Flow control problems in dynamical distribution networks[1]

Claudio De Persis

Institute of Technology and Management
Faculty of Mathematics and Natural Sciences
University of Groningen

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[1] Joint work with **Mathias Bürger**, University of Stuttgart, Germany.

Outline

- Output agreement under time-varying disturbances
- Necessary conditions
- Sufficient conditions
    - Incremental passive systems
    - Dynamic controllers
    - Static disturbances
- Power Generation Control

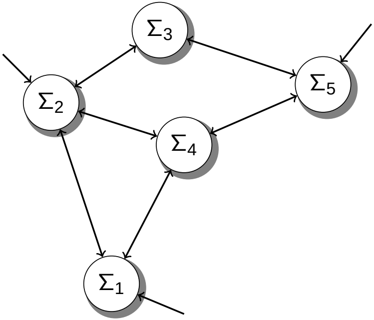
Output agreement under time-varying disturbances

Cooperative Control and Flow Networks

Network systems

Dynamical systems interconnected **physically** and via **information exchange**

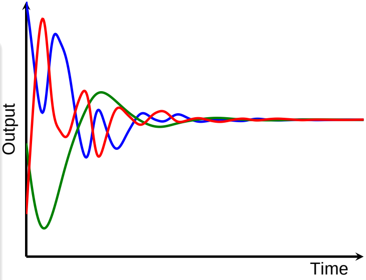
- Formation Control
- Distributed Optimization
- Circuits Agreement



Flow networks

Vital in contemporary society

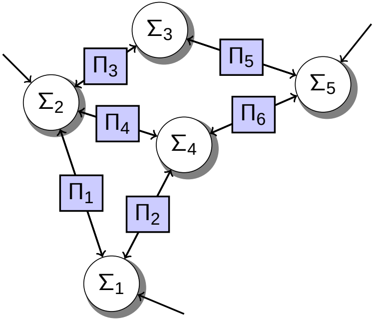
- Exchange material
- Subject to external disturbances
- Output synchronization
- Optimality



Cooperative Control and Flow Networks

Node dynamics

$$\Sigma_i : \dot{x}_i = f_i(x_i, u_i, w_i)$$
$$y_i = h_i(x_i, w_i), i = 1, 2, \ldots, n$$



Exo-systems $\dot{w}_i = s_i(w_i)$
$(s_i(w_i) - s_i(w_i'))^T (w_i - w_i') \leq 0$

Edge controllers – Dynamic coupling

$$\Pi_k : \dot{\eta}_k = \phi_k(\eta_k, \nu_k)$$
$$\lambda_k = \psi_k(\eta_k, \nu_k), \quad k = 1, 2, \ldots, m,$$

Couplings

- $\nu = -(B \otimes I_p)^T y$ *(diffusive couplings, relative outputs)*
- $u = (B \otimes I_p)\lambda$ *(physical coupling)*

B: edge-node signed **incidence matrix** of the network.

Couplings

$\nu = -(B \otimes I_p)^T y$ (Relative measurements)

$$\text{Edge } k \; : \quad \nu_k = -(b_{ik} y_i + b_{jk} y_j) = \begin{cases} y_i - y_j & b_{ik} = -1 \\ y_j - y_i & b_{ik} = +1 \end{cases}$$

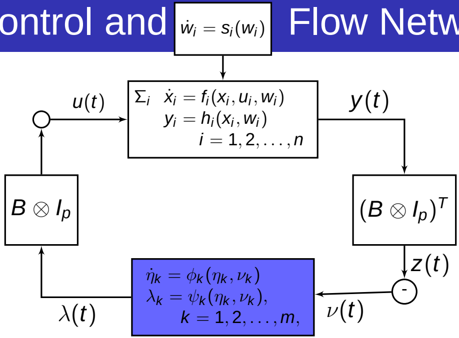
$u = (B \otimes I_p)^T \lambda$ (Balance equation)

$$\text{Node } i \; : \quad u_i = \sum_k b_{ik} \lambda_k$$

Signed **incidence matrix** $B = [b_{ik}]$ $(n \times m)$ matrix

$$b_{ik} = \begin{cases} +1 & \text{node } i \text{ **positive** end of link } k \\ -1 & \text{node } i \text{ **negative** end of link } k \\ 0 & \text{otherwise} \end{cases}$$

