Report 99-044

Report on the Workshop
Stochastic Models from Statistical Physics

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STOCHASTIC MODELS FROM STATISTICAL PHYSICS

EURANDOM, Eindhoven, The Netherlands
April 19 – 23, 1999

organised jointly by the DFG Schwerpunkt programme
'Interacting Stochastic Systems of High Complexity'
and the EURANDOM research programme
'Interacting Stochastic Systems'

The workshop was intended to present the current state of the art at the interface of probability and statistical physics. It was aimed at a rather wide audience of probabilists working in or around this area of mathematics, with particular care for the young generation of students and post-doctoral researchers getting started.

To meet this aim, the presentation was structured on three levels:

- 4 mini-courses, composed of three hours of lectures each, were designed to give an overview on some of the central topics and questions in the field:

  Bolthausen discussed recent progress in the mathematical understanding of mean-field spin glass models and, in particular, explained a rigorous mathematical framework in which some of the heuristic results of physicists can be formulated. Newman discussed the dynamics of disordered lattice systems (focusing on the large-time behaviour of gradient dynamics in infinite systems) and their equilibrium properties (stressing the central role of the structure of ground states). Slade gave a review on branched polymers and lattice trees, where the central new aspect is the construction of continuum scaling limits and their identification with super-Brownian motion. Yau reviewed new methods for estimating the speed of convergence to equilibrium in a large class of conservative stochastic dynamics.

- These mini-courses were complemented by 11 one-hour lectures on recent results on specific topics in these and closely related areas. Here the theory of percolation, statistical mechanics, and particle systems on general graphs emerged as a rapidly developing and promising new subject. Some important new results concerning the Gibbsian theory and renormalisation illustrated a considerable maturing of this area. Several talks were devoted to various aspects of the dynamics of spin systems, in particular, questions of metastability and aging are now seen as pertinent issues to which probability theory can make substantial contributions.

- Wednesday was entirely devoted to contributed talks by junior researchers. 15 papers had been selected from a total of some 30 submissions. The overall high quality of both the presentations and the results presented testify to the liveliness of the field and its capacity to attract brilliant students.

Monday April 19:

8:15 – 8:45	registration
8:45 – 9:00	opening by the director of EURANDOM and welcome by the organising committee
chair: H.-O. Georgii	
9:00 – 10:30	G. Slade I
10:30 – 11:00	coffee
11:00 – 12:30	C. Newman I
12:30 – 15:00	lunch break
chair: D. Dürr	
15:00 – 16:00	E. Orlandi
16:00 – 16:30	tea
16:30 – 17:30	V. Gayrard
17:30 – 18:30	A. Stacey
18:30	social event: drinks

Tuesday April 20:

chair: A. Martin-Löf	
9:00 – 10:30	G. Slade II
10:30 – 11:00	coffee
11:00 – 12:30	C. Newman II
12:30 – 15:00	lunch break
chair: H. Leschke.	
15:00 – 16:00	A.-S. Sznitman
16:00 – 16:30	tea
16:30 – 17:30	E. Scoppola
17:30 – 18:30	A. van Enter

Thursday April 22:

chair: G. Kersting

9:00 – 10:30	E. Bolthausen I
10:30 – 11:00	coffee
11:00 – 12:30	H.-T. Yau I
12:30 – 15:00	lunch break

chair: J. Gärtner

15:00 – 16:00	S. Olla
16:00 – 16:30	tea
16:30 – 17:30	O. Häggström
17:30 – 18:30	J. Steif

Friday April 23:

chair: F. Götze

9:00 – 10:30	E. Bolthausen II
10:30 – 11:00	coffee
11:00 – 12:30	H.-T. Yau II
12:30 – 13:30	short (!) lunch break

chair: A. Wakolbinger

13:30 – 14:30	E. Olivieri
14:30 – 15:30	G. Ben Arous
15:30 – 16:00	closing and tea

A. Klenke, *Weak convergence and linear systems duality.*
W. König, *Almost sure asymptotics for the continuous parabolic Anderson model.*
M. Löwe, *Critical fluctuations in the Hopfield model.*
J. Lorinczi, *A Gibbsian description of* $P(\phi)_1$*-processes.*
F. Redig, *Variational principle for weakly Gibbsian measures.*
Y. Velenik, *The droplet on the wall.*
J. Yukich, *Large deviation principles for nearly additive processes.*

A conditional limit theorem for Gibbs measures in the Ising-model

S. Adams

In the Ising-model on the lattice $\mathbb{Z}$ in dimension 1 we prove that the Gibbs measures conditioned on a microcanonical constraint for the energy converge to the Gibbs measures which maximize the entropy relative to the constraint. This result answers the equivalence of Gibbs ensembles on the level of measures in a formulation, which would mean that the microcanonical and the grand canonical Gibbs distributions in finite boxes have the same infinite-volume limits. In our result we do not need periodic boundary conditions like in [Csi87] and [Geo93] and we have the result for the whole lattice $\mathbb{Z}$ instead of $\mathbb{Z}^+$ in [Men93].

