On-demand caches and content-oriented networks
Stochastic content-service systems

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Outline

1. Introduction
2. Single cache analysis
3. Cache network analysis
4. Conclusion
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Context: Data communication networks

Traditional Client—Server architectures

1 Host-to-Host communication model
2 Data flows
   • Kelly or Jackson’s Queueing models

(a) Client–Server model

- $\lambda$ Requests/s
- $\mu_0$
- Queue
- $\mu$
- $\lambda \theta$ bits/s
- $\lambda$ rqts/s
- Server
- Size $\theta$ bits
Context: Content-oriented networks

1. Host-to-Content communication model
2. Data acceleration, Server load reduction, Popular contents, Self-adaptation
   - Caching (Storage & Policy) + Queueing models
   - “On-demand” policies: Least Recently Used (LRU), FIFO, Random, Timeout

(b) Client–(Cache)–Server model, $\nu \leq \lambda$
Context: Model of Content-oriented networks

Client—Time-To-Live (TTL)-based Cache network—Server models

1. (TTL)-based Caching + Queueing models
   - Simple, Tractable, and Extensible

(c) Client—(TTL Cache)—Server model, \( \nu \leq \lambda \)
Link with existing cache models

“Space” representations of LRU, FIFO, Random policies
- Describe the position of an item in the cache
- Exact analysis via Markov chains (King 1972, Gelenbe 1973)

“Time” representations of cache replacement policies
- Describe the remaining time of an item in the cache
Link with existing cache models

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- Describe the remaining time of an item in the cache
TTL approach: Towards a unifying cache model?

Simple and accurate models for existing cache networks

- “Che approximation” of LRU caches → Deterministic (Det.) \( R \)-TTL model
- FIFO and Random caches → Det. and Exponential \( \Sigma \)-TTL models
- Amazon ElastiCache, Squid web caches → Det. \( \min (\Sigma, R) \)-TTL models

TTL-based caches within real systems

- (Modern) Domain Name System caches run Det. \( \Sigma \)-TTL policy.
- (OpenFlow) Software-defined switches run Det. \( \min (\Sigma, R) \)-TTL policy.
Why TTL models are further interesting?

New caching replacement policies: **TTL as control parameter** for

- User QoE metrics: Delay, Downloading time
- Server load, Network QoS, Power save, etc.
- Content-service differentiation (Premium offer, Real-Time Apps, etc.)
- Elastic storage provisioning and management (Cloud, Data center)
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Not covered here: Questions are welcome!

- Optimization and control
- Behaviour of TTL-based cache and $G/M/1$ queue in tandem
Challenges of the performance evaluation

In this talk (PhD thesis)

1. Workload model
   - Independent Reference Model (IRM) or Poisson assumption
   - (Markov-) Renewal, (non-) Stationary request models

2. Cache model and performance
   - per-content/demand metrics of interest (hit & occupancy probabilities)
   - global cache performance (hit ratio)
   - Filtering (cache misses)

3. Extension to cache networks
   - Filtering (cache misses)
   - Splitting and Aggregating (cache routing)

Assumptions (very easy to relax!)

- Downloading delays ≪ Request time scales (or Infinite bandwidth)
- Infinite cache capacity (only TTL causes a file eviction)
Outline

1 Introduction

2 Single cache analysis
   - TTL-based concept
   - Description of basic TTL-based policies
   - Single TTL-based cache and single file

3 Cache network analysis

4 Conclusion
**A nice property of TTL-based models**

- **Figure**: From capacity-driven to TTL-based caches

- Infinite capacity $\Rightarrow$ Decoupling $\Rightarrow$ Focus on SINGLE content item

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**Σ-TTL policy**

**Algorithm**

- On cache miss: add content, assign timer $T$ to it
- On cache hit: use remaining value of $T$
- **Only when** timer $T$ expires: remove content

![Diagram of Σ-TTL policy](image)

(a) Σ-TTL policy, $r = 0$
**R-TTL policy**

**Algorithm**
- On cache miss: add content, assign timer $T$ to it
- On cache hit: re-initialize timer $T$
- **Only when** timer $T$ expires: remove content

![Diagram](image)

(b) $R$-TTL policy, $r = 1$
**min \((\Sigma, \mathcal{R})\)-TTL policy**

**Algorithm: a straightforward generalization**

- On cache miss: apply \(\mathcal{R}\)-TTL policy with probability \(r\)
- Otherwise: apply \(\Sigma\)-TTL policy with probability \(1 - r\)

where \(r = \mathbb{P}_0 (Q^{(1)} < Q^{(0)})\) and
\(Q^{(1)}\) and \(Q^{(0)}\) are sojourn times in \(\mathcal{R}\)-TTL and \(\Sigma\)-TTL models respectively.

\[
\begin{align*}
\text{(c) } & \text{\(\Sigma\)-TTL with prob. } 1 - r \\
\text{(d) } & \text{\(\mathcal{R}\)-TTL with prob } r
\end{align*}
\]
Analysis of $G/G/\Sigma$ and $G/G/\mathcal{R}$-TTL cache models