Cooperative Control and Flow Networks



Output Agreement Problem

Given nonlinear dynamical systems Σ_i under couplings

$$\nu = -(B \otimes I_p)^T y, \quad u = (B \otimes I_p)\lambda$$

find controllers Γ_i such that every solution $(w(t), x(t), \eta(t))$ is bounded and satisfies

$$\lim_{t\to\infty} (B^T \otimes I_p)\, y(t) = \mathbf{0} \Leftrightarrow \lim_{t\to\infty} \|y_i(t) - y_j(t)\| = \mathbf{0}, \forall i, j$$

Necessary conditions

Necessary conditions

There exists ω-limit set $\Omega(\mathcal{W} \times \mathcal{X} \times \Xi)$ which is nonempty, compact, invariant and uniformly attracts $\mathcal{W} \times \mathcal{X} \times \Xi$ under the flow of

$$\begin{aligned} \dot{w} &= s(w) \\ \dot{x} &= f(x, (B \otimes I_p)\lambda, w) \\ \dot{\eta} &= \phi(\eta, -(B^T \otimes I_p)h(x, w)). \end{aligned}$$

Moreover

$$\Omega(\mathcal{W} \times \mathcal{X} \times \Xi) \subset \{(w, x, \eta) : (B^T \otimes I_p)h(x, w) = \mathbf{0}\}.$$

By invariance, for every $\dot{w} = s(w)$ originating from $\mathcal{W}$, there must exist (x^w, u^w) s.t.

$$\textbf{(RE)} \quad \begin{aligned} \dot{x}^w &= f(x^w, u^w, w) \\ \mathbf{0} &= (B^T \otimes I_p)h(x^w, w) \end{aligned}$$

and η^w s.t.

$$\textbf{(IMP)} \quad \begin{aligned} \dot{\eta}^w &= \phi(\eta^w, \mathbf{0}) \\ \lambda^w &= \psi(\eta^w, \mathbf{0}) \end{aligned}$$

with $u^w = (B \otimes I_p)\lambda^w$.

Necessary conditions

(RE) regulator equations

If $\exists \pi(\cdot) : \mathcal{W} \to \mathcal{X}, \sigma(\cdot) : \mathcal{W} \to \mathcal{X}$ s.t. $x^w = \pi(w)$ and $\eta^w = \sigma(w)$ then (RE) becomes

$$\begin{aligned} \frac{\partial \pi}{\partial w} s(w) &= f(\pi(w), (B \otimes I_p)\psi(\sigma(w)), w) \\ 0 &= (B^T \otimes I_p) h(\pi(w), w) \end{aligned}$$

and (IMP)

$$\begin{aligned} \frac{\partial \sigma}{\partial w} s(w) &= \phi(\sigma(w), \mathbf{0}) \\ \lambda^w &= \psi(\sigma(w)) \end{aligned}$$

that make the manifold

$$\begin{aligned} \mathcal{M} &= \{(x, \eta, w) : x = \pi(w),\ \eta = \sigma(w)\} \\ &\subset \{(x, \eta, w) : (B^T \otimes I_p) h(x, w) = \mathbf{0}\} \end{aligned}$$

invariant

Necessary conditions

(IMP) implies that the controller

$$\begin{array}{rcl} \dot{\eta} & = & \phi(\eta, \nu) \\ \lambda & = & \psi(\eta, \nu) \end{array}$$

with

$$\nu = \mathbf{0}, \quad \eta(0) = \sigma(w(0))$$

generates the feedforward input

$$u^w = (B \otimes I_p)\psi(\eta, \mathbf{0})$$

Controllers on the edges

To decompose the controller into controllers on the edges, we introduce

$$\begin{array}{rcl} \dot{\eta}_k & = & \phi(\eta_k, \nu_k) \\ \lambda_k & = & \psi_k(\eta_k, \nu_k), \quad k = 1, 2, \ldots, m \end{array}$$