[Csi87] Csiszar, I., Cover, T.M., Choi, B.S., *Conditional limit theorems under Markov conditioning*, IEEE Trans. Inform. Theory **33**, 788-801, 1987.

[Geo93] Georgii, H.O., *Large Deviations and Maximum Entropy Principle For Interacting Random Fields an* $\mathbb{Z}^d$, The Annals of Probability **21**, No. 4, 1845-1875, 1993.

[Men93] Menzel, P., *A limit theorem for one-dimensional Gibbs measures under conditions on the empirical field* , Stochastic Processes and their Applications **44**, 347-359, 1993.

Diffraction Theory of Stochastic Structures

M. Baake

Random tilings and lattice gases are possible models for the description of solids with built-in disorder, and in particular of those with entropically based stabilization mechanisms. Rather little is known about the kinematic diffraction theory of the corresponding unbounded measures, and even less is known in rigorous terms.

It is the aim of this contribution to derive some results with methods from statistical mechanics and ergodic theory. Exactly solvable models with known correlation functions or systems of Bernoulli and Markov type serve as toy models to display the possible scenarios, and to make some folklore results rigorous. Contrary to common belief, the continuous part of the diffraction spectra, and the singular continouous part in particular, turns out to be very significant and might shed some light on known problems of practical structure determination and refinement.

Aging of spherical Glasses and Random Matrices

G. Ben Arous

Sompolinski and Zippelius (1981) propose the study of dynamical systems whose invariant measures are the Gibbs measures for (hard to analyze) statistical physics models of interest. In the course of doing so, physicists often report of an "aging" phenomenon. For example, aging is expected to happen for the Sherrington-Kirkpatrick model, a disordered mean-field model with a very complex phase transition in equilibrium at low temperature. We shall study the Langevin dynamics for a simplified spherical version of this model. The induced rotational symmetry of the spherical model reduces the dynamics in question to an N-dimensional coupled system of Ornstein-Uhlenbeck processes whose random drift parameters are the eigenvalues of certain random matrices. We obtain the limiting dynamics for N approaching infinity, and by analyzing its long time behavior, explain what is aging (mathematically speaking), what causes this phenomenon, and what is its relationship with the phase transition of the corresponding equilibrium invariant measures.

Probabilty cascades and an abstract cavity method

E. Bolthausen

Parisi first described for the Sherrington-Kirkpatrick model a solution in the low temperature regime, which has a very interesting hierarchical structure. Later on, there had been other approaches, based on the so-called cavity method, which led to the same conclusion (see [2]). The correctness of the Parisi solution has not been rigorously proved.

In a paper with Alain-Sol Sznitman [1], we clarified the abstract mathematical structure behind this solution, and also of the cavity method. This hierarchical structure is best described by a clustering process which has a simple Markovian property. We also were able to give an abstract version of the cavity method.

The clustering process is easiest in the case of the generalized random energy model (GREM), first invented by Derrida. The limiting GREM had been discussed by Ruelle [3]. In our clustering description, this has a straightforward generalization to the case where there is no longer a finite number of branching stages. The GREM is not able to catch some features which turn out to be of crucial importance for the SK-model, but which can also be formulated within the clustering process, together with the abstract cavity method.

The mini-course gave an introduction to these topics. It has however to be emphasized that no new results on the SK model are proved.

References

[1] Bolthausen, E. & Sznitman, A.-S.: *On Ruelle's probability cascades and an abstract cavity method.* Com. Math. Phys. **197**, 247-276 (1998)

[2] Mézard, M., Parisi, G. & Virasoro, M.A.: *Spin glass theory and beyond.* World Scientific, Singapore, 1987

[3] Ruelle, D.: *A mathematical reformulation of Derrida's REM and GREM.* Com. Math. Phys. **108**, 225-239 (1987)

On the Volume of the 'Super-Sausage'

J. Engländer

Let α and β be positive constants. Let X be the supercritical super-Brownian motion corresponding to the evolution equation $u_t = \frac{1}{2}\Delta + \beta u - \alpha u^2$ in $\mathbb{R}^d$ and let Z be the binary branching Brownian-motion with branching rate β. For $t \geq 0$, let $R(t) = \overline{\bigcup_{s=0}^{t} \mathrm{supp} X(s)}$, that is $R(t)$ is the (accumulated) support of X up to time t. For $t \geq 0$ and $a > 0$, let $R^a(t) = \bigcup_{x \in R(t)} \bar{B}(x,a)$. We call $R^a(t)$ the *super-Brownian sausage* corresponding to the supercritical super-Brownian motion X. For $t \geq 0$, let $\hat{R}(t) = \overline{\bigcup_{s=0}^{t} \mathrm{supp} Z(s)}$, that is $\hat{R}(t)$ is the (accumulated) support of Z up to time t. For $t \geq 0$ and $a > 0$, let $\hat{R}^a(t) = \bigcup_{x \in R(t)} \bar{B}(x,a)$. We call $\hat{R}^a(t)$ the *branching Brownian sausage* corresponding to Z. We prove the following two asymptotics:

$$\begin{aligned} & \lim_{t\to\infty} \frac{1}{t} \log E_{\delta_0}[\exp(-\nu |R^a(t)|) |\, X \text{ survives}] \\ = & \lim_{t\to\infty} \frac{1}{t} \log \hat{E}_{\delta_0} \exp(-\nu |\hat{R}^a(t)|) = -\beta. \end{aligned}$$

for all $d \geq 2$ and all $a, \alpha, \nu > 0$.

On the impossibility of a Gibbs representation for the dilute Ising model in the Griffiths phase

A.C.D. van Enter

We consider the quenched dilute nearest neighbor Ising model at low temperatures and with strong dilution. Each site can either contain a plus or minus spin, or be empty. The magnetization of this model was shown to be non-analytic as a function of the magnetic field by Griffiths; this is the famous Griffiths singularity. We consider the quenched measure describing this three-valued lattice system and show that it is not a Gibbs measure.

Fluctuations of a random walk with randomly placed nodes

N. Gantert

based on joint work with Ofer Zeitouni

Abstract

Suppose that the integers are assigned random variables $\{\omega_x\}$, taking values in the unit interval. The sequence $\omega = (\omega_x)_{x\in\mathbb{Z}}$ serves as an environment. For every fixed ω, let $X = (X_n)_{n\geq 0}$ denote the Markov chain on $\mathbb{Z}$ starting at $X_0 = 0$, with $P_\omega[X_{n+1} = x+1|X_n = x] = \omega_x = 1 - P_\omega[X_{n+1} = x-1|X_n = x]$. The process $(X_n)_{n\geq 0}$ is called random walk in random environment (RWRE). Let the distribution η of $(\omega_x)_{x\in\mathbb{Z}}$ be a product measure with marginal α. Assume that $\langle\rho\rangle := \int ((1-\omega_0)/\omega_0)\,\alpha(d\omega_0) < 1$. Then, the RWRE has a positive speed $v_\alpha := (1-\langle\rho\rangle)/(1+\langle\rho\rangle)$, i.e. $X_n/n \to v_\alpha$, $P_\omega - a.s.$, for η-almost all ω. Let $\omega_{\min} := \min\{u : u \in \operatorname{supp}\alpha\}$ and $\omega_{\max} := \max\{u : u \in \operatorname{supp}\alpha\}$. Assume that $\omega_{\min} < 1/2 < \omega_{\max}$. Then, there is a unique $s > 1$ such that $\int ((1-\omega_0)/\omega_0)^s \alpha(d\omega_0) = 1$. It is shown in [1] that in this case, for $v < v_\alpha$,

$$\limsup_{n\to\infty} \frac{1}{n^{1-1/s}} \log P_\omega[X_n < nv] = 0 \tag{1}$$

and conjectured that

$$\liminf_{n\to\infty} \frac{1}{n^{1-1/s}} \log P_\omega[X_n < nv] = -\infty\,. \tag{2}$$

The conjecture can be proved in the particular case where α is concentrated on $\{p, 1\}$ with $p < 1/2$. This particular case of a RWRE, also denoted "two-coins case" or "random walk with randomly placed nodes" is easier to analyse, since the RWRE is in this case a simple random walk with drift on the (random) intervals without nodes and goes to the right at the nodes, i.e. at the locations x where $\omega_x = 1$. For the random walk with randomly placed nodes, one can also extend the results to environment distributions η which are stationary and ergodic instead of being product measures.

References

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[2] N. Gantert and O. Zeitouni, "Large deviations for one-dimensional random walk in a random environment - a survey", to appear, 1999.

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METASTABILITY IN STOCHASTIC DYNAMICS OF DISORDERED MEAN-FIELD MODELS

V. Gayrard

I report on a joint work with Anton Bovier, Markus Klein, and Michael Eckhoff in which we study a class of Markov chains that describe reversible stochastic dynamics of a large class of disordered mean field models at low temperatures. Our main purpose is to give a precise relation between the metastable time scales in the problem to the properties of the rate functions of the corresponding Gibbs measures. We derive the analog of the Wentzell-Freidlin theory in this case, showing that any transition can be decomposed, with probability exponentially close to one, into a deterministic sequence of "admissible transitions". For these admissible transitions we give upper and lower bounds on the expected transition times that differ only by a constant factor. The distributions of the rescaled transition times are shown to converge to the exponential distribution.

