Asymptotic properties of object-sharing systems

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YEQT VIII, November 2014, Eindhoven

1. joint work with Christine Fricker (Inria) and Vincent Jost (CNRS)

Question : Who has already used a bike-sharing system ? What was your experience ?

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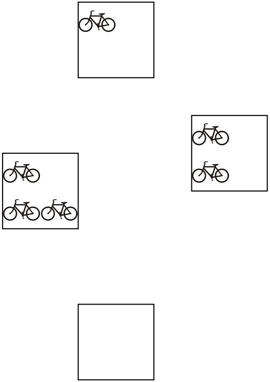
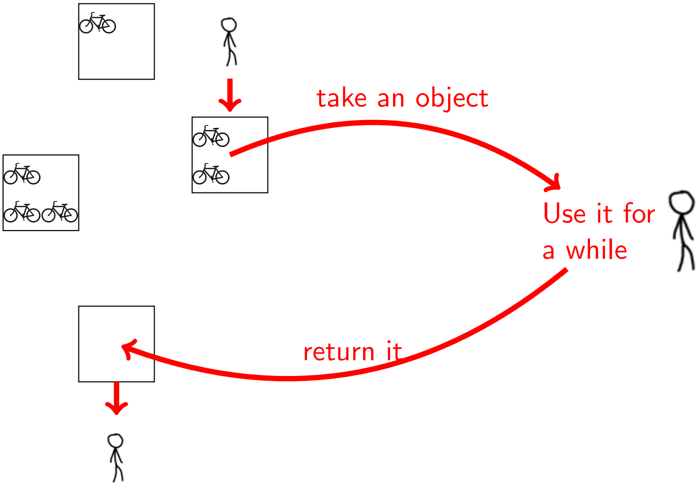
- Problems : lack of resources.

Object-sharing systems



Object-sharing systems



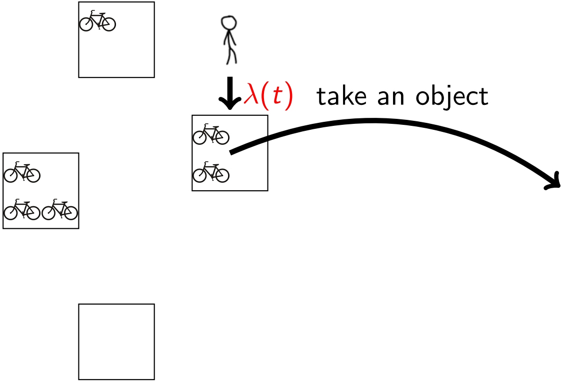
I will focus on large object-sharing systems

Map of Velib' stations in Paris (France).

Example of Velib' :

- 20 000 bikes
- 1 200 stations.

Object-sharing system as closed-queuing networks



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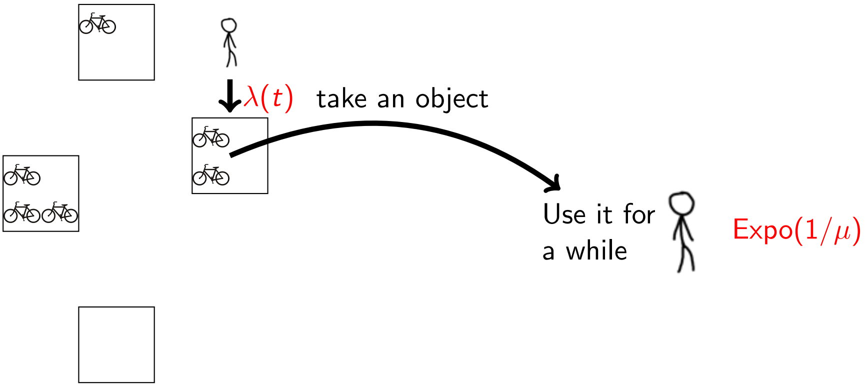

Scaling : $N \to \infty$ stations, s objects per station.

