Managing Call Centers with Many Strategic Agents

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Work Schedule Flexibility

86% of the "Best Companies to Work For" offer employees some type of flexible schedule.
- Fortune, 2014.

81% of employers allowed some workers to periodically change their starting and quitting times.
- National Study of Employers (USA), 2013.

In the US, all federal employees now have the right to request more flexible work options.
- White House memo issued on June 23, 2014.

In the UK, the right to request flexible working hours is now extended to all workers.
- Flexible Working law issued on July 1, 2014.

“Flexitime” at Hydro-Québec

“Flexitime” schedule:

- 30% of call center agents have flexible work hours
- Must be there during core periods (2 periods of 2 hours each)
- Are otherwise free to choose their time schedules
- Don’t need to inform their managers ahead of time
- Must be there for a total of 70 hours per 2 weeks

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Major headaches for Hydro-Québec's call center managers!

Research Questions

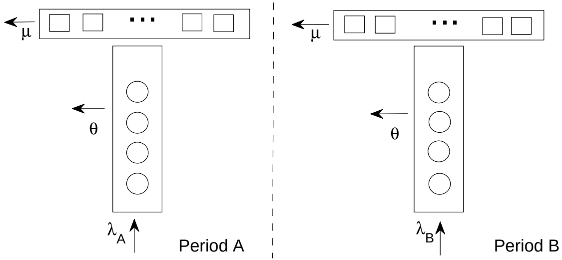
- What is the impact of strategic agent behavior on the system?
- What are optimal operational decisions with strategic agents?
- Can we align the objectives of the system manager and the agents?

Related Literature

- Staffing and routing in many-server systems
- Queueing games
- Strategic servers in queues
  - Gopalakrishnan, R., Dorouli, S., Ward, A. and A. Weirman. 2014. Routing and staffing when servers are strategic.
  - Gurvich, I., Lariviere, M. and A. Moreno-Garcia. 2014. Staffing service systems when capacity has a mind of its own.

Staffing Decisions with Non-Strategic Agents

Queueing Framework



- Poisson arrival processes with rates λ_A, λ_B
- No overlap between periods A and B
- I.I.D. exponential service times with rate $\mu = 1$
- I.I.D. exponential abandonment times with rate θ

System Manager's Problem with Non-Strategic Agents

Staffing Costs

- c: Fixed compensation
- r: Variable compensation rate
- t_A, t_B: Total compensations

$$t_A = c + r \cdot \frac{\lambda_A}{n_A} \quad \text{and} \quad t_B = c + r \cdot \frac{\lambda_B}{n_B}.$$

Customer-Related Costs

- p: Abandonment penalty cost ($ per customer who abandons)
- h: Delay cost ($ per customer per minute wait)

Fluid Approximation to the System Manager's Problem

For $t = A, B$:

- n_t: Number of servers in period t
- q_t: Queue length in period t
- η_t: Total abandonment rate in period t

$$\eta_t = (\lambda_t - n_t)^+ \text{ and } q_t = \frac{\eta_t}{\theta}.$$

The system manager determines optimal staffing levels n_A^* and n_B^*:

$$\min_{n_A \geq 0, n_B \geq 0} \Bigg\{ n_A \left(c + r\frac{\lambda_A}{n_A} \right) + n_B \left(c + r\frac{\lambda_B}{n_B} \right) + \left(p + \frac{h}{\theta} \right) (\lambda_A - n_A)^+ + \left(p + \frac{h}{\theta} \right) (\lambda_B - n_B)^+ \Bigg\}$$

Bassamboo and Randhawa (2010).

Fluid-Based Prescriptions with Non-Strategic Agents

Recall: c = staffing cost; p = abandonment cost; h = delay cost.

Optimal Staffing Levels

(i) if $c \leq p + h/\theta$, then $n_A^* = \lambda_A$ and $n_B^* = \lambda_B$,

(ii) if $c > p + h/\theta$, then $n_A^* = n_B^* = 0$.