The genealogy of a cluster in the multitype voter model

J. Geiger

joint work with J.T. Cox

The genealogy of a cluster in the multitype voter model can be defined in terms of a family of dual coalescing random walks. We represent the genealogy of a cluster as a point process in a size-time plane and show that in high dimensions the genealogy of the cluster at the origin has a weak Poisson limit. The limiting point process is the same as for the genealogy of the size-biased Galton-Watson tree. Moreover, our results show that the branching mechanism and the spatial effects of the voter model can be separated on a macroscopic scale.

A Direct Simulation Method For The Coagulation-Fragmentation Equations With Multiplicative Coagulation Kernels

F. Guias

We present a fast method of direct simulation for the stochastic process corresponding to coagulation-fragmentation dynamics with multiplicative coagulation kernels. Convergence results and error estimates are proved in the case where the corresponding deterministic equations are known to have a unique solution, as well as a relative compactness property of the family of stochastic processes in more general situations. Simulation results are discussed also in cases in which convergence results to the deterministic equations are not known.

Percolation on nonamenable graphs

O. Haggstrom

There has been a substantial recent interest in percolation theory beyond the usual $\mathbf{Z}^d$ setting, with particular emphasis on nonamenable transitive graphs. Many new phenomena occur compared to the $\mathbf{Z}^d$ case, most of them being related to the possible nonuniqueness of infinite clusters. I review some recent results in this field, obtained by Benjamini, Lyons, Peres, Schonmann, Schramm, myself, and others.

Surface tension and the Ornstein-Zernike behaviour for the 2D Blume-Capel model

O. Hryniv*

joint work with R. Kotecky

Considering spin configurations $\sigma \in X \equiv \{-1,0,1\}^{\mathbb{Z}^2}$, the Hamiltonian of the Blume-Capel model, in a finite volume $\Lambda \subset \mathbb{Z}^2$ and under fixed boundary conditions $\overline{\sigma} \in \{-1,0,1\}^{\mathbb{Z}^2}$, is

$$\begin{aligned} H_\Lambda(\sigma|\overline{\sigma}) &= J \sum_{\substack{\langle x,y\rangle \\ x,y\in\Lambda^\circ}} (\sigma(x)-\sigma(y))^2 + J \sum_{\substack{\langle x,y\rangle \\ x\in\Lambda^\circ, y\in\partial\Lambda}} (\sigma(x)-\overline{\sigma}(y))^2 \\ &\qquad - \lambda \sum_{x\in\Lambda^\circ} \sigma(x)^2 - h \sum_{x\in\Lambda^\circ} \sigma(x). \end{aligned}$$

Here, the first two sums are over pairs of nearest neighbour sites, $\partial\Lambda$ denotes the inner boundary of the set Λ, $\partial\Lambda = \{x \in \Lambda : \exists y \in \mathbb{Z}^2 \setminus \Lambda, |y-x| = 1\}$, and Λ° is the interior of Λ, $\Lambda^\circ = \Lambda \setminus \partial\Lambda$; the real parameters λ and h are called external fields, and $J > 0$ is the coupling constant.

The phase diagram, for fixed temperature $T = 1/\beta$ and $J > 0$, features the triple point $(\lambda, h) = (\lambda_0(T), 0)$, $\lambda_0(T) \downarrow 0$ as $T \downarrow 0$, at which all three phases (predominantly plus, zero, or minus) coexist, and from which three lines of coexistence of two phases emanate. Here we consider $h = 0$ and $\lambda > \lambda_0(T)$, i.e., the case of coexistence of the plus and minus phases. It happens that the behaviour of the system depends on the value of λ, namely, there exist two increasing functions $\lambda_1(T)$ and $\lambda_2(T)$ such that $\lambda_1(T) \downarrow 2J$ and $\lambda_2(T) \downarrow 4J$ as $T \downarrow 0$ and the behaviour of the system is different in every of three intervals

$$\Delta_0^{(T)} = (\lambda_0(T), \lambda_1(T)), \qquad \Delta_1^{(T)} = (\lambda_1(T), \lambda_2(T)), \qquad \Delta_2^{(T)} = (\lambda_2(T), +\infty).$$

For any fixed $\lambda \in \Delta_i^{(T)}$ and $T \geq 0$ small enough, the situation is rather simple (with essentially one ground state) and can be analysed using standard perturbation methods. On the contrary, for $\lambda = \lambda_i(T)$, $i = 1,2$, the system interpolates between two different types of behaviour. This "transition" creates in its turn an additional combinatorial complexity and requires to develop another technique for its analysis.