- File requests form a **stationary point process**, $\{N(t), t \geq 0\}$

\[ Q = T_1 \]

\[ Q = T_2 \]

\[ \text{sojourn time } Q = T_1 \]

\[ \text{sojourn time } Q = T_2 \]

\[ \text{data in cache} \]

\[ \text{data in cache} \]

\[ \text{Z hits} \]

\[ \text{Z hits} \]

\[ \text{inter-miss time } Y \]

\[ \text{inter-miss time } Y \]

\[ m_0 \]

\[ m_1 \]

\[ t_0 \]

\[ t_1 \]

\[ \ldots \]

\[ t_{Z-1} \]

\[ t_Z \]

\[ t_{Z+1} \]

\[ m_2 \]

\[ (e) \text{ Case } r = 0, \ G/G/\Sigma\text{-TTL} \]

\[ (f) \text{ Case } r = 1, \ G/G/\mathcal{R}\text{-TTL} \]
TTL-based caches under Poisson requests

- $F(t) = \mathbb{P}(X < t) = 1 - e^{-\lambda t}$, $F^*(s) = \mathbb{E}[e^{-sX}] = \frac{\lambda}{\lambda + s}$
- $M(t) = \mathbb{E}[N(t)] = \lambda t$, $T(t) = \mathbb{P}(T < t)$, $T^*(s) = \mathbb{E}[e^{-sT}]$
- e.g.: Exp. $T^*(s) = \frac{\mu}{\mu + s}$, $\mu = 1/\mathbb{E}[T]$; Det. $T^*(s) = e^{-sD}$, $T = D$;
TTL-based caches under Poisson requests

- \( F(t) = \mathbb{P}(X < t) = 1 - e^{-\lambda t} \), \( F^*(s) = \mathbb{E}[e^{-sX}] = \frac{\lambda}{\lambda + s} \)
- \( M(t) = \mathbb{E}[N(t)] = \lambda t \), \( T(t) = \mathbb{P}(T < t) \), \( T^*(s) = \mathbb{E}[e^{-sT}] \)
- e.g.: Exp. \( T^*(s) = \frac{\mu}{\mu + s}, \mu = 1/\mathbb{E}[T] \); Det. \( T^*(s) = e^{-sD}, T = D \);

Proposition (Exact Performance and Miss stream, Fofack et al. 2012, 2013)

*Hit and Occupancy probability (P.A.S.T.A):* \( H_P = O_P \)

\[
H_P^{(0)} = 1 - \frac{1}{1 + \lambda \mathbb{E}[T]}; \quad H_P^{(1)} = 1 - T^*(\lambda)
\]

*Miss stream is a Renewal process, and* \( G^{(r)*}(s) = \mathbb{E}[e^{-sY}] \) *the LST of the CDF* \( G^{(r)}(t) = \mathbb{P}(Y < t) \) *is given by*

\[
\begin{align*}
G^{(0)*}(s) &= 1 - (1 - F^*(s)) \times \frac{\lambda}{s}(1 - T^*(s)), \quad \text{if } r = 0 \\
G^{(1)*}(s) &= F^*(s)(1 - T^*(s)), \quad \text{if } r = 1 \\
G^{(r)*}(s) &= (1 - r)G^{(0)*}(s) + rG^{(1)*}(s), \quad \text{if } r = \mathbb{P}^0 \left( Q^{(1)} < Q^{(0)} \right)
\end{align*}
\]
General performance metrics of basic TTL-based caches

Theorem (Fofack et al. 2014, PhD thesis)

Under general stationary assumption, $P^0$ the Palm probability, $E^0$ the expectation w.r.t. $P^0$,

$$H_P = r E^0 [F(T_1)] + (1 - r) \left(1 - (1 + E^0 [N(\bar{T}_1)])^{-1}\right)$$

$$O_P = (E^0[X_1]^{-1} \times (1 - H_P)) \times E^0[Q], \quad \text{“Little’s Law for caches”}$$

where $E^0 [F(T_1)] = P^0 (X_1 < T_1)$, and $Q$ is the sojourn time:

$$Q = \begin{cases} 
Q^{(0)} = T_1, & \text{if } r = 0 \\
Q^{(1)} = T_1 \times 1\{X_1 > T_1\} + (X_1 + \bar{Q}) \times 1\{X_1 < T_1\}, & \text{if } r = 1 \\
Q^{(r)} = \min \left( Q^{(0)}, Q^{(1)} \right), & \text{for min } (\Sigma, R) - TTL \text{ model } \text{if } r = P^0 \left( Q^{(1)} < Q^{(0)} \right).
\end{cases}$$
Miss process under general stationary request correlations

- Only an approximation of the CDF of the first inter-miss time.

\[ Y_1 = Q + \tilde{X} \approx \mathbb{E}^0[Q] + \tilde{X} \]
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Exact analysis of “Pure” TTL-based cache networks

Negligible content/chunk size w.r.t cache size (DNS cache hierarchy)

- Assumption: Markov-Renewal traffic model
- Filtering, Aggregating & Splitting produce Markov Renewal Processes