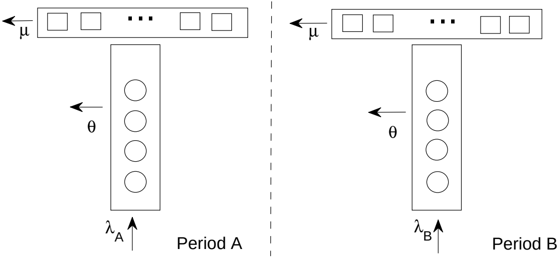
In (i), both A and B are critically loaded. In this case,

$$\alpha^* = \frac{n_A^*}{n_A^* + n_B^*} = \frac{\lambda_A}{\lambda_A + \lambda_B} \text{ and } \rho_A^* = \rho_B^* = 1,$$

where $\rho_t = \lambda_t / n_t$.

Staffing Decisions with Strategic Agents

Strategic Agent Behavior



- Agents have individual valuations for working in either period
- Strategic agents select working period of their choice

A Two-Stage Sequential Game

Sequence of events

- Stage 1: System manager selects total staffing level n
- Stage 2: Agents in pool of size n select working period

An equilibrium arises in the system.

We determine this equilibrium by using backward induction.

Second Stage: Agent's Problem

Individual Agent Preferences

- n: Total number of agents in the system (given)
- n_A, n_B: To be determined by the agents
- Independent and heterogeneous agents
- v_A, v_B: agent valuations

$$v_A(n_A) = c + r \cdot \frac{\lambda_A}{n_A} \quad \text{and} \quad v_B(n_B) = c + (r + X) \cdot \frac{\lambda_B}{n_B},$$

where X is a random variable (cdf F, ccdf $\bar{F}$).

Agent selects period A if, and only if, $v_A(n_A) \geq v_B(n_B)$.

Subgame Perfect Nash Equilibrium

Let α^e be the equilibrium proportion of agents who select A.

Existence and Uniqueness of Equilibrium

We show that there exists a unique equilibrium threshold, t^e, such that

$$\alpha^e = P(X \leq t^e) = F(t^e).$$

We also show that t^e is the unique solution of:

$$r\frac{\lambda_A \bar{F}(t^e)}{\lambda_B F(t^e)} - r - t^e = 0.$$

Impact of Strategic Agent Behavior

Recall that we have at optimum in the non-strategic case:

- $\alpha^* = \lambda_A/(\lambda_A + \lambda_B)$ is the optimal proportion of agents who are in A
- $\rho_A^* = \rho_B^* = 1$

Comparison with the Non-Strategic Case

We show that, for any value of n, the following hold:

(i) $F(0) = \alpha^* \Leftrightarrow \alpha^e = \alpha^*$

(ii) $F(0) < \alpha^* \Leftrightarrow \alpha^e < \alpha^*$

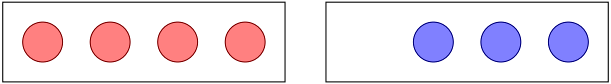
(iii) $F(0) > \alpha^* \Leftrightarrow \alpha^e > \alpha^*$

where $F(0) = P(X \leq 0)$.

Explanation: Case (i)

Suppose that the staffing levels in periods A and B are as follows:

Period A　　　　Period B



$n_A = \lambda_A$　　　　$n_B = \lambda_B$

Then, let agents select their periods.

The proportion of agents who choose period A is:

$$P(v_B \leq v_A) = P\left(c + (r + X)\frac{\lambda_B}{n_B} \leq c + r\frac{\lambda_A}{n_A}\right) = P(X \leq 0) = F(0).$$

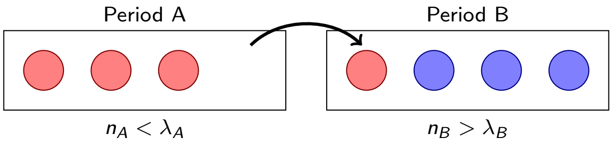
So, for the above to be an equilibrium state, we need to have:

$$F(0) = \frac{\lambda_A}{\lambda_A + \lambda_B} = \alpha^*.$$

In this case, $\alpha^e = F(0) = \alpha^*$.

