The Nearest Item Heuristic for Carousel Systems

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Abstract

A carousel is a computer controlled warehousing system, which is widely used to store small and medium sized goods. One of the most important performance characteristics of such systems is the response time, which mostly depends on the travel time. In this paper we consider some reasonable heuristics for order picking. In particular we establish properties of the Nearest Item (NI) heuristic. We derive tight upper bounds for the travel time and a closed form expression for its mean value.

1 Introduction

A carousel is an automated warehousing system consisting of a large number of shelfs or drawers rotating in a closed loop in either direction. Such systems are used for storage and retrieval of small and medium sized goods. The picker has a fixed position in front of the carousel, which rotates the required items to the picker. The advantage of such systems is that the picker has time for sorting, packing, labeling *etc.*, while the carousel is rotating.

An order is a list of items to be picked. Ideally, the items should be picked in a sequence minimising the total pick time, which is the travel time plus the pure pick time. The latter obviously does not depend on the pick strategy. Hence, we only have to consider the travel time in order to minimise the total pick time.

Bartoldi and Platzman [1] and Stern [3] study the optimal pick strategy for a carousel system. They show that there are only $2n$ candidate sequences, where n is the number of positions to be retrieved. It implies that an optimal route can always be found in linear time. Rouwenhorst *et al.* [2] provide some stochastic upper bounds for the optimal route. Their upper bounds are proved to be rather tight. Nevertheless, neither the probability distribution nor tight upper bounds for the minimal travel time have yet been not obtained.

In their paper Bartoldi and Platzman [1] also consider some simple heuristics for a carousel system. One of these heuristics is the Nearest Item (NI) heuristic, where the next item to be picked is always the nearest one. In particular, the authors prove that the travel distance under the NI heuristic is never greater than one rotation of the carousel. In the present paper we improve this upper bound and we show that the new upper bound is tight. Also, we obtain a closed form expression for the mean travel time under the assumption of uniformly distributed pick positions.

The paper is organised as follows. In the next section we introduce the model and some notation. In Section 3 we study upper bounds for the travel time under the NI heuristic. In particular, we improve an upper bound of Bartoldi and Platzman [1]. In Section 4 we derive a closed-form expression for the mean travel time under the NI heuristic. In the final section we briefly discuss our results.

2 Carousel model

Following Bartoldi and Platzman [1] and Rouwenhorst *et al.* [2] we represent a carousel as a circle of length 1. We suppose that the pick positions are uniformly distributed, and we denote the shortest distance between the positions y and z on a carousel by $\rho(y,z)$ (see Fig. 1). We assume that the acceleration time of the

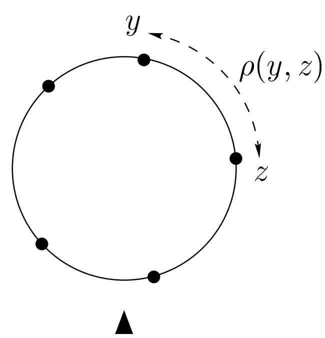

Figure 1: A carousel system.

carousel is negligible or assigned to pick time. Hence, the travel distance can be identified with the travel time.

The presentation will become more clear, when we act as if the picker travels to the pick positions instead of the other way around. In the sequel we shall denote by

$$\omega = (\omega_0, \omega_1, \ldots, \omega_n) \in [0,1)^{n+1}$$

a list of $n+1$ positions, where ω_0 is the start position of the picker and ω_i with $i = 1, 2, \ldots, n$, is the ith pick position. This completes the model description. In the next section we will explore the NI heuristic.

3 NI heuristic: upper bounds

The main object in this section is to establish an upper bound for the NI heuristic and to prove its tightness. The NI heuristic can be described as follows (*cf.* Bartoldi and Platzman [1]):

Step 1: Always rotate to the nearest item to be retrieved.

An important feature of the NI heuristic is that it has the following 'recursive' property:

Property 3.1 *The remaining part of the NI heuristic is equal to the NI heuristic for the rest of the items with the picker's current position as starting point.*

To study the NI heuristic we will compare it with the Shorter Direction (SD) heuristic, which is described in Bartoldi and Platzman [1] as follows:

Step 1: Evaluate the length of the route that simply rotates clockwise, and the length of the route that simply rotates counter-clockwise.