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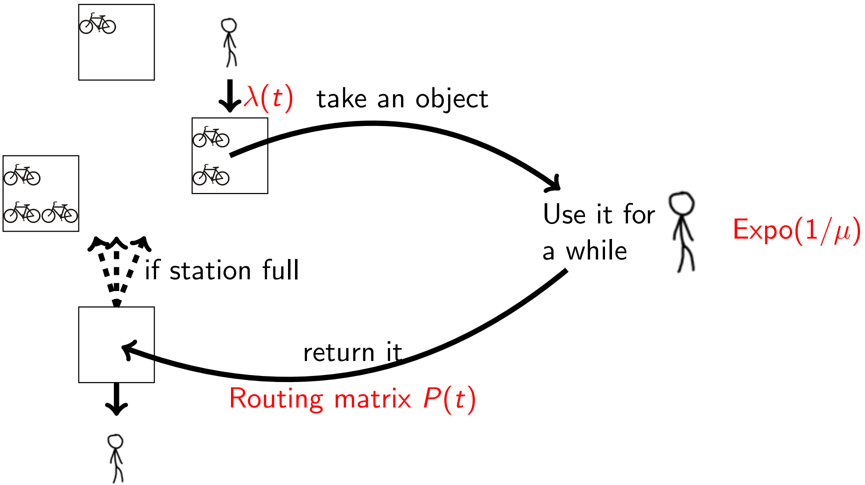


Scaling : $N \to \infty$ stations, s objects per station.

Main message

Theoretical results : When the system is large :

- if the stations have finite capacities, stations behave as independent M/M/1/K queues.
- if the stations have infinite capacities, there are problems of concentration.

Practical considerations :

- Performance is poor, even for a symmetric city (but simple incentives like a two-choice rule can help a lot).
- Frustrating users can help :
  - It is better to have stations of finite capacities.
  - Frustrating some users can improve the overall usage.
  - We show that the optimal fleet size is not

Outline

The homogeneous model

- All stations are identical.

Motivation :

- Impact of random choices
- Close-form results
- "Best-case analysis"

"Theorem"

Asymptotically, stations are independent and behaves as a $M/M/1/K$.

Distribution of x_i, the fraction of station with i bikes

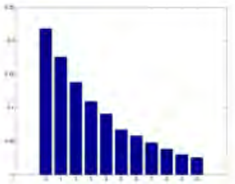
Theorem

There exists ρ, such that in steady state*, as N goes to infinity :*

$$x_i \propto \rho^i.$$

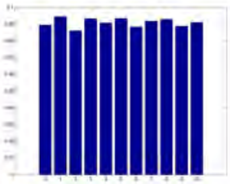
$\rho \leq 1$ *iff* $s \leq \frac{C}{2} + \frac{\lambda}{\mu}$ *where s be the average number of bikes per stations.*

$s < \frac{C}{2} + \frac{\lambda}{\mu}$



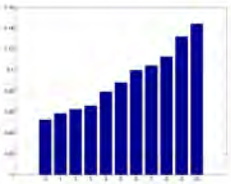
$\rho < 1$

$s = \frac{C}{2} + \frac{\lambda}{\mu}$



$\rho = 1$

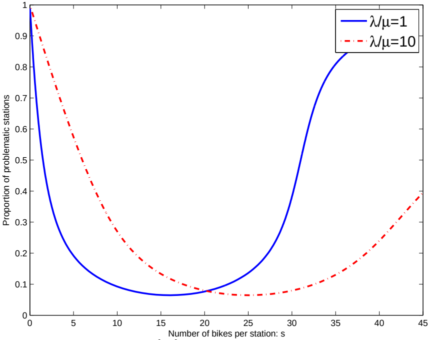
$s > \frac{C}{2} + \frac{\lambda}{\mu}$



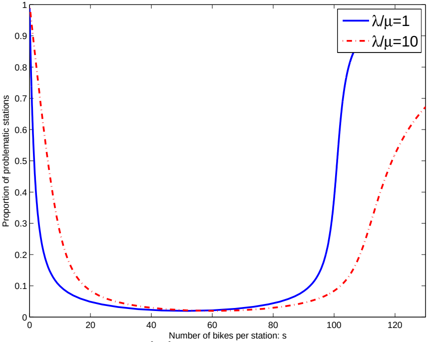
$\rho < 1$

Consequences : optimal performance for $s \approx C/2$

y-axis : Prop. of problematic stations. x-axis : number of bikes/station s.