Sufficient conditions

A class of dynamical systems

Systems

$$\Sigma_i : \quad \begin{array}{ll} \dot{x}_i & = f_i(x_i, u_i, w_i) \\ y_i & = h_i(x_i, w_i), i = 1, 2, \dots, n \end{array}$$

are **incrementally passive** if there exists a C^1 regular storage function $V_i : \mathbb{R}_{\geq 0} \times \mathbb{R}^{r_i} \times \mathbb{R}^{r_i} \to \mathbb{R}_{\geq 0}$ such that

$$\frac{\partial V_i}{\partial t} + \frac{\partial V_i}{\partial x_i} f_i(x_i, u_i, w_i) + \frac{\partial V_i}{\partial x_i'} f_i(x_i', u_i', w_i) \leq (y_i - y_i')^T (u_i - u_i')$$

Interpretation

- $V_i(t) = \frac{1}{2}(x_i(t) - x_i'(t))^T Q_i (x_i(t) - x_i'(t))$ distance of solutions $x_i(t), x_i'(t)$
- $\Delta y_i^T \Delta u_i = (y_i - y_i')^T (u_i - u_i')$ “incremental power injection”
- Incremental dissipation inequality yields

$$\frac{d}{dt} V_i(t) \leq \Delta y_i^T(t) \Delta u_i(t)$$

A class of dynamical systems

Systems

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Example 1 Linear systems

$$\begin{aligned} \dot{x}_i &= A_i x_i + G_i u_i + P_i w_i \\ y_i &= C_i x_i, \end{aligned}$$

that are **passive** are incrementally passive with $V_i = \frac{1}{2}(x_i - x_i')^T Q_i (x_i - x_i')$ and

$$Q_i = Q_i^T > 0 \text{ s.t. } A_i^T Q_i + Q_i A_i \leq 0 \text{ and } Q_i G_i = C_i^T$$

A class of dynamical systems

Systems

$$\Sigma_i : \quad \begin{array}{ll} \dot{x}_i & = f_i(x_i, u_i, w_i) \\ y_i & = h_i(x_i, w_i), i = 1, 2, \dots, n \end{array}$$

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$$\frac{\partial V_i}{\partial t} + \frac{\partial V_i}{\partial x_i} f_i(x_i, u_i, w_i) + \frac{\partial V_i}{\partial x_i'} f_i(x_i', u_i', w_i) \leq (y_i - y_i')^T (u_i - u_i')$$

Example 2 Nonlinear systems

$$\begin{array}{rcl} \dot{x}_i & = & f_i(x_i) + G_i u_i + P_i w_i \\ y_i & = & C_i x_i \end{array}$$

with $f_i(x_i) = \nabla F_i(x_i)$, $F_i(x_i)$ twice continuously differentiable and concave, and $G_i = C_i^T$ are incrementally passive. By concavity of $F_i(x_i)$, $(x_i - x_i')^T (f_i(x_i) - f_i(x_i')) \leq 0$, and $V_i = \frac{1}{2}(x_i - x_i')^T (x_i - x_i')$

A class of dynamical systems

Systems

$$\Sigma_i : \quad \begin{aligned} \dot{x}_i &= f_i(x_i, u_i, w_i) \\ y_i &= h_i(x_i, w_i), i = 1, 2, \ldots, n \end{aligned}$$

are **incrementally passive** if there exists a C^1 regular storage function $V_i : \mathbb{R}_{\geq 0} \times \mathbb{R}^{r_i} \times \mathbb{R}^{r_i} \to \mathbb{R}_{\geq 0}$ such that

$$\frac{\partial V_i}{\partial t} + \frac{\partial V_i}{\partial x_i} f_i(x_i, u_i, w_i) + \frac{\partial V_i}{\partial x_i'} f_i(x_i', u_i', w_i) \leq (y_i - y_i')^T (u_i - u_i')$$