Step 2: Choose the shorter of the two routes from step 1.

By applying the NI heuristic to retrieve a list of n items, the picker will subsequently visit the positions $\omega_{i_1}, \omega_{i_2}, \ldots, \omega_{i_n}$. For convenience we denote

$$x_l = \omega_{i_l},\ l = 1, 2, \ldots, n;\ x_0 = \omega_0.$$

We also introduce the following random variables:

T_n^{NI} – the travel time to retrieve n items under the NI heuristic;

T_n^{SD} – the travel time to retrieve n items under the SD heuristic.

These random variables are of course functions of the elementary random event $\omega \in [0,1)^{n+1}$. Since the NI heuristic seems to be slightly more subtle than the SD heuristic, one can expect that it performs better with high probability. In fact, we will prove that the NI heuristic is never worse than the SD heuristic.

Lemma 3.2 *For any $\omega \in [0,1)^{n+1}$ it holds that $T_n^{NI}(\omega) \leq T_n^{SD}(\omega)$.*

Proof. We will present a proof by induction to n. It is clear that for any $\omega \in [0,1)^2$ we have $T_1^{NI}(\omega) = T_1^{SD}(\omega) = \rho(x_0, x_1)$. Now suppose that for some $n = 1, 2, \ldots$ we have $T_n^{NI}(\omega) \leq T_n^{SD}(\omega)$, $\omega \in [0,1)^{n+1}$. Then we will prove that $T_{n+1}^{NI}(\omega) \leq T_{n+1}^{SD}(\omega)$, $\omega \in [0,1)^{n+2}$. The proof is illustrated in Fig. 2. First, recall that under the SD heuristic the carousel always rotates in the same direction. There are only two possible routes of that kind, and their lengths differ only in the first segment. Therefore, choosing the shorter direction actually means choosing the shorter first interval. Hence, the algorithm for the SD heuristic can be formulated as follows:

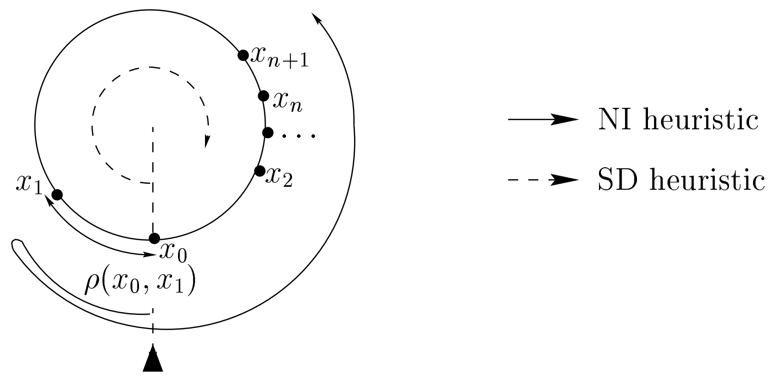

Figure 2: An illustration for the proof of Lemma 3.2.

Step 1: Rotate to the nearest item.

Step 2: Proceed further in the same direction.

It means that the NI and SD heuristic start with the same segment of length $\rho(x_0, x_1)$. After the first step the picker is at position x_1 and n items remain to be picked. Thus, the current situation can be described by $\omega' \in [0,1)^{n+1}$. The remaining travel time under the SD heuristic cannot be shorter than $T_n^{SD}(\omega')$, since by definition $T_n^{SD}(\omega')$ is the minimal travel time needed to pick n items by proceeding in the same direction. Hence,

$$\rho(x_0, x_1) + T_n^{SD}(\omega') \le T_{n+1}^{SD}(\omega). \tag{1}$$

Further, due to property 3.1 we have

$$T_{n+1}^{NI}(\omega) = \rho(x_0, x_1) + T_n^{NI}(\omega'). \tag{2}$$

From (2), the induction assumption and (1) it follows that

$$T_{n+1}^{NI}(\omega) = \rho(x_0, x_1) + T_n^{NI}(\omega') \le \rho(x_0, x_1) + T_n^{SD}(\omega') \le T_{n+1}^{SD}(\omega),$$

which completes the proof. $\square$

In order to pick n items under the NI heuristic, n segments of the carousel should be covered. Their lengths are $\rho(x_0, x_1)$, $\rho(x_1, x_2)$, ..., $\rho(x_{n-1}, x_n)$. Note that they do not necessarily coincide with gaps between two adjacent items, since under the NI heuristic the carousel can rotate in different directions (see Fig. 2). Bartoldi and Platzman [1] showed that T_n^{NI} is always less than 1 for all n. Now we shall use Lemma 1 to prove the following stronger assertion.