(a) $C = 30$.



(b) $C = 100$.

Fraction of problematic stations (=empty+full) minimal for $s = \lambda/\mu + C/2$

- Prop. of problematic stations is at least $2/(C+1)$ (6.5% for $C = 30$)

Improvement by dynamic pricing : "two choices" rule

- Users can observe the occupation of stations.
- Users choose the least loaded among 2 stations close to destination to return the bike (ex : force by pricing)

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Paradigm known as "*the power of two choices*" :

- Comes from balls and bills [Azar et al. 94]
- Drastic improvement of service time in server farm [Vvedenskaya 96, Mitzenmacher 96]

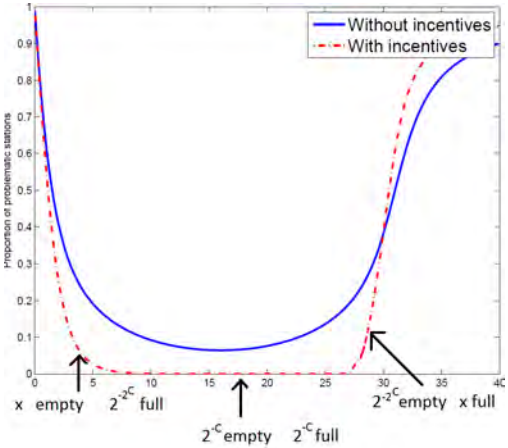
Question : what is the effect on bike-sharing systems ?
Characteristics :

1. Finite capacity of stations.
2. Strong geometry : choice among neighbors.

Two choices – finite capacity but no geometry

With no geometry, we can solve in close-form.

- Proof uses mean field argument.

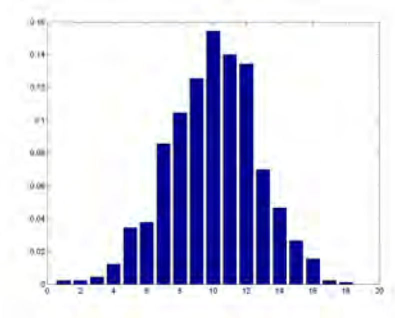


Choosing two stations at random, improves perf. from $1/C$ to $\sqrt{C}2^{-C/2}$

Two choices – taking geometry into account

Mean field do not apply (geometry) :(.

- Existing results for balls and bins (see [Kenthapadi et al. 06])
- Only numerical results exists for server farms (ex : [Mitzenmacher 96])



We rely on simulation

Occupancy of stations
x-axis = occupation of station.
y-axis : proportion of stations.

Recall : with no incentives, the distribution would be uniform.

Empirically :

- with geometry 2D : proportion of problematic stations is $\approx \sqrt{C}2^{-C/2}$. (recall : with no-geometry : 2^{-C}, with no incentive : $1/C$).

Outline

We assume that as N goes to infinity, the parameters (λ_i, p_i) of the station have a limiting distribution.

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"Theorem"

When the stations have finite capacities, a station behaves as a M/M/1/K.

