Report 99-024

EURANDOM Workshop

Ergodic Theory

November, 12-14, 1998

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Foreword

This booklet contains the summaries of the lectures delivered during the first Eurandom workshop on ergodic theory, from November 12 to November 14, 1998. It is our intention to hold this workshop on a yearly basis in November, on a small scale with around thirty participants. The electronic addresses of the participants can be found at the beginning of their respective abstracts. It is fortunate that Eurandom provides us with a unique opportunity for ergodic theorists to meet once a year in pleasant and stimulating surroundings to discuss the most recent problems and results in this area; we hope to encourage more participation by younger persons in the coming meetings. The meeting for 1999 is scheduled for November 25 to November 27. Please send any suggestions or requests for participation to one of the five organizers.

Karma Dajani
Michel Dekking
Mike Keane
Cor Kraaikamp
Ronald Meester

Program

Thursday November 12

12.00-13.15 lunch

13.30-14.30 *Charles Radin* (Austin)
'Aperiodic tilings'

14.30-15.30 *Karoly Simon* (CWI, Amsterdam)
'Hausdorff measure of fractals with overlapping cylinders'

15.30-15.45 tea

15.45-16.45 *Mike Keane* (CWI, Amsterdam)
'Homomorphisms of noncommutative Bernoulli schemes'

16.45-17.45 *Ronald Meester* (Utrecht)
'Convergence of continued fraction type algorithms and generators'

Friday, November 13

9.00- 10.00 *Karma Dajani* (Utrecht)
'Entropy and relative growth of sequences'

10.00-10.30 coffee

10.30-11.30 *Bernard Host* (Paris)
'Some results of uniform distribution on the torus T^d'

11.30-12.30 *Meir Smorodinsky* (Tel Aviv)
'Processes that can or cannot be generated 'on line' by an indepedent process'

12.30-14.15 lunch

14.15-15.15 *Jeff Steif* (Gothenburg)
'Finitary codings and phase transitions for Markov random fields'

15.15-15.30 tea

15.30-16.30 *Wolfgang Krieger* (Heidelberg)
'Presentations of subshifts and semigroup-invariants'

16.30-17.30 *Klaus Schmidt* (Wien)
'Recurrence of stationary random walks'

Saturday, November 14

9.00- 10.00 *Cor Kraaikamp* (Delft)
'A Gauss-Kusmin theorem for optimal S-expansions'

10.00-10.30 coffee

10.30-11.30 *Bob Burton* (Corvallis)
'An ergodic theory approach to quasi-Monte Carlo problems in financial mathematics'

11.30-12.30 *Bryna Kra* (IHES)
'Commutative groups of circle diffeomorphisms; local vs. global'

12.30-14.15 lunch

14.15-15.15 *Mark Pollicott* (Manchester)
'Computing Hausdorff dimension for non-linear iterated function schemes'

15.15-15.30 tea

15.30-16.30 *Jean-Paul Thouvenot* (Paris)
'An application of ergodic theory to probability theory'

16.30-17.30 *Francois Ledrappier* (Parijs)
'Ergodic properties of some linear actions'

On saturday we will have a dinner in 'El Tejatro', which is located in the 'cellar' of the 'Stadsschouwburg' (= City theater).

List of participants Bob Burton, Michel Dekking, Karma Dajani, Bernard Host, Mike Keane, Bryna Kra, Cor Kraaikamp, Wolfgang Krieger, Francois Ledrappier, Ronald Meester, Mark Pollicott, Karoly Simon, Meir Smorodinsky, Jeff Steif, Jean-Paul Thouvenot

Abstracts

Bob Burton (Corvallis, burton@MATH.ORST.EDU)
'An ergodic theory approach to quasi-Monte Carlo problems in financial mathematics'

Abstract
A common problem arising in financial mathematics and other places is to find a sequence of points in $[0,1]^d$, $\{x\} = (x_0, x_1, x_2, E)$ that are good sample points for approximating integrals. In Monte Carlo Integration the points are chosen in an iid manner. In quasi-Monte Carlo the points are chosen to be more dispersed and thus give smaller error. The approach taken over the last years has been to find good sequences in $[0,1]$ and then to try to find ways to join them into ordered d-tuples, possibly using randomization. In work in progress with Aimee Johnson, we apply the techniques of ergodic theory to this problem, especially using finite rank transformations to generate good sequences.

