p)$. We show that $\sigma^*(p)$ is strictly decreasing with $p \in (0, p_c(d))$, and $\sigma^*(p) = 0$ if $p \in (p_c(d), 1)$ (where $p_c(d)$ is the critical density for homogeneous percolation in $\mathbb{L}$).

Other results, including uniqueness of the critical point and decays of the sub- and supercritical cluster distributions will be given, and a connection to a model of collapsing lattice animals will be made.

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