Finite capacities regime

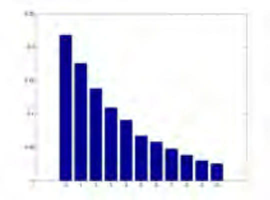
Theorem (Propagation of chaos-like result)

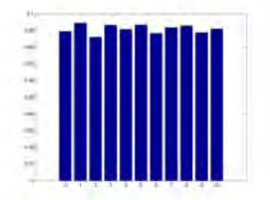
There exists a function $\rho(p)$ such that for all k, if stations $1, \dots k$ have parameter $p_1, \dots p_k$, then, as N goes to infinity :

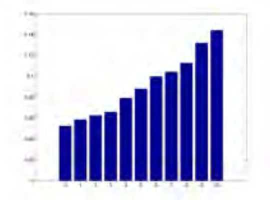
$$P(\#\{\text{bikes in stations j}\} = i_j \text{ for } j = 1..k) \propto \prod_{j=1}^{k} \rho(p_j)^{i_j}$$

Depending on popularity, stations have different behaviors :

Popular start $\rightarrow$ Popular destination

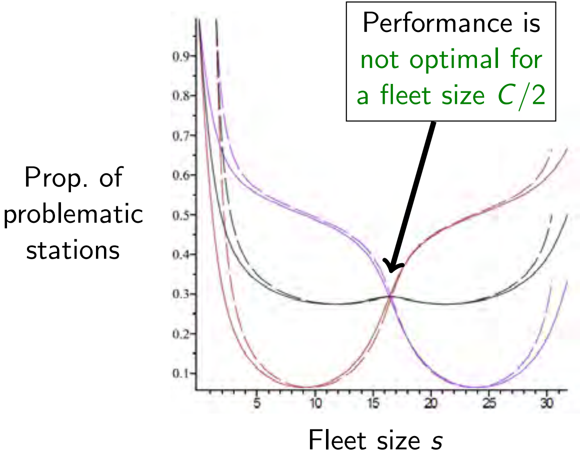






Finite-capacity : numerical example

Two types of stations : popular and non-popular for arrivals : $\lambda_1/\lambda_2 = 2$.



Infinite capacities can worsen the situation

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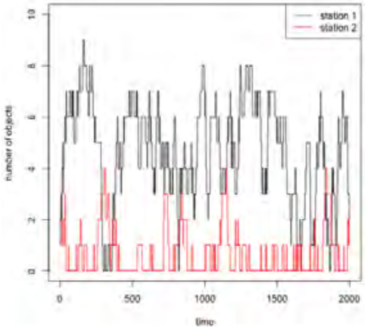
Theorem (Malyshev-Yakovlev 96)

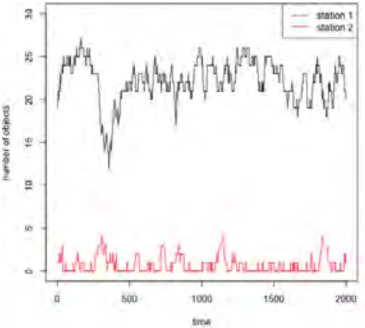
When the stations have infinite capacity, then there exists s_c :

- *if $s < s_c$, a station behaves as a $M/M/1/K$.*
- *if $s > s_c$, bikes will accumulate in a few stations.*

Example with $\mu = 1$, $p = (2, 1, 1, 1, 1, 1, 1, 1, 1)/10$:



$s = 1 < s_c$



$s = 3 > s_c$

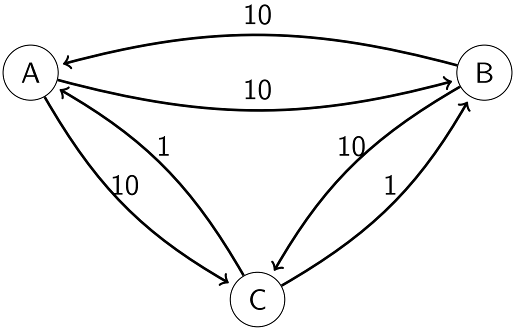
Outline

Having finite capacities prevent saturation of the demand. What if we could frustrate some demand ?

Model : we have a trip demand $\Lambda_{ij}(t)$ and an accepted demand $\lambda_{ij}(t)$.