Our aim is to study the asymptotic behaviour of the partition function describing the interface between plus and minus phases inclined by an angle θ. Namely, let σ^a, $a = 1,2$, denote the basic column configurations:

$$\sigma^a \equiv (\sigma_t^a)_{t=-\infty}^{+\infty},$$

where

$$\sigma_t^1 \equiv \begin{cases} +1, & t > 0, \\ -1, & t \leq 0, \end{cases} \qquad \sigma_t^2 \equiv \begin{cases} +1, & t > 0, \\ 0, & t = 0, \\ -1, & t < 0, \end{cases}$$

and ω_k be the (vertical) shift, $(\omega_k(\sigma))_t = \sigma_{t-k}$. Now, for any θ, $|\theta| < \frac{\pi}{2}$, define the θ-inclined basic configurations of the type a, $a = 1,2$, according to the following formulas:

$$\sigma^{\theta,a} = (\sigma_x^{\theta,a})_{x\in\mathbb{Z}^2}, \quad \text{where} \quad \left(\sigma_{(k,l)}^{\theta,a}\right)_{l=-\infty}^{+\infty} \equiv \omega_{[k\tan\theta]}(\sigma^a).$$

Finally, introduce the mixed boundary conditions $\sigma^{\theta,a,b} = \sigma^{\theta,a,b}(x)$, $x \in \mathbb{Z}^2$, via

$$\sigma^{\theta,a,b}(x) \equiv \begin{cases} \sigma_x^{\theta,a}, & x_1 < 0, \\ \sigma_x^{\theta,b}, & x_1 \geq 0. \end{cases}$$

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The random energy model under Metropolis dynamics

M. Isopi

In finite volume we have a finite state reversible Markov chain and therefore we cannot spot an anomalous rate of convergence as it may occur in infinite volume. On the other hand, for disordered systems the constants which determine the rate of convergence are random variables. It is thus natural to investigate their asymptotic behaviour in the thermodynamic limit. In particular one may question the selfaveraging property. This means whether those constants, properly normalized, converge to a deterministic constant. For the spectral gap of the Random Energy Model under Metropolis dynamics we derive upper and lower bounds which are sharp in exponential order. Thus the log of the gap converges to a deterministic function of the temperature. The log-Sobolev constant has the same behaviour.
Most of this is joint work with L. R. Fontes, Y. Kohayakawa and P. Picco.

Weak Convergence and Linear Systems Duality

A. Klenke

joint work with Ted Cox and Ed Perkins

For a class of interacting particle systems on a countable set, including the so-called linear systems, self-duality has proved to be a strong tool to show longtime convergence to an invariant state ν_θ from a constant initial state θ. This convergence was extended to a class of translation invariant random initial states through the Liggett-Spitzer coupling. Here we drop the assumption of translation invariance of the initial state. Instead, we assume only that there exists a global density θ in a certain L^2-sense. We use the duality to carry out a comparison argument and show convergence to the same ν_θ as above. We treat some examples in more detail: the parabolic Anderson model, the mutually catalytic branching model, and the smoothing and potlatch processes.

ALMOST SURE ASYMPTOTICS FOR THE CONTINUOUS PARABOLIC ANDERSON MODEL[1]

W. König[2]

1 The continuous parabolic Anderson problem

We consider the parabolic Anderson problem

$$\begin{aligned} \partial_t u(t,x) &= \kappa\Delta u(t,x) + \xi(x)u(t,x), && (t,x) \in (0,\infty)\times\mathbb{R}^d, \\ u(0,x) &= 1, && x \in \mathbb{R}^d, \end{aligned} \tag{1.1}$$

where $\kappa > 0$ is a diffusion constant, and $\xi = \{\xi(x); x \in \mathbb{R}^d\}$ is a random shift-invariant potential. We shall consider the two cases of a Gaussian field and a shot noise Poisson field. We are particularly interested in the influence of high peaks of the field ξ on the asymptotic behavior of the solution $u(t,0)$. Our goal here is to investigate the second-order asymptotics for $u(t,0)$ almost surely w.r.t. the field ξ and to relate it to the geometry of the high peaks of the field.

Rough logarithmic asymptotics for the moments and for the almost sure behavior of $u(t,0)$ have been derived by Carmona and Molchanov (1995). The second-order asymptotics for the moments have been described by Gärtner and König (1998). The discrete analogon (i.e., the moment asymptotics and the almost sure asymptotics for the parabolic Anderson model on $\mathbb{Z}^d$ rather than on $\mathbb{R}^d$) has been investigated by Gärtner and Molchanov (1998). A general discussion of the parabolic Anderson model and the phenomenon of intermittency can be found in Carmona and Molchanov (1994). Let us also mention that in the monograph by Sznitman (1998) related problems are considered, but the field ξ there is assumed to be bounded from above, and this leads to different phenomena and requires different methods.