Karma Dajani (Utrecht, dajani@math.uu.nl)
'Entropy and relative growth of sequences'
(Joint work with Wieb Bosma and Cor Kraaikamp).

Abstract
Expansions that furnish increasingly good approximations to real numbers are usually related to dynamical systems. Although comparing dynamical systems seems difficult in general, Lochs was able in 1964 to relate the relative speed of approximation of decimal and regular continued fractions (almost everywhere) to the quotient of the entropies of their dynamical systems. He used detailed knowledge of the continued fraction operator. In this talk we show that his result follows from certain general properties that the continued fraction map satisfies. Essential in this new proof is the Theorem of Shannon-McMillan-Breiman. Other expansions, such as the alternating Lüroth series, have similar properties, and this allows us to generalize Lochs' result. Even in cases where the expansion operator satisfies weaker conditions, the same correlation between relative speed of approximation and entropy quotient seems to exist. We were able to prove this in the case of g-adic expansions (by elementary but technical means), and furnish numerical evidence in other cases. If this hypothesis is correct, it provides a means to approximate hitherto unknown values of entropy. We apply this to an example of Bolyai expansions.

Bernard Host (Paris, host@univ-mlv.fr)
'Some results of uniform distribution on the torus T^d'

Abstract
This paper consists in a generalization to the d-torus T^d of previous results of the author: Given a probability measure μ on and an endomorphism T of T^d, we explore the relations between three properties: The uniform distribution of the sequence $(T^n t)$ for μ-almost all t; the behaviour of μ relatively to the translations by some rational subgroups of T^d; and the entropy of μ for another endomorphism S of T^d.

Mike Keane (CWI, Amsterdam, M.S.Keane@cwi.nl)
'Homomorphisms of noncommutative Bernoulli schemes'
(Joint work with T. Hamachi).

Abstract
In this lecture we discuss ongoing work with T. Hamachi of Kyushu University. In classical ergodic theory the classification of dynamical systems has played a central role over the past forty years. Key results in this area include the classification of (commutative) Bernoulli Schemes both in the measure-theoretic category and in the finitary category. Recently we have discovered a method enabling us to extend part of this classification to the noncommutative setting. In this lecture we schetch the method which is based on the idea of a fiber cocycle over a commutative homomorphism, and leads after much technical work to the homomorphism theorem for noncommutative Bernoulli Schemes of different entropies; this work is yet incomplete and in progress. We also give some ideas and questions concerning the central isomorphism problem, which remains open.

Bryna Kra (IHES, kra@math.univ-mlv.fr)
'Commutative groups of circle diffeomorphisms: local vs. global'
(Joint work with Y. Katznelson and D. Ornstein).

Abstract
We obtain a version of Herman's theorem on the differentiability of the conjugation for commuting groups of circle diffeomorphisms. We show that the loss of differentiability from that of a group of commuting, orientation preserving C^κ ($\kappa > 2$) diffeomorphisms of the circle to the differentiability of the conjugating map is limited by the joint Diophantine properties of the rotation numbers of a finite subset. We start with the local case, meaning when the diffeomorphisms are perturbations of rotations. For the general case, we first show that the conjugation is C^1 and then use the local result to prove the general statement.

Cor Kraaikamp (Delft, c.kraaikamp@its.tudelft.nl)
'A Gauss-Kusmin theorem for optimal continued fractions'
(Joint work with Karma Dajani).

Abstract
One of the first – and still one of the most important – results in the metrical theory of continued fractions is the so-called Gauss-Kusmin theorem. Let $\xi \in [0,1)$, and let $[0; d_1, d_2, \cdots, d_n, \cdots]$ be the *regular continued fraction* (RCF) expansion of ξ, then it was observed by Gauss in 1800 that for $z \in [0,1]$

$$\lim_{n\to\infty} \lambda(\{\xi \in [0,1);\, T^n\xi \leq z\}) = \frac{\log 2}{\log(1+z)}.$$