- Generous policy : $\lambda_{ij}(t) := \Lambda_{ij(t)}$
- Possible control $\lambda_{ij}(t) \leq \Lambda_{ij}(t)$

Frustrating demand can improve the balance of objects



Users want to go to C. Almost nobody wants to go to A or B.

	Rate of trips (infinite capacities, infinite vehicles)
Generous policy	≈ 6 trips / time unit

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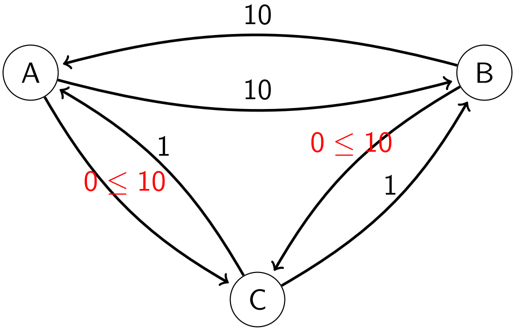

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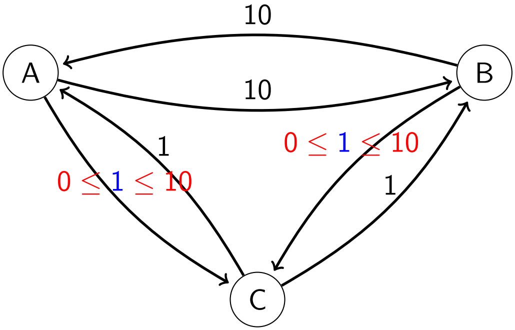

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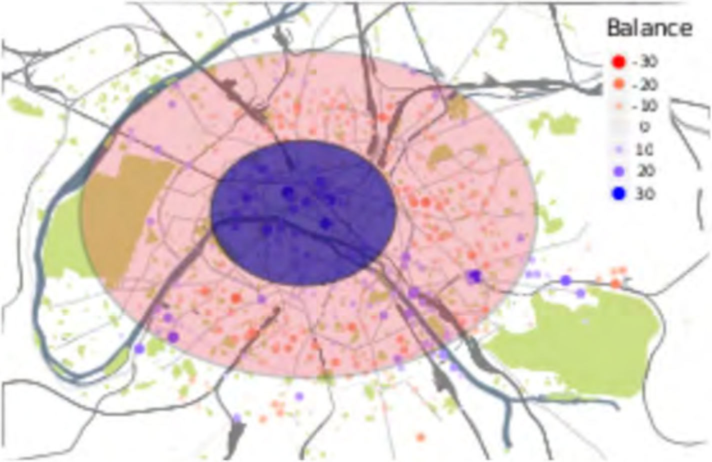
	Rate of trips (infinite capacities, infinite vehicles)
Generous policy	≈ 6 trips / time unit
Frustrating policy	20 trips / time unit
Optimal circulation	24 trips / time unit

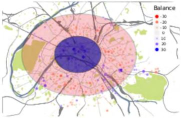
We can explore dynamic scenarios [Waserhole/Jost 2012]