According to the Feynman-Kac formula, the solution u is given by

$$u(t,x) = \mathbb{E}_x \exp\left\{\int_0^t \xi(W_s)\,ds\right\}, \qquad x \in \mathbb{R}^d, t > 0, \tag{1.2}$$

where $\mathbb{E}_x$ denotes expectation w.r.t. a Brownian motion $(W_s)_{s\in[0,\infty)}$ on $\mathbb{R}^d$ with generator $\kappa\Delta$, starting at $x \in \mathbb{R}^d$. Thus, in order to describe the asymptotic almost sure behavior of $u(t,0)$, one needs to find the 'best strategy' of the Brownian motion to contribute maximally to the expectation of $e^{\int_0^t \xi(W_s)\,ds}$. Let us first explain the idea of our approach in a rough and informal manner.

If h_t denotes the height of the highest peak of the field ξ in the cube $Q_t = [-t,t]^d$, then the Brownian motion will run in short time to the location of this peak and spend the rest of the time until t close to its top. This behavior will lead to a first-order term $\frac{1}{t}\log u(t,0) \approx h_t$, and h_t will turn out to be non-random. On the other hand, the diffusive nature of the motion forces it to keep on moving, therefore the relation between the geometry of the peak and the size of the diffusion constant κ will determine the correction to this asymptotic contribution coming from this path. We shall see that the shape of the peak will be close to a deterministic parabola $p(\cdot)$ which is given in terms of some quantities that define the field. The second-order term will then be given in terms of the principal eigenvalue of the harmonic oscillator $\kappa\Delta + p$. This eigenvalue can be thought of measuring how well the motion manages to stay close to the top of the peak.

[1]joint work with Jürgen Gärtner, Berlin, and Stanislav Molchanov, Charlotte

[2]partially supported by D. Dawson's Max Planck Award for International Cooperation

(i) The actual value of h_t is

$$h_t = \begin{cases} \sqrt{2d\sigma^2 \log t}, & \text{Gaussian case,} \\ \frac{d\sigma^2 \log t}{\log\log t}(1+o(1)), & \text{Poisson case,} \end{cases} \qquad t \to \infty. \tag{3.6}$$

In the Poisson case this approximation is too rough for the second-order asymptotics. In general, one cannot expect to find an explicit expression replacing h_t in (3.5). This asymptotics does not only depend on σ^2 and Σ, but is sensitive to changes of B in $\{x\colon B(x) > \sigma^2/2\}$. Namely, one can see that this remark applies to the moment asymptotics (see the following remark) which is crucial for our derivation of the upper bound in (3.5).

(ii) Let us contrast the almost sure behavior to the behavior of the moments of $u(t,0)$ which has been described by Gärtner and König (1998). The result of that paper, specialized to the Gaussian and Poisson situation, reads as follows.

$$\frac{1}{t}\log\langle u(t,0)\rangle = \frac{H(t)}{t} - (\chi + o(1))\sqrt{H'(t)}, \qquad t \to \infty. \tag{3.7}$$

Thus, the asymptotics of the moments of $u(t,0)$ are rather different from the almost sure asymptotics which reflects the high irregularity of the fields.

(iii) An inspection of our proof shows that the r.h.s. of (3.5) can be interpreted in the following way: The first-order term h_t is roughly equal to $\max_{Q_t} \xi$ (recall that $Q_t = [-t,t]^d$) and the entire r.h.s. of (3.5) is roughly equal to the principle eigenvalue of the harmonic oscillator $\kappa\Delta + h_t B$. (Indeed, a direct calculation shows that $-\chi\sqrt{h_t}$ is equal to the principal eigenvector of $\kappa\Delta - h_t|\Sigma \cdot|^2/(2\sigma^2)$ in $\mathbb{R}^d$, the eigenfunction being a certain Gaussian density.) As a consequence, the dependence of $u(t,0)$ on the diffusion constant κ enters the second-order term only (via χ).

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A Gibbsian description to $P(\phi)_1$ processes

J. Lorinczi

We consider Brownian motions perturbed by suitable potentials and ask the question whether their stationary path measures can be described as Gibbs measures. Our main result is that, for a rich set of potentials (i.e., Kato-class potentials) and a full-measure subset of boundary conditions Gibbs measures do exist and are moreover unique. The set of allowed boundary path configurations is identified explicitly. One application of this result is in the direction of constructing Gibbs measures also for pair potentials (in which case the Feynman-Kac formula does not work). We illustrate these results on some specific examples.

Critical Fluctuations in the Hopfield Model

M. Löwe

I report about joint work with Barbara Gentz (WIAS Berlin). We investigate the fluctuations of the order parameter in the Hopfield model of spin glasses and neural networks at the critical temperature $1/\beta_c = 1$. The number of patterns $M(N)$ is allowed to grow with the number N of spins but the growth rate is subject to the constraint $M(N)^{13}/N \to 0$. As the system size N increases, on a set of large probability the distribution of the appropriately scaled order parameter under the Gibbs measure comes arbitrarily close (in a metric which generates the weak topology) to a non-Gaussian measure which depends on the realization of the random patterns. This random measure is given explicitly by its (random) density.