Here λ is the Lebesgue measure and the RCF-operator $T : [0,1) \to [0,1)$ is defined by

$$T\xi := \frac{1}{\xi} - \lfloor \frac{1}{\xi} \rfloor,\ \xi \neq 0;\ T0 := 0,$$

where $\lfloor . \rfloor$ denotes the *floor* - or *entier* function. It is not known how Gauss found this result, but later, in a letter dated January 30, 1812, Gauss asked Laplace to give an estimate of the error term $r_n(z)$, defined by

$$r_n(z) := \lambda(T^{-n}[0,z]) - \frac{\log(1+z)}{\log 2},\ n \geq 1.$$

It was Kusmin in 1928 who was the first to prove Gauss' result and at the same time to answer his question. Kusmin showed that $r_n(z) = \mathcal{O}(q^{\sqrt{n}})$, $n \to \infty$, with $q \in (0,1)$, uniform in z. Independently, Paul Lévy showed one year later that $r_n(z) = \mathcal{O}(q^n)$, $n \to \infty$, with $q = 0.7\ldots$, uniform in z. Lévy's result, but with a better constant, was obtained by P. Szüsz in 1961 using Kusmin's approach. From that time on, a great number of such *Gauss-Kusmin theorems* followed. In this talk a Gauss-Kusmin theorem for a completely different continued fraction expansion – the Optimal Continued Fraction (OCF) expansion – is described. In order to do so, first a Gauss-Kusmin theorem is derived for the natural extension of the ergodic system underlying Hurwitz' Singular Continued Fraction (SCF) (and similarly for the continued fraction to the nearer integer (NICF)). Since the NICF, SCF and OCF are all examples of maximal S-expansions, it follows from an old result of the speaker that the SCF and OCF are metrically isomorphic. This isomorphism is then used to carry over the results for the SCF to any other maximal S-expansion, in particular to the OCF.

Wolfgang Krieger (Heidelberg, krieger@math.uni-heidelberg.de)
'Presentations of subshifts and semigroup-invariants'

Abstract
Let Σ be a finite state space and consider a subshift $C \subset \Sigma^{\mathbf{Z}}$. Say that a $\sigma \in \Sigma$ is left (right)-instantaneous if it has a past (future) that is compatible with its entire future (past). Say that a presentation of a subshift is (bilaterally) instantaneous if all its states are right and left instantaneous. By example it is shown that the property of a presentation to be instantaneous is not an invatiant property. An invariant sufficient condition (PB) for the existence of an instantaneouspresentation is given. By example it is shown that there are coded systems [1] that do not have property (PB).

Further invariant properties (P1) and (P2) of subshifts are described: A subshift has property (P1) if in a sufficiently high block-presentation, given any two admissable words a and b with more than two symbols, that have the same initial symbol and the same final symbol, and that have the same context, one obtains by removing the initial symbols and the final symbols from a and b words that have the same context. A subshift has property (P2) if it has property (P1) and if in a sufficiently high block-presentation admissable words have the same context, provided they have the same initial symbol and the same final symbol, and have the same left context, and the same right context, and provided they share a future that is compatible with their entire past, and share a past that is compatible with their entire future.

One can present a subshift by specifying a transition matrix

$$(\alpha_{\sigma,\sigma'})_{\sigma,\sigma'\in\Sigma}$$

where the entries are elements of a semi-group (with zero) and the admissible words $(\sigma(i))_{1\leq i\leq n}$ of the subshift are given by the condition

$$\prod_{1\leq i\leq n} \alpha_{\sigma(i),\sigma(i+1)} \neq 0 .$$

In [2] an invariant property (A) of subshifts is described that allows us to invariantly associate to a subshift a semi-group (with zero).

Theorem 1 *Let $\mathcal{M}_C$ be a monoid (with zero) associated to a subshift C with properties (A), (P1), (P2), (PA), (PB). Then there exists an instantaneous presentation of C by a transition matrix $(\alpha_{\sigma,\sigma'})_{\sigma,\sigma'\in\Sigma}$, with $\alpha_{\sigma,\sigma'} \in \mathcal{M}_C$.*

[1] F. Blanchard, G. Hansel, *Systèmes Codés*, Theoretical Computer Science **44** (1986), 17-49.

[2] W. Krieger, *On a syntactically Defined Invariant of Symbolic Dynamics*, preprint.

Francois Ledrappier (Paris, ledrappi@math.polytechnique.fr)
'Ergodic properties of some linear actions'

Abstract
We consider the natural linear action of a discrete group of (2×2) matrices of determinant 1 on the plane. For lattices and subgroup of lattices with abelian quotient, ergodicity and the nature of orbits are known from the ergodic properties of the horocycle flow on surfaces through the famous double coset trick. We extend this idea to the distribution properties of orbits.