- Passive systems naturally arise in many applications
- Passivity is preserved under the coupling conditions
- Incremental passivity provides a framework for comparing solutions

Controllers

Controllers at the edges

$$\begin{aligned} \dot{\eta}_k &= \phi(\eta_k, \nu_k) \\ \lambda_k &= \psi_k(\eta_k), \quad k = 1, 2, \ldots, m \end{aligned}$$

are **incrementally passive**, i.e. there exist regular storage functions $W_k(\eta_k, \eta_k')$, with $W_k : \mathbb{R}^{q_k} \times \mathbb{R}^{q_k} \to \mathbb{R}_{\geq 0}$ such that

$$\frac{\partial W_k}{\partial \eta_k} \phi_k(\eta_k, \nu_k) + \frac{\partial W_k}{\partial \eta_k'} \phi_k(\eta_k', \nu_k') \leq (\lambda_k - \lambda_k')^T (\nu_k - \nu_k').$$

Example Let $(w_i - w_i')^T (s_i(w_i) - s_i(w_i')) \leq 0$. Then

$$\begin{aligned} \dot{\eta}_k &= \underbrace{s(\eta_k) + M_k^T v_k}_{\phi_k(\eta_k, \nu_k)} \\ \lambda_k &= \underbrace{M_k \eta_k}_{\psi_k(\eta_k)} \end{aligned}$$

with

$$W_k(\eta_k, \eta_k') = (\eta_k - \eta_k')^T (\eta_k - \eta_k')/2$$

Main result

Theorem

- There exist bounded solutions to **(RE)+(IMP)**
- Incremental passivity of Σ_i and Π_k holds

Then the controllers

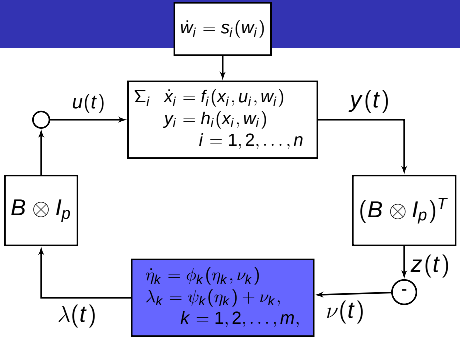
$$\Pi_k \; : \quad \begin{array}{rcl} \dot{\eta}_k & = & \phi(\eta_k, \nu_k) \\ \lambda_k & = & \psi_k(\eta_k) + \nu_k, \quad k = 1, 2, \ldots, m \end{array}$$

with

$$\nu = -(B \otimes I_p)^T y, \quad u = (B \otimes I_p)\lambda$$

solve the *Output Agreement Problem*.

Proof - Sketch



- Systems on forward and feedback path are incrementally passive
- Coupling preserves incremental passivity
- Direct term ν_k in $\lambda_k = \psi_k(\eta_k) + \nu_k$ adds additional output damping

The overall storage function $U(x, x^w, \eta, \eta^w) = \sum_i V_i(x_i, x_i^w) + \sum_k W_k(\eta_k, \eta_k^w)$ satisfies

$$\begin{aligned} \dot{U} &\leq (y - y^w)^T(u - u^w) + (\lambda - \lambda^w)^T(\nu - \nu^w) - (\nu - \nu^w)^T(\nu - \nu^w) \\ &\leq -||(B^T \otimes I_p)y||^2 \end{aligned}$$

Static Disturbances

Static disturbances

Theorem

- $\dot{w}_i = 0$
- There exist (x^w, u^w) constant solutions to **(RE)**
- Let $\lambda^w = \psi(\eta^w)$ be a solution to $(B \otimes I_p)\lambda^w = u^w$ satisfying

$$(\psi_k(\eta) - \psi_k(\eta'))^T(\eta - \eta') \geq c\|\eta - \eta'\|^2, \quad \forall \eta, \eta' \quad \text{strong monotonicity}$$

Then the controllers

$$\Pi_k \ : \quad \begin{array}{rcl} \dot{\eta}_k & = & \nu_k \\ \lambda_k & = & \psi_k(\eta_k) + \nu_k, \quad k = 1, 2, \ldots, m \end{array}$$

with

$$\nu = -(B \otimes I_p)^T y, \quad u = (B \otimes I_p)\lambda$$

solve the *Output Agreement Problem*.