Disordered Systems: In and Out of Equilibrium

C. M. Newman

In the mini-course we introduce and discuss some of the basic random processes of interest in Equilibrium and Non-Equilibrium Statistical Physics. The focus is on Ising models (including spin glasses) and related percolation models and mostly considers some easily formulated issues that arise at zero temperature. Despite the simple formulation, there are many interesting open problems. Equilibrium issues include the numbers of ground states and related questions concerning geodesics in first-passage percolation and numbers of trees in minimal spanning forests. Nonequlibirum issues concern the large time behavior starting from a deeply quenched (i.e., uniformly random) initial configuration.

Renormalization group maps: Gibbsianess and convergence under strong mixing conditions

E. Olivieri

We study a renormalization group map: the block averaging transformation applied to Gibbs measures relative to a class of finite range lattice gas, when suitable mixing conditions are satisfied. Using block decimation, cluster expansion and detailed comparison between ensembles, we are able to prove Gibbsianness and convergence to a trivial fixed point. Our results apply to standard Ising model at any temperature above the critical one and arbitrary magnetic field.

Hydrodynamics and Fluctuations of the Asymmetric Exclusion Process

S. Olla

We consider an asymmetric exclusion process in dimension $d \geq 3$ under diffusive rescaling starting from the Bernoulli product measure with density $0 < \alpha < 1$. We prove that the density fluctuation field Y_t^N converges to a generalized Ornstein–Uhlenbeck process, which is formally the solution of the stochastic differential equation $dY_t = \mathcal{A}Y_t dt + dB_t^\nabla$, where $\mathcal{A}$ is a second order differential operator and B_t^∇ is a mean zero Gaussian field with known covariances.
In collaboration with C. Landim and C.C.Chang

Random Field Ising Models with Kac type of Interaction.

E. Orlandi
joined work with Marzio Cassandro and Pierre Picco

I present the results contained in the paper

"Typical configurations for one dimensional random field Kac model" Marzio Cassandro, Enza Orlandi, Pierre Picco to appear in Ann. Prob.

and some extensions of them at which we are still working toghether with Maria Eulalia Vares. In the first paper we study the typical profiles of a one dimensional random field Kac model. We give upper and lower bounds of the space scale where the profiles are constant. The results hold almost surely with respect to the realizations of the random field. The analysis is based on a bloc-spin construction, deviation techniques for the local empirical order parameters, and concentration inequalities for the realizations of the random magnetic field. For the upper bound, we exhibit a scale related to the law of the iterated logarithm, where the random field makes an almost sure fluctuation that obliges the system to break its rigidity. For the lower bound, we prove that on a smaller scale the fluctuations are not strong enough to allow this transition.
In the second paper (still in progress) we give large deviations type of results for the same model on the scale where the random field becomes relevant to determine the optimal profiles which minimize a suitable random functional. On this scale the use of cluster expansion becomes essential.

Cluster and polymer expansions in quantum statistical models of continuouse systems

A. Rebenko

A Euclidean Gibbs state is constructed in the thermodynamic limit for the continuous quantum systems with Bose and Fermi statistics for sufficientely small mass of quantum particles using cluster and polymer expansions methods.

Variational Principle for weakly Gibbsian Measures

F. Redig

We consider restrictions of low temperature Gibbs-measures on two-dimensional spin configurations to a one-dimensional layer. We prove that there exists a translation invariant potential making these (in general non-Gibbsian) restricted measures into "weakly Gibbsian" measures. We also prove that the thermodynamic quantities (energy, pressure) for this potential exist and that the weakly Gibbsian measures satisfy a variational principle.

Metastability and nucleation for conservative dynamics

E. Scoppola

joint work with F. den Hollander and E. Olivieri

In this paper we study metastability and nucleation for the two-dimensional lattice gas with Kawasaki dynamics at low temperature and low density. Particles perform independent random walks on $\mathbb{Z}^2$ and inside a *finite* but arbitrary box Λ_0 feel exclusion as well as a binding energy U with particles at neighboring sites. The initial configuration is chosen such that Λ_0 is empty while outside Λ_0 particles are placed according to a Poisson random field with density $\rho = e^{-\Delta\beta}$ for some $\Delta \in (U, 2U)$. That is to say, initially the system is in equilibrium with density ρ *conditioned* on Λ_0 being empty. For large β, in equilibrium without this condition Λ_0 is fully occupied because of the binding energy. We investigate how the transition from empty to full takes place under the dynamics. In particular, we identify the size and shape of the critical droplet, the time of its creation, and the typical trajectory prior to its creation, in the limit as $\beta \to \infty$. The choice $\Delta \in (U, 2U)$ corresponds to the situation where the critical droplet has side length $l_c \in (1, \infty)$, i.e., the system is metastable.