Theorem 3.3 *For any $\omega \in [0,1)^{n+1}$ and any $k = 1, 2, \dots, n$, the total length of the k largest segments under the NI heuristic never exceeds $1 - 1/2^k$.*

Proof. Consider the NI heuristics starting in an arbitrary point $x_0 \in [0,1)$. Let $1 \le l_1 < l_2 < \ldots < l_k \le n$ be the indices of the k largest segments in the order we cover them. The sequence of corresponding lengths $\rho(x_{l_1-1}, x_{l_1})$, $\rho(x_{l_2-1}, x_{l_2})$, …, $\rho(x_{l_k-1}, x_{l_k})$ is of course not necessarily monotone.

We proceed with the NI heuristic until facing the first segment l_1. Now the picker is at point x_{l_1-1}, and there are still $n - l_1 + 1$ positions to be visited.

Consider the case that $\rho(x_{l_1-1}, x_{l_1}) \ge 1/2^k$. If we pick the remaining $n - l_1 + 1$ items under the SD heuristic starting at point x_{l_1-1}, then the travel time cannot exceed $1 - 1/2^k$. Then, from Property 3.1 and Lemma 3.2 it follows that the remaining travel time under the NI heuristic also does not exceed $1 - 1/2^k$. Recall that l_1 is the first one of the k largest segments faced under the NI heuristic. Hence, all k largest segments are included in the remaining path. So, their total length cannot be greater than $1 - 1/2^k$.

Now, assume that $\rho(x_{l_1-1}, x_{l_1}) < 1/2^k$. Then we proceed further until interval l_2 is faced. If $\rho(x_{l_2-1}, x_{l_2}) \ge 1/2^{k-1}$, then we can use similar arguments as above to conclude that the total length of the remaining $k-1$ of the k largest segments is not greater than $1 - 1/2^{k-1}$, and it immediately follows that the total length of k largest segments does not exceed

$$\rho(x_{l_1-1}, x_{l_1}) + 1 - 1/2^{k-1} < 1/2^k + 1 - 1/2^{k-1} = 1 - 1/2^k.$$

If $\rho(x_{l_2-1}, x_{l_2}) < 1/2^{k-1}$, then we proceed with the NI heuristic and repeat the same arguments. Finally, two cases are possible:

1. There exists an $i = 2, 3, \ldots, k$ such that $\rho(x_{l_j-1}, x_{l_j}) < 1/2^{k-j+1}$, $j = 1, 2, \ldots, i-1$, and $\rho(x_{l_i-1}, x_{l_i}) \ge 1/2^{k-i+1}$. In this case the remaining path under the NI heuristic is not longer than $1 - 1/2^{k-i+1}$, and therefore the total length of the k largest segments does not exceed

$$\begin{aligned}\sum_{j=1}^{i-1} \rho(x_{l_j-1}, x_{l_j}) + 1 - \frac{1}{2^{k-i+1}} &< \frac{1}{2^k} + \frac{1}{2^{k-1}} + \ldots + \frac{1}{2^{k-i+2}} + 1 - \frac{1}{2^{k-i+1}} \\ &= 1 - \frac{1}{2^k}.\end{aligned}$$

2. For each $i = 2, 3, \ldots, k$ we have $\rho(x_{l_i-1}, x_{l_i}) < 1/2^{k-i+1}$. Then the total length of the k largest segments is less than

$$\frac{1}{2^k} + \frac{1}{2^{k-1}} + \ldots + \frac{1}{2} = 1 - \frac{1}{2^k}.$$

Thus, in both cases the assertion of the theorem holds. □

Since the complete travel time is identical to the total length of the n largest segments, an upper bound for the travel time under the NI heuristic immediately follows from Theorem 3.3.