Ronald Meester (Utrecht, meester@math.uu.nl)

'Convergence of continued fraction type algorithms and generators'

(Joint work with Cor Kraaikamp).

Abstract

The concept of convergence of continued fraction type algorithms has been defined a number of times in the literature. In this talk the relation between these definitions is investigated, and it is shown that these definitions do not always coincide. We relate the definitions to the question whether or not the natural partition of the underlying dynamical system is a generator. It turns out that the 'right' definition of convergence is equivalent to this partition being a generator. The second definition of convergence is shown to be equivalent only under extra conditions on the transformation. These extra conditions are typically found to be satisfied when the second definition is used in the literature.

Mark Pollicott (Manchester, mp@ma.man.ac.uk)

'Computing Hausdorff dimension for non-linear iterated function schemes'

(Joint work with Oliver Jenkinson)

Abstract

This talk dealt with the problem of calculating the (Hausdorff) Dimension of sets generated by non-linear contractions (e.g. iterated function schemes).

A motivating example is the case of the set E_2 consisting of those numbers in the unit interval whose continued fraction expansion contains only the terms 1 or 2. More complicated examples include (hyperbolic) Julia sets.

By using ideas of Bowen, Ruelle and others one can express the dimension implicitly in terms of the pressure of an underlying dynamical system. More precisely, if $T : X \to X$ is a C^2 expanding conformal map then the dimension s of X is given by $P(-s \log |T'|) = 0$. This was used by McMullen (Amer. J. Math., 1998) to calculate the dimension of a number of canonical examples.

In our approach, we make the additional assumption that the transformation T is real analytic. The characterization of s in terms of the pressure is easily recast in terms of being a zero of a certain function which is the determinant of an operator (and closely related to a zeta function). However, using work of Grothendieck (Memoirs Amer. Math. Soc, 1956) this determinant can be expressed in terms of a rapidly converging series, whose terms are each explicitly expressable in terms of a finite number of period points for T.

In particular, by truncating the series we get simple functions whose zeros converge to the zero of the series (i.e. the dimension) at a super-exponential rate. For the particular case of E_2 we can easily compute its dimension to 25 decimal places using only the first 8 terms in the series, taking us up to the level of the best known estimates with (apparently) less computor power required than in previous approachs.

Charles Radin (Austin, radin@fireant.ma.utexas.edu)
'Aperiodic tilings'

Abstract
The two subjects of substitution subshifts, and subshifts of finite type, have been studied for many years showing little in common. Aperiodic tiling refers to the mathematics which has been developing out of the discovery 30 years ago that for subshifts with Z^2 action, rather than Z action, the two subjects are intimately related. The most important results have been theorems of isomorphism between substitution subshifts with Z^d action and certain subshifts of finite type with Z^d action (for $d > 1$), in particular the rather general theorem of this type published 10 years ago by S. Mozes.

Tiling dynamical systems are geometric generalizations of subshifts, wherein the elements of the alphabet are polyhedra instead of abstract symbols. In particular there are natural analogues called substitution tiling systems and finite type tiling systems, now with R^d action. Their theory has developed along lines similar to that for subshifts, but with interesting geometric features which sometimes even throws light on the older subshift theory. (An analog of the theorem of Mozes for tiling dynamical systems was published by C. Goodman-Strauss this year.) The goal of the lecture will be to expose the use of geometric ideas in this part of ergodic theory, for instance in understanding the role of different forms of isomorphism between subshifts.

Klaus Schmidt (Vienna, klaus.schmidt@univie.ac.at)
'Recurrence of stationary walks'

Abstract
Let T be a measure-preserving and ergodic automorphism of a probability space $(X, \mathcal{S}, \mu)$. By modifying an old argument we obtain a sufficient condition for recurrence of the d-dimensional stationary random walk defined by a Borel map $f: X \longmapsto \mathbf{R}^{\mathbf{d}}$, $d \geq 1$, in terms of the asumptotic distributions of the maps $(f + fT + \ldots + fT^{n-1})/n^{1/d}$, $n \geq 1$. If $d = 2$, and if $f: X \longmapsto \mathbf{R}^{\mathbf{2}}$ satisfies the central limit theorem with respect to T (i.e. if the sequence $(f + fT + \ldots + fT^{n-1})/\sqrt{n}$ converges in distribution to a Gaussian law on $\mathbf{R}^{\mathbf{2}}$), then our condition implies that the two-dimensional random walk defined by f is recurrent.