Because particles are *conserved* under Kawasaki dynamics, the analysis of metastability and nucleation is more difficult than for Ising spins under Glauber dynamics. The key point is to show that at low density the gas outside Λ_0 can be treated as a reservoir that creates particles with rate ρ at sites on the interior boundary of Λ_0 and annihilates particles at sites on the exterior boundary of Λ_0. Once this has been achieved, the problem reduces to *local* metastable behavior inside Λ_0, and standard techniques from non-conservative dynamics can be applied. The dynamics inside Λ_0 is still conservative, but this obstacle can be circumvented via local geometric arguments.

Statistical Mechanics and Super-Brownian Motion

G. Slade

Brownian motion has played an important role in statistical mechanics for a long time. Super-Brownian motion is a more recent construct, useful for the description of tree-based mass distributions. The mini-course will give an introduction to the variant of super-Brownian motion known as integrated super-Brownian excursion (ISE), and will describe how it arises in the critical behaviour of lattice trees and percolation in high dimensions. The mini-course will discuss the results of various collaborations with C. Borgs, J.T. Chayes, E. Derbez, T. Hara and R. van der Hofstad.

The contact process and branching random walk on trees

A. Stacey

The contact process on certain trees (including all regular trees of degree at least three) has two phase transitions, with an intermediate phase of weak survival in which the process can survive for ever but drifts off to infinity. It is known that this behaviour cannot happen on $\mathbb{Z}^d$. It has been conjectured that there is a weak survival phase on any tree satisfying a certain growth condition. It has also been conjectured that that contact process has two phase transitions on a given graph if and only if the (simple symmetric nearest neighbour) branching random walk does. Both these conjectures are shown to be false. Use is made of new results about the behaviour of the contact process on large finite trees; and the behaviour of the branching random walk on Galton-Watson trees. Other processes, such as percolation and the Ising model, also exhibit different behaviour depending on the underlying graph; the connections – in this respect between these process and the contact process are discussed.

Robust Phase Transitions for Heisenberg and Other Models on General Trees

J. E. Steif

joint work with Robin Pemantle

We study the classical Heisenberg model (and to a lesser extent the q–state Potts model) on a general tree. In the Heisenberg model, the state space is the d–dimensional unit sphere and the interactions are proportional to the cosines of the angles between neighboring spins. The phenomenon of interest here is the classification of phase transition (non-uniqueness of the Gibbs state) according to whether it is robust. We show that for both the Heisenberg and Potts models, occurrence of robust phase transition is determined by the geometry (branching number or Hausdorff dimension) of the tree. We compute the critical value for robust phase transition for both of these models exactly in terms of the branching number. In some cases, such as the $q \geq 3$ Potts model, robust phase transition and usual phase transition do not coincide, while in other cases, such as the Heisenberg models, we conjecture that robust phase transition and usual phase transition are equivalent.

A law of large numbers for random walks in random environment

A.-S. Sznitman

This talk will present a strong law of large numbers recently derived by M. Zerner (ETH Zürich) and myself for multi-dimensional random walks in random environment. We show that a condition introduced by S. Kalikow in 1981 to prove transience in a multi-dimensional context (the only known such criterion up to now), in fact implies a strong law of large numbers with non-zero velocity.

The droplet on a wall

Y. Velenik

We review various works on the problem of the wetting transition in the canonical ensemble, i.e. the interaction between an equilibrium crystal and a substrate, including results for SOS models in $d = 1$ (de Coninck et al.), $d \geq 2$ (Bolthausen-Ioffe), and for the Ising model in $d = 2$ (Pfister-Velenik) and $d > 2$ (Bodineau-Ioffe-Velenik).

Relaxation to Equlibrium for Conservative Dynamics

H.-T. Yau

We review the martingale method for determining the logarithmic Sobolev constants for various models, such as the Bernoulli-Laplace model, the symmetric simple exclusion and the random transposition model. We then use the logarithmic Sobolev inequality to determine the relaxation time to equilibrium for state spaces with finite cardinality.
For processes on the whole space Z^d, the decay rates to equilibria are governed by power laws. We review a recent method which determines the decay rate in the variance sense. For an arbitrary local function u, the decay rate to equilibrium for the zero range dynamics on the d-dimensional integer lattice is $C_u t^{-d/2} + o(t^{-d/2})$ in the variance sense. The constant C_u is also computed explicitly.

Large Deviation Principles for Nearly Additive Processes

J. E. Yukich

joint work with T. Seppalainen

We prove a large deviation principle for a process indexed by rectangles of the multidimensional integer lattice or Euclidean space, under approximate additivity and regularity hypotheses. The rate function is the convex dual of the limiting logarithmic moment generating function. In some applications the rate function can be expressed in terms of relative entropy. The general result applies to processes in statistical mechanics and Euclidean combinatorial optimization. Examples include the free energy of a short-range spin glass model, the cluster number for site percolation, the length of the minimal tour through a random point set, and the length of a minimal spanning tree through a random point set.