Network topology

- Blue “sees” network as Tree
- Green “sees” network as Polytree

Server, $S_1$

Cache, $C_1$

Cache, $C_4$

Cache, $C_5$

Users

Server, $S_2$

Server, $S_1$
Per-content TTL-based cache network analysis

(i) Linear

(j) Star

(k) Linear star

(l) Polytree

(m) Feed-forward

Figure: Per-content directed acyclic graph routing topologies

Sequential methodology

- Apply sequentially the single TTL-based cache analysis
“Constrained” TTL-based cache networks

Constraints on cache capacities (Video-on demand systems)

- **Issue:** Dependency among cache states and TTLs
- **Why?** Saturation of capacity constraints \( \sum_{i=1}^{K_n} O_{P,i,n} = C_n, \forall n \)
- **Example:** Heterogeneous networks of LRU, FIFO, RND caches

Recursive methodology, Fofack et al. 2014 (Valuetools)

- Accurate and Polynomial time algorithm (with quadratic speed of convergence)
(Possible) Applications of TTL-based models

On real caching systems

1. Recently...
   - Dynamic Page Caching mechanism (Akamaï)
   - WebRTC protocol (Google)

2. Past few years...
   - New concepts: Information-Centric Networking (CCN, NDN architectures)
   - (Mature) technos: push-based (CDNs, WWW) and pull-based (DNS, P2P)

In other systems

- Green networks and Smart grids (Idle mode, Power saving protocols)
- Economics (Product warranty), Physics (Geiger-Müller counters)
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Take home message

1. Cache networks
   - Think “TTL model” for cache analysis
   - Proved results on TTL caches are valid for most caches!
     e.g. If $F(t)$ is concave then Deterministic TTL performs the best.
   - Deploy your “TTL model” using LRU, RND, FIFO or Pra-TTL

2. New research opportunities
   - Cache network optimization and control (Current work)
   - Spatio-temporal diversity in content access on mobile networks?
Take home message

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Thank you!
Miss process under general independent requests

- Arrivals are i.i.d $\implies$ Renewal process
- $F(t) = \mathbb{P}(X < t), \ M(t) = \mathbb{E}[N(t)], \ T(t) = \mathbb{P}(T < t)$

Proposition (Exact CDF of the inter-miss times, Fofack et al. 2012, 2013)

**Hit and Occupancy probability:** $H_P \neq O_P = \lambda (1 - H_P) \mathbb{E}[Q]$

$$H_P^{(0)} = 1 - \frac{1}{1 + \mathbb{E}[M(T)]}; \quad H_P^{(1)} = \mathbb{E}[F(T)]$$

**Miss stream is a Renewal process, and the CDF $G^{(r)}(t) = \mathbb{P}(Y < t)$ is given by**

1. $G^{(0)}(t) = F(t) - \int_0^t (1 - T(x))dM(x)(1 - F(t - x))$ \quad (7)
2. $G^{(1)}(t) = F(t) - \int_0^t (1 - T(x))dF(x) + \int_0^t (1 - T(x))dF(x)G^{(1)}(t - x)$ \quad (8)
3. $G^{(r)}(t) = (1 - r)G^{(0)}(t) + rG^{(1)}(t), \quad \text{for min}(\Sigma, R) - TTL \text{ model}$ \quad (9)
Miss process under Markov-correlated requests

- Arrivals occur at jumps of a DTMC \(((t_k, \xi_k)_{k \geq 0})\) on \(S = \{1, 2, \ldots, J\}\).
- Requests form a stationary Markov Renewal Process (MRP), \( [F(t)]_{i,j} := \mathbb{P}(X_k < t, \xi_{k+1} = j \mid \xi_k = i), \ T_i(t) = \mathbb{P}(T < t \mid \xi_k = i) \)

**Proposition (Exact Kernel of the miss process, PhD thesis)**

**Miss stream is a Markov Renewal Process, with the kernel \(G^{(r)}(t)\) given by**

\[
G^{(0)}(t) &= F(t) - \int_0^t dR(x)(1 - F(t - x)) \tag{10} \\
G^{(1)}(t) &= F(t) - L(t) + \int_0^t dL(x)G^{(1)}(t - x) \tag{11} \\
G^{(r)}(t) &= (1 - r)G^{(0)}(t) + rG^{(1)}(t), \text{ for } \min(\Sigma, \mathcal{R})-TTL \text{ model} \tag{12}
\]

\[
[L(t)]_{i,j} := \int_0^t (1 - T_i(x))dF_{i,j}(x), \ [R(t)]_{i,j} := \int_0^t (1 - T_i(x))dM_{i,j}(x)
\]
Hint: under (Markov) Renewal Assumption.

\[ Y = \begin{cases} 
X_1 + \cdots + X_{N(T_1)+1} & \text{if } r = 0 \\
X_1 \mathbf{1}(X_1 > T_1) + (X_1 + \tilde{Y}) \mathbf{1}(X_1 < T_1) & \text{if } r = 1.
\end{cases} \]