Karoly Simon (CWI, Amsterdam, Karoly.Simon@cwi.nl)
'Hausdorff measure of fractals with overlapping cylinders'

Abstract
We consider some one parameter family of self-similar fractals on the Real line with overlapping cylinders. It is known from some earlier works of Pollicott, Solomyak and Simon that typically the fact that the cylinders intersect each other does not imply the drop of the dimension (if we assume a so called transversality condition). In this talk we will investigate how the overlapping effects the appropriate dimensional Hausdorf measure.

Meir Smorodinsky (Tel Aviv, meir@math.tau.ac.il)
'Processes that can or cannot be generated 'on line' by an indepedent process'

Abstract
We consider the set of stochastic processes $\{X\}$ which are tail-trivial. We shall say that $X = (X_n)_{-\infty<n<\infty}$ and $Y = (Y_n)_{-\infty<n<\infty}$ are *equivalent* if they can be joined in such a way that for all n

$$\sigma\{\cdots X_{n-1}, X_n\} =_{mod0} \sigma\{\cdots Y_{n-1}, Y_n\}.$$

Let $U = (U_n)_{-\infty<n<\infty}$ be an i.i.d. process with uniform distribution. We shall say that $X = (X_n)_{-\infty<n<\infty}$ is *generated on line by an indepedent process* or call it *substandard* if X and U can be joined in such a way that for all n

1. U_n is jointly independent of $U_{n-1}, U_{n-2}, ...\&X_{n-1}, X_{n-2}\cdots$.
2. There exist a measurable function $f_n : (I, X_{n-1}, X_{n-2}, \cdots) \to X_n$ such that

$$X_n = f_n(U_n, X_{n-1}, X_{n-2}, \cdots).$$

3. $\sigma\{\cdots X_{n-1}, X_n\} \subset \sigma\{\cdots U_{n-1}, U_n\}$.

Denote by $\sigma_n(X)$ the sigma-algebra $\sigma\{\cdots X_{n-1}, X_n\}$. We call a process dyadic if the conditional distribution of $\sigma_n(X)$ given $\sigma_{n-1}(X)$ is 2 equally probable points. We give a new proof of a result of Vershik :
Theorem 1 *There exist a dyadic process that is not sub-standard.*
A process is called *homogeneous* if for every n its *nth* conditional measure is uniform: either atomic with some number $p_n > 1$ of atoms of equal mass, or non-atomic and hence isomorphic to Lebesgue measure on the unit interval. If for all n we are in the atomic case then the *r.f.* is called **p**-*adic*, where **p** is the sequence (p_n). If every p_n is 2 it is called *dyadic.* If all the conditional measures are non-atomic then the process itself is called non-atomic.
Theorem 2:(joint work with J. Feldman)*An homogeneous process is equivalent to an independent process if it is sub-standard.*

Jeff Steif (Gothenburg, steif@math.chalmers.se)
'Finitary codings and phase transitions for Markov random fields'
(Joint work with J. van den Berg).

Abstract
We study the existence of finitary codings (also called finitary homomorphisms or finitary factor maps) from a finite-valued i.i.d. process to certain random fields. For Markov random fields we show, using ideas of Marton and Shields, that the presence of a phase transition is an obstruction for the existence of the above coding: this yields a large class of Bernoulli shifts for which no such coding exists.

Conversely, we show that for the stationary distribution of a monotone exponentially ergodic probabilistic cellular automaton such a coding does exist. The construction of the coding is partially inspired by the Propp-Wilson algorithm for exact simulation.

In particular, combining our results with a theorem of Martinelli and Olivieri, we obtain the fact that for the plus state for the ferromagnetic Ising model on $\mathbf{Z}^{\mathbf{d}}$, $d \geq 2$, there is such a coding when the interaction parameter is below its critical value and there is no such coding when the interaction parameter is above its critical value.

Jean-Paul Thouvenot (Paris, kalikow@ccr.jussieu.fr)
'An application of ergodic theory to probability theory'

No abstract submitted.