Static disturbances

- Static disturbances $\Rightarrow (\eta^w, \lambda^w)$ constant with

$$\begin{aligned} \dot{\eta}_k^w &= 0 \\ \lambda_k^w &= \psi_k(\eta_k^w), \quad k = 1, 2, \ldots, m \end{aligned}$$

- Incremental storage function

$$W_k(\eta_k, \eta_k^w) = \Psi_k(\eta_k) - \Psi_k(\eta_k^w) - \nabla\Psi_k^T(\eta_k^w)(\eta_k - \eta_k^w)$$

where $\nabla\Psi_k(\eta_k) = \psi_k(\eta_k)$.

Bregman *The relaxation method of finding the common points of convex sets and its application to the solution of problems in convex programming* USSR Comput. Math. Math. Phys. 1967
Jayawardhana *et al.* *Passivity of nonlinear incremental systems: Application to PI stabilization of nonlinear RLC circuits* SCL 2007

- Strong monotonicity of ψ_k implies Ψ_k strictly convex and therefore $W_k(\eta_k, \eta_k^w)$ has a global minimum and is regular.
- "Incremental passivity" with respect to constant (η^w, λ^w)

Economically efficient frequency regulation in power networks

Swing equations

Bus dynamics (swing equations)

$$\begin{aligned} \dot{\delta}_i &= \omega_i \\ M_i \dot{\omega}_i &= u_i - \textstyle\sum_{j \in \mathcal{N}_i} V_i V_j B_{ij} \sin\left(\delta_i - \delta_j\right) - A_i \omega_i - P_i^l \end{aligned}$$

where

δ_i	rotor angle wrt synchronously rotating reference frame at bus i,
ω_i	Frequency deviation at bus i,
V_i	Voltage at bus i,
M_i	Moment of inertia at bus i,
A_i	Damping coefficient at bus i,
$\mathcal{N}_i$	Set of buses connected to bus i,
B_{ij}	Susceptance of the line between buses i and j,
P_i^l	Power demand at bus i,
u_i	Controllable power generation at bus i.

Swing equations

Swing equations

$$\begin{aligned}\dot{\delta}_i &= \omega_i \\ M_i\dot{\omega}_i &= u_i - \textstyle\sum_{j\in\mathcal{N}_i} V_iV_jB_{ij}\sin\left(\delta_i - \delta_j\right) - A_i\omega_i - P_i^l\end{aligned}$$

Assumptions

- Lines are lossless, i.e. the conductance is zero.
- Nodal voltages V_i are constant.
- Phase angle of the voltage = rotor angle δ_i

Swing equations

Swing equations

$$\begin{aligned} \dot{\delta}_i &= \omega_i \\ M_i \dot{\omega}_i &= u_i - \textstyle\sum_{j \in \mathcal{N}_i} V_i V_j B_{ij} \sin\left(\delta_i - \delta_j\right) - A_i \omega_i - P_i^l \end{aligned}$$

can be written in compact form letting

$$\eta = B^T \delta$$

to obtain

$$\begin{aligned} \dot{\eta} &= B^T \omega \\ M \dot{\omega} &= u - B \Gamma \mathbf{sin}(\eta) - A\omega - P^l \end{aligned}$$

where

- $\Gamma = \mathrm{diag}\{\gamma_1, \ldots, \gamma_m\}$, $\gamma_k = V_i V_j B_{ij} = V_j V_i B_{ji}$, $k \sim \{i, j\}$
- $\mathbf{sin}(\eta) = (\sin \eta_1 \ldots \sin \eta_m)^T$