Corollary 3.4 *For each* $\omega \in [0,1)^{n+1}$ *the travel time under the NI heuristic satisfies*

$$T_n^{NI}(\omega) \leq 1 - 1/2^n. \tag{3}$$

Let us give an example to show that Corollary 3.4 provides a tight upper bound.

Example 3.5 Let $n = 5$, and let the starting position of the picker be $x_0 = 0$. The items to be picked are located at the positions $1/32$, $3/32$, $7/32$, $15/32$ and $31/32 - \varepsilon$, where ε is positive and arbitrarily small (see Fig. 3).

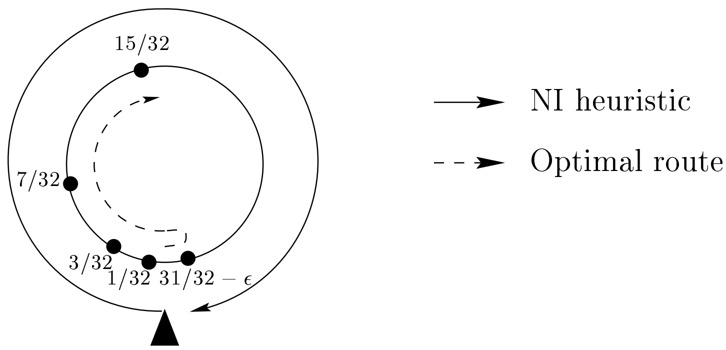

Figure 3: An example for which the travel time is arbitrarily close to the upper bound.

Then the travel distance under the NI heuristic is

$$\frac{1}{32} + \frac{2}{32} + \frac{4}{32} + \frac{8}{32} + \left(\frac{16}{32} - \varepsilon\right) = \frac{31}{32} - \varepsilon = 1 - \frac{1}{2^5} - \varepsilon.$$

The upper bound $1 - 1/2^5$ is tight, since ε is arbitrarily small. A similar example can be easily constructed for any n.

Remark 3.6 In Example 3.5 the travel time does not really achieve its upper bound. However, if the picker starts at point $x_0 = 0$ and needs to pick only one item at point $x_1 = 1/2$, then the travel time is equal to its upper bound $1/2$. For $n > 1$ the upper bound can also be achieved, if we assume that when the travel times to the nearest items clockwise and counter-clockwise are exactly the same, the picker always proceeds, say, clockwise. Now, if we put $\varepsilon = 0$ in the example above, then the travel time will be exactly $1 - 1/2^5$.

Note that Example 3.5 is 'the worst' we can construct. Indeed, from the proof of Theorem 3.3 it follows that if the first segment is smaller or greater than $1/2^n$, then the travel time to pick n items under the NI heuristic is less than $1 - 1/2^n$. The only case when the upper bound can be achieved is when $\rho(x_0, x_1) = 1/2^n$. Then after the first step, the picker is at position x_1 and $n-1$ items remain to be picked. Due to Property 3.1 we can use similar arguments to show that the upper bound can only be achieved if $\rho(x_1, x_2) = 1/2^{n-1}$. The same can be done for each of the n steps under the NI heuristic. It implies that the upper bound can be achieved if and only if the l-th segment has length $1/2^{n-l+1}$ for all $l = 1, 2, \ldots, n$.

Fig. 3 also shows that the NI strategy is sometimes far from optimal. Indeed, in the case under consideration the optimal sequence is: $31/32 - \varepsilon$, $1/32$, $3/32$, $7/32$, $15/32$. The total length of this route is

$$\left(\frac{1}{32} + \varepsilon\right) + \left(\frac{1}{32} + \varepsilon\right) + \frac{1}{32} + \frac{2}{32} + \frac{4}{32} + \frac{8}{32} = \frac{17}{32} + 2\varepsilon,$$

which is much less than $31/32 - \varepsilon$, when ε is small.