Case (ii): System Imbalance

Now, what happens if $F(0) < \alpha^*$?



The proportion of agents who choose period A is:

$$P(v_B \leq v_A) = P\left(c + (r + X)\frac{\lambda_B}{n_B} \leq c + r\frac{\lambda_A}{n_A}\right) = P(X \leq 0) = F(0) < \alpha^*.$$

In this case, some agents in A will now select B instead.

So, our initial state cannot be an equilibrium, and we have:

$$F(0) < \alpha^e < \alpha^*.$$

In this case, period A becomes overloaded and period B underloaded.

First Stage:
System Manager's Problem

Fluid Approximation with Strategic Agents

In this stage, assume that α^e is given.

The system manager determines the optimal total staffing level n:

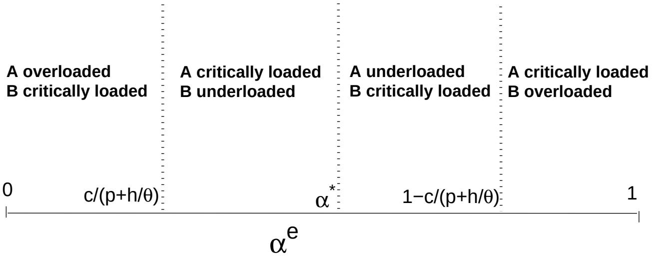
$$\min_{n \geq 0} \left\{ n\alpha^e \left(c + r\frac{\lambda_A}{n\alpha^e} \right) + n(1-\alpha^e) \left(c + r\frac{\lambda_B}{n(1-\alpha^e)} \right) + \left(p + \frac{h}{\theta} \right) (\lambda_A - n\alpha^e)^+ + \left(p + \frac{h}{\theta} \right) (\lambda_B - n(1-\alpha^e))^+ \right\}$$

Fluid-Based Prescriptions: Asymptotically Optimal Regimes

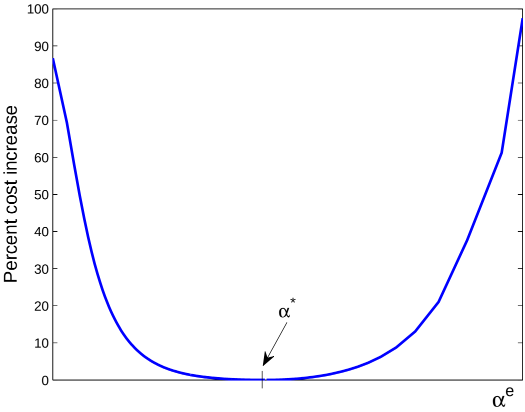
It is never optimal to have:

- Both *A* and *B* underloaded
- Both *A* and *B* overloaded

So, have at least one period critically loaded.



Cost Impact of Strategic Agents



- $c = 0.5, p = 0.7, h = 1$, and $\mu = \theta = 1$
- $\lambda_A = 150$ and $\lambda_B = 250$
- X uniform on (x_l, x_u)
- Vary r, x_l, and x_u to vary α^e

System Controls

A Coordinating Compensation Scheme

Let

$$r_A = r + \beta_A \text{ and } r_B = r + \beta_B$$

be the variable compensation rates for periods A and B.

Unequal Compensation Rates

We show that the compensation scheme with

$$\beta_A = (1 - \alpha^*)F^{-1}(\alpha^*) \text{ and } \beta_B = -\alpha^* F^{-1}(\alpha^*),$$

is a budget-neutral compensation scheme for which $\alpha^e(r_A, r_B) = \alpha^*$.

The system manager can control the system by increasing compensation in the period which is busier at equilibrium.

Conclusions

- We proposed a model for strategic agents
- We quantified the impact of strategic agent behavior
- We derived optimal staffing levels with strategic agents
- We developed a compensation scheme that makes agents behave in line with the system manager's objective

Extensions

We also considered the following extensions:

- Social welfare
- Different agent valuation models
- Routing Decisions
- What if the system manager doesn't honor or only partially honors agents' choices?
- General abandonment
- Heterogeneous service rates

Thank You!