Economically efficient frequency regulation

Swing equations

$$\begin{aligned} \dot{\eta} &= B^T \omega \\ M\dot{\omega} &= u - B\Gamma \mathbf{sin}(\eta) - A\omega - P^l \end{aligned}$$

Economically efficient power generation Design control u, such that

- $\omega(t) \rightarrow \mathbf{0}$ (demand-supply balancing)
- $u(t) \rightarrow u^*$ (steady state generation)

where u^* is the economically efficient generation solution to

$$\min_{u,v} C(u) = \min_{u,v} \sum_{i \in \mathcal{V}} \frac{1}{2} q_i u_i^2$$
$$\text{s.t.} \quad 0 = u - D\Gamma v - P^l, \quad \mathbf{sin}(\eta) = v$$

Li *et al.* *Connecting automatic generation control and economic dispatch from an optimization view*. **ACC 2014.**
Simpson-Porco *et al. Synchronization and power sharing for droop-controlled inverters in islanded microgrids* Automatica 2013.
Andreasson *et al. Distributed vs. centralized power systems frequency control under unknown load changes* ECC 2013.

A solution to the regulator equation

Since $M\dot{\omega} = u - B\Gamma\mathbf{sin}(\eta) - A\omega - P^l$, **(RE)** write as

$$\begin{aligned} \dot{\eta}^w &= B^T\omega^w = \mathbf{0} \\ \mathbf{0} &= u^w - B\Gamma\mathbf{sin}(\eta^w) - P^l \end{aligned}$$

We are interested in u^w arising from the network optimization problem

$$\begin{aligned} &\min_{u,v} C(u) = \min_{u,v} \sum_{i\in\mathcal{V}} C_i(u_i) = \min_{u,v} \sum_{i\in\mathcal{V}} \frac{1}{2} q_i u_i^2 \\ \text{s.t.} \quad & 0 = u - B\Gamma v - P^l, \quad \mathbf{sin}(\eta) = v \end{aligned}$$

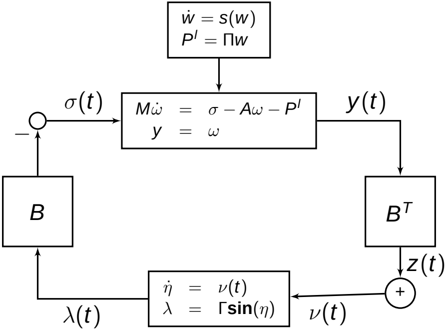
The optimal u^w is

$$u^w = Q^{-1}\frac{\mathbf{1}_n\mathbf{1}_n^T P^l}{\mathbf{1}_n^T Q^{-1}\mathbf{1}_n}$$

provided the **feasibility condition** holds

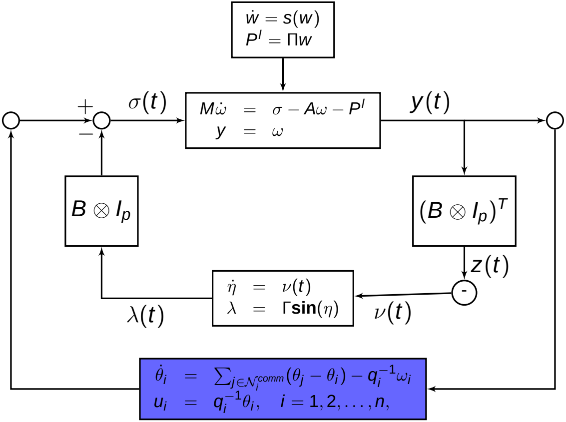
$$\exists \eta^w \in \mathcal{R}(B^T) \cap (-\pi/2, \pi/2)^m \text{ s.t. } B\Gamma\mathbf{sin}(\eta^w) = (Q^{-1}\frac{\mathbf{1}_n\mathbf{1}_n^T}{\mathbf{1}_n^T Q^{-1}\mathbf{1}_n} - I_n)P^l$$