Due to (3) we can roughly estimate the probability that the pick sequences under the NI and SD heuristics coincide. In other words, we are speaking about the probability that the carousel never changes direction under the NI heuristic. For example, this always happens when ω is such that $T_n^{SD}(\omega) < 1/2$. The probability of this event is $1/2^{n-1}$. This value can be treated as a rough lower bound for the probability that the NI and SD heuristics coincide. Further, according to (3) the travel time under the NI heuristic can not exceed $1 - 1/2^n$. It means that the NI and SD heuristics can coincide only for ω's satisfying $T^{SD}(\omega) \leq 1 - 1/2^n$. Thus, we have:

$$\frac{1}{2^{n-1}} \leq \text{Prob}\{\text{NI concides with SD}\} \leq 2 \cdot \left(1 - \frac{1}{2^n}\right)^n - \left(1 - \frac{1}{2^{n-1}}\right)^n. \tag{4}$$

For $n = 1$ both bounds in (4) are equal to 1. If n tends to infinity, then the lower bound tends to 0, but the upper bound tends to 1. It is mentioned in Bartoldi and Platzman [1] that both heuristics will coincide with high probability if the item density is large, *i.e.* if n is increasing. Thus, the upper bound in (4) can be used as a rough estimation, when n is large.

4 Mean travel time under the NI heuristic

To derive a formula for the mean travel time under the NI heuristic we will develop a procedure exploiting property 3.1. According to this property the remaining part of the NI heuristic after the first step is equal to the NI heuristic for the other $n-1$ items with the picker's current position as starting point. The expected travel time of the first step can be found straightforwardly. However, the expectation of the remaining travel time is not just the mean travel time under the NI heuristic for $n-1$ items, because we also need to take into consideration the empty space at one side of the picker's position (namely, the first segment, which is clearly without any items to be picked). So, we can obtain a recursive equation for the mean travel time conditioned on the empty space at one side of the picker's position. Denote by $E(T_n^{NI}|t)$ the mean travel time under the NI heuristic, given that at one side of the picker's starting point there is an empty space of size t. Then the mean travel time under the NI heuristic is just equal to $E(T_n^{NI}|0)$:

$$E(T_n^{NI}) = E(T_n^{NI}|0). \tag{5}$$

Our object now is to derive a formula for $E(T_n^{NI}|t)$, $t \in [0,1)$.

The case $t \geq 1/2$ is trivial, since in this case the carousel will rotate in one direction only. It is easy to see that there are n segments to cover, and the average length of each segment is $(1-t)/(n+1)$. Thus, we have:

$$E(T_n^{NI}|t) = \frac{n}{n+1}(1-t), \quad 1/2 \leq t \leq 1. \tag{6}$$

Let us now consider $t < 1/2$. We will derive a recursive equation for $E(T_n^{NI}|t)$ by conditioning on the location of the nearest item. Let $f_n(y|t)$ denote the density of the travel time to the nearest item given that there is an empty space of size t near the starting point. There are two possible cases, which are shown in Fig. 4. For $y \leq t$ we have

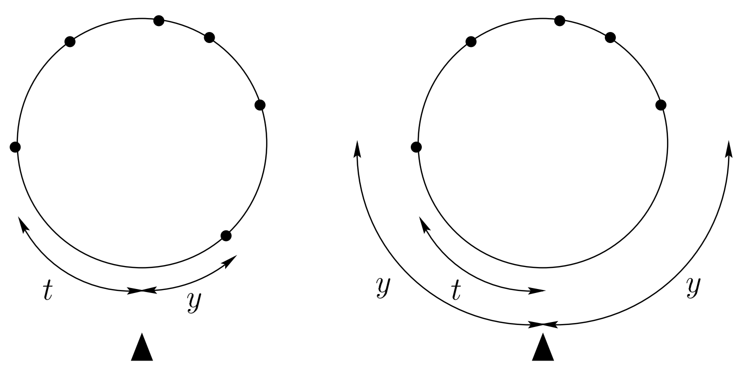

Figure 4: Two possible locations of the nearest item.