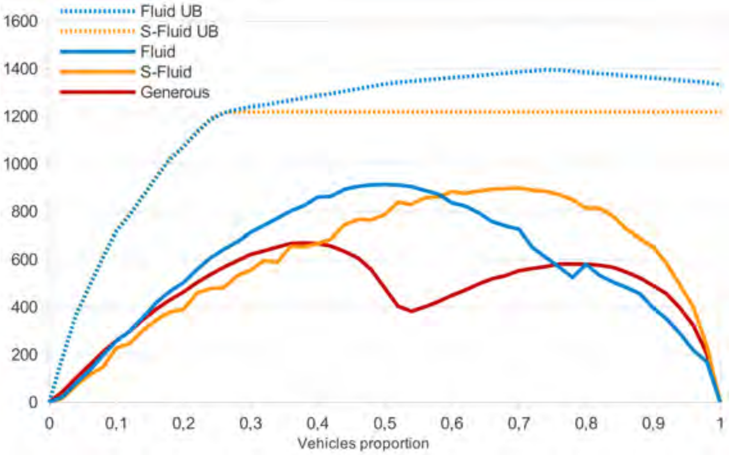
Tides in Paris



Static time-varying frustration of user can improve the situation

Trips per Second





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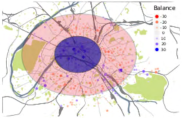
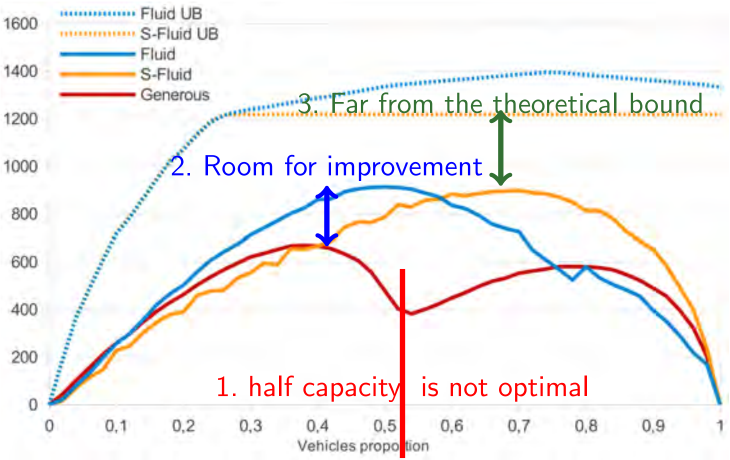
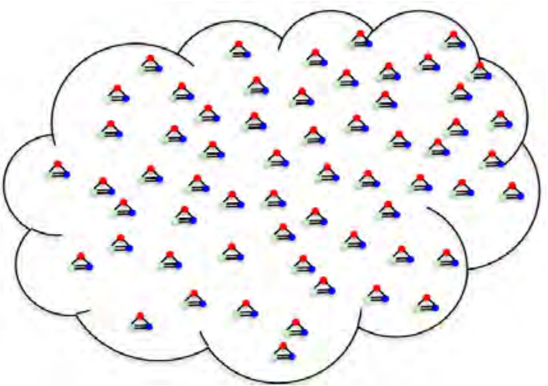




Outline

Methodological comments : Asymptotic based on mean-field approximation



- Basic models are reversible.
  - Saddle-points methods can also be used.

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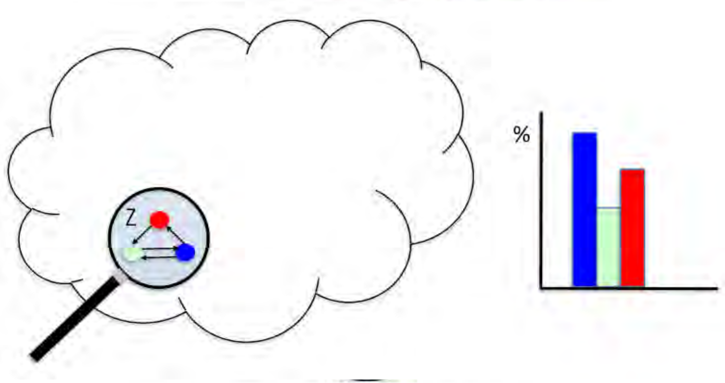

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Take-away message

Asymptotic results for a large class of object-sharing network.

- Performance poor, even for symmetric : $1/C$ problematic stations.
- Simple incentives can help a lot : 2^{-C}.
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Remarks and future work :

- Metrics are not easy to define.
- Visualization of traces and Influence of geometry ?
- Analyze transient and steady-state behavior.

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> If an ideal symmetric system works poorly, do not expect perfect service in a real system ;)

References

- C Fricker, N Gast. *Incentives and redistribution in homogeneous bike-sharing systems with stations of finite capacity*. EURO Journal on Transportation and Logistics. 2014.
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