Incremental passivity of the swing equations



- Forward path (strictly) incrementally passive with $V(\omega, \omega^w) = \|\omega - \omega^w\|_M^2/2$ ($\omega^w = \mathbf{0}$)
- Feedback path "incrementally passive" with $W(\eta, \eta^w) = \Psi(\eta) - \Psi(\eta^w) - \nabla\Psi^T(\eta^w)(\eta - \eta^w)$, $\nabla\Psi(\eta) = \Gamma\mathbf{sin}(\eta)$ and $\dot{\eta}^w = B^T\omega^w = \mathbf{0}$ (η in a neighborhood of η^w)
- If $\sigma = u - B\lambda$, closed-loop "incrementally passive" from input u, u^w to output ω, ω^w.

Main result



Controller

$$\begin{aligned} \dot{\theta} &= -L^{comm}\theta - Q^{-1}\omega \\ u &= Q^{-1}\theta \end{aligned}$$

guarantees $\omega \to \omega^w = \mathbf{0}$ and $u \to u^w$.

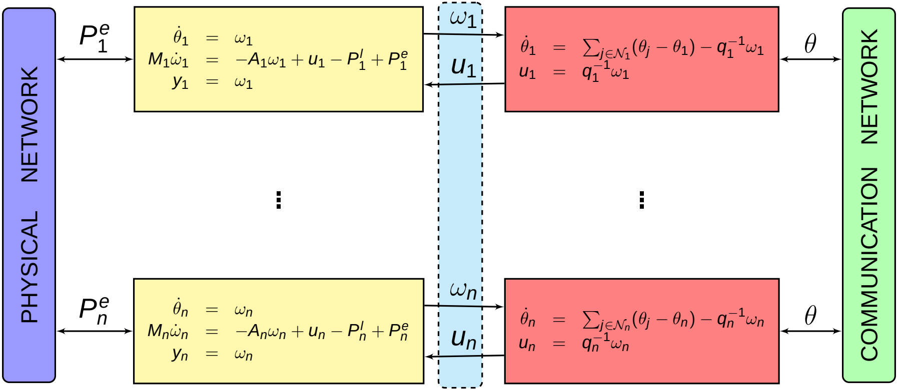
Proof – Sketch

Controller

$$\begin{aligned} \dot{\theta} &= -L^{comm}\theta + Q^{-1}\mu \\ u &= Q^{-1}\theta \end{aligned}$$

- satisfies **(IMP)** with $\theta^w(0) = \frac{\mathbf{1}_n \mathbf{1}_n^T P^l}{\mathbf{1}_n^T Q^{-1} \mathbf{1}_n}$ and $\nu = \mathbf{0}$
- is incrementally passive with $\Theta(\theta, \theta^w) = \|\theta - \theta^w\|^2/2$
- Coupling condition $\mu = -\omega$

Power networks as a Cyber-Physical System



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De Persis-Postoyan. A Lyapunov redesign of coordination algorithms for cyberphysical systems. arXiv:1404.0576.
De Persis-Tesi. Resilient Control under Denial-of-Service. arXiv:1311.5143.

Conclusion

Conclusion

- Framework for control of network systems under time-varying disturbances
- Incremental passivity and internal-model controllers
- Economic efficiency and power networks

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Bürger-De Persis-Trip. *An Internal Model Approach to (Optimal) Frequency Regulation in Power Grids*. `arxiv.org/abs/1403.7019` and in Proceedings MTNS 2014, Groningen.

Future reserch

Future reserch

Coordination in the presence of disturbances

- Contractive, differentially passive, incrementally stable systems
  - Bürger-De Persis. *Further result about dynamic coupling for nonlinear output agreement*. Submitted, 2014.
- Hybrid and uncertain exosystems
- Other applications (formation control, biochemical networks)
  - De Persis-Jayawardhana. *On the internal model principle in the coordination of nonlinear systems*. **IEEE Transactions on Control of Network Systems**, 2014.
- Cyber-physical systems