$$f_n(y|t) = n(1-t-y)^{n-1}/(1-t)^n,$$

and after this step there will be empty space of size $t+y$. For $t < y < 1/2$ it holds that

$$f_n(y|t) = 2n(1-2y)^{n-1}/(1-t)^n,$$

and after such a step there will be empty space of size $2y$. Now we use the full expectation formula:

$$\begin{aligned} E(T_n^{NI}|t) &= \int_0^t \frac{n(1-t-y)^{n-1}}{(1-t)^n}\left[E(T_{n-1}^{NI}|t+y)+y\right]dy \\ &+ \int_t^{1/2} \frac{2n(1-2y)^{n-1}}{(1-t)^n}\left[E(T_{n-1}^{NI}|2y)+y\right]dy, \quad 0 \leq t < 1/2. \end{aligned} \tag{7}$$

To find a solution for equation (7) we first introduce the functions

$$D_n(t) = E(T_n^{NI}|t)(1-t)^n, \quad n = 0, 1, \ldots.$$

Here

$$D_0(t) = E(T_0^{NI}|t)(1-t)^0 \equiv 0.$$

Also from (6) it directly follows that for all $n = 1, 2, \ldots$ we have

$$D_n(t) = \frac{n}{n+1}\,(1-t)^{n+1}, \quad 1/2 \le t \le 1. \tag{8}$$

Now we can rewrite equation (7) in the following form:

$$\begin{aligned} D_n(t) &= \int_0^t nD_{n-1}(t+y)\,dy + \int_t^{1/2} 2nD_{n-1}(2y)\,dy \\ &+ \int_0^t n(1-t-y)^{n-1}y\,dy + \int_t^{1/2} 2n(1-2y)^{n-1}y\,dy, \quad 0 \le t < 1/2. \end{aligned} \tag{9}$$

The last two integrals in (9) can be easily calculated. Putting $\tau = y + t$ in the first integral and $\tau = 2y$ in the second one, we simplify equation (9) to:

$$D_n(t) = \int_t^1 nD_{n-1}(\tau)\,d\tau + \frac{(1-t)}{n+1}^{n+1} - \frac{(1-2t)}{2(n+1)}^{n+1}, \quad 0 \le t < 1/2. \tag{10}$$

Since $D_0(t) \equiv 0$ we obtain from (10) that

$$D_1(t) = \frac{(1-t)}{2}^2 - \frac{(1-2t)}{4}^2, \quad 0 \le t < 1/2. \tag{11}$$

To proceed we make the following assumption:

$$D_n(t) = a_n(1-t)^{n+1} + b_n(1-2t)^{n+1}, \quad n = 1, 2, \ldots;\ 0 \le t < 1/2, \tag{12}$$

where a_n and b_n are coefficients depending on n. If (12) is correct, then according to (10) and (11) we obtain the recursion

$$a_n = \frac{n}{n+1}\,a_{n-1} + \frac{1}{n+1}; \quad a_1 = 1/2. \tag{13}$$

The coefficients a_n, $n = 1, 2, \ldots$ are uniquely determined by (13), and it is easy to check that $a_n = n/(n+1)$ satisfies (13). The recursion for the coefficients b_n, $n = 1, 2, \ldots$, also follows from (10) and (11):

$$b_n = \frac{n}{2(n+1)}b_{n-1} - \frac{1}{2(n+1)};\ b_1 = 1/4. \tag{14}$$

From (14) we derive

$$2(n+1)b_n = nb_{n-1} - 1.$$

Denote $b_n' = (n+1)b_n$. Then we have

$$b_n' = \frac{1}{2}\,b_{n-1}' - \frac{1}{2}; \quad b_1' = -1/2.$$

Thus,

$$b'_n = \frac{1}{2^{n-1}}\, b'_1 - \sum_{i=1}^{n-1} \frac{1}{2^i} = -\sum_{i=1}^{n} \frac{1}{2^i} = \frac{1}{2^n} - 1,$$

and we finally obtain

$$b_n = \frac{1}{n+1}\left(\frac{1}{2^n} - 1\right), \quad n = 1, 2, \ldots .$$

Thus, assumption (12) implies that

$$D_n(t) = \frac{n}{n+1}(1-t)^{n+1} - \frac{1}{n+1}\left(1 - \frac{1}{2^n}\right)(1-2t)^{n+1}, \quad n = 1, 2, \ldots;\ 0 \le t < 1/2. \tag{15}$$

We now check that (15) indeed satisfies (10). Substitution of (8) and (15) into (10) yields

$$\begin{aligned} D_{n+1}(t) &= \int_t^1 n(1-\tau)^{n+1}\, d\tau \\ &+ \int_t^{1/2} \left(1 - \frac{1}{2^n}\right)(1-2\tau)^{n+1}\, d\tau + \frac{(1-t)^{n+2}}{n+2} - \frac{(1-2t)}{2(n+2)}^{n+2} \\ &= \frac{n+1}{n+2}(1-t)^{n+2} - \frac{1}{n+2}\left(1 - \frac{1}{2^{n+1}}\right)(1-2t)^{n+2}, \quad 0 \le t < 1/2, \end{aligned}$$

which coincides with (15). Our results are summarised in the following theorem:

Theorem 4.1 *For all $n = 1, 2, \ldots$ we have:*

$$E(T_n^{NI}|t) = \begin{cases} \dfrac{n}{n+1}(1-t) - \left(1 - \dfrac{1}{2^n}\right) \dfrac{(1-2t)^{n+1}}{(n+1)(1-t)^n}, & 0 \le t < 1/2; \\ \dfrac{n}{n+1}(1-t), & 1/2 \le t \le 1; \end{cases} \tag{16}$$

$$E(T_n^{NI}) = \frac{n}{n+1} - \left(1 - \frac{1}{2^n}\right)\frac{1}{n+1}. \tag{17}$$

Remark 4.2 We can use the same procedure to obtain higher moments of T_n^{NI}. To obtain the second moment, say, we need to consider the conditional expectation $E([T_n^{NI}]^2|t)$, for which we can find an explicit expression for $1/2 \le t \le 1$ and derive a recursive equation similar to (7) for $0 \le t < 1/2$. The solution for this recursive equation can be found by similar calculations, but the resulting expression becomes definitely more complex.

Let us compare the mean performance of the NI and SD heuristics. One can see (cf. Rouwenhorst *et al.* [2]) that

$$P(T_n^{SD} < t) = \begin{cases} 2t^n, & 0 \le t \le 1/2, \\ 2t^n - (2t-1)^n, & 1/2 < t \le 1. \end{cases}$$

Hence, it is easy to compute that

$$E(T_n^{SD}) = \frac{n}{n+1} - \frac{1}{2}\frac{1}{n+1}.$$

If the carousel just rotates in the same arbitrarily chosen direction, then the mean travel time is clearly $n/(n+1)$, since there are n segments to cover, and $1/(n+1)$ is the average length of each segment. If the SD heristic is applied, then the mean travel time will be reduced by $1/2$ of an average segment. By applying the NI heuristic, we can reduce the mean travel time by a fraction $1-1/2^n$ of an average segment. Obviously, when n is large the difference between different heuristics becomes negligible.

5 Discussion

One can distinguish two main directions for studying the performance of carousel systems. The first one concerns the analysis of the optimal order picking strategy. However, a detailed analysis of probabilistic characteristics of the response time is quite complicated (cf. Rouwenhorst *et al.* [2]).

The other main direction is developing and studying simple heuristics for order picking in carousel systems. In practice they can be very useful, because they provide reasonable control without much (computational) effort. Probabilistic properties of such heuristics sometimes can be obtained analytically. So, in real life one may prefer simple heuristics because (a) they don't require much effort, and (b) their properties are well-understood.

The present paper can be classified in the second direction. We studied in detail the NI heuristic. We provided a tight upper bound for the travel time. Moreover, in Section 4 we developed a procedure to obtain the mean travel time under the NI heuristic. Most likely, the same procedure can also be used for the detailed probabilistic analysis of simple heuristics for order picking (cf. Remark 4.2).

The analysis becomes much more complicated for a system of two carousels operated by one picker. The additional and crucial feature of this system is the presence of switch-over times. Clearly the travel time now significantly depends on the switch-over time. For example, if the switch-over time is close to zero, then it actually reduces to the case of a single carousel system. On the other hand, if the switch-over time is large, then it will strongly effect the travel time. To the best of our knowledge, multiple carousel systems have not been studied in the literature so far. Recent research, however, indicates that the techniques developed in this paper to find a tight upper bound for the travel time under the NI heuristic, as well as its mean value, are also promising for the analysis of simple heuristics for order picking in multiple carousel systems.

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