

The Behavior of Fund Managers with Benchmarks

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ABSTRACT

Recent papers have formulated a model of portfolio choice for a fund manager, as agent for investors who incent the manager to evaluate portfolio returns relative to a benchmark portfolio designated by them. The papers make the ad-hoc assumptions that the manager chooses a portfolio that maximizes a fixed expected exponential utility of the return in excess of the benchmark return, and that returns are normally distributed. In what follows, I dub this the *extant model*.

This paper provides a deeper explanation for both the manager's use of exponential utility and the specific degree of risk aversion used by the manager when choosing a portfolio, whether returns are normally distributed or not. In this deeper model, however, the principals' choice of benchmark influences the manager's degree of risk aversion — an effect that is totally absent in the extant model.

1 Introduction

It is common practice for the performance of a fund manager, acting as an agent on behalf of a group of investors as principal, to be evaluated relative to the returns on some benchmark portfolio designated by the principal. As a result, Brennan [2] and Becker, et.al [1] consider the portfolio choice of a fund manager who maximizes the expected exponential utility of her portfolio's return in excess of the exogenous benchmark's return. The manager's degree of risk aversion is not restricted by the model itself, and is assumed to be independent of the designated benchmark. This model, as detailed in section 2, is dubbed the *extant model* in what follows.

The purpose of this article is to utilize a model of portfolio choice in which the manager *derives* the exponential utility function from a deeper hypothesis, regardless of the functional form of the excess return distribution. The deeper model is quite reasonably based on the manager's desire to maximize the probability of outperforming the average benchmark return over typical contractual time periods. While the model supports the aforementioned papers' use of exponential utility, it does not support their implicit assumption that the manager's degree of risk aversion is independent of the benchmark chosen. *In fact, the model provides the basis for a "Lucas Critique" of those papers: the principals should realize that their choice of benchmark will influence the manager's degree of risk aversion, and that this in turn will affect the portfolio selected by the manager.*

2 The Extant Model

The aforementioned papers' specifications are nested in the following general model: A manager chooses a portfolio p with return R_p that maximizes the following expected exponential (a.k.a. CARA) utility of the difference between its return and a benchmark portfolio's return R_b , i.e.

$$\max_p E[U(R_p - R_b)] = \max_p E[-e^{-\gamma(R_p - R_b)}] \equiv \max_p -\log E[e^{-\gamma(R_p - R_b)}] \quad (1)$$

Although the above logarithmic representation of the problem is unusual, it will prove to be the most useful in what is to follow. Becker, et.al (op.cit) restricted attention to benchmark and manager portfolios composed of a “market portfolio” with return R_m and a riskless asset with return R_f . More formally, they assume that

$$R_b \equiv hR_m + (1 - h)R_f \quad (2)$$

$$R_p \equiv xR_m + (1 - x)R_f.$$

The papers assume that returns are normally distributed. Assuming that the principal’s choice of the specific benchmark portfolio in (2) is governed by standard portfolio theory, the 2-fund separation theorem predicts that the principal will want to maximize expected utility of *own* terminal wealth, and hence will choose some h -weighted combination of the “tangency” portfolio with return R_m and the riskless asset with return R_f . For example, Brennan (op.cit, eqn.(4) shows that the exact h will be inversely related to the ordinary investor’s degree of risk aversion.

But what about the manager, who is forced to use the principal’s benchmark? Despite this difference with standard portfolio theory, the papers make the implied assumption that it applies to the manager as well, with only the argument of the utility function changed. Hence, Becker,et.al substitute R_p in (2) into (1) as well, resulting in:

$$\max_x -\log E[e^{-\gamma(x-h)(R_m - R_f)}] \quad (3)$$

Let $r_m = R_m - R_f$ denote the market portfolio’s return in excess of the riskless return. Note

from elementary statistics that problem (3) requires the manager to find the portfolio weight x that maximizes -1 times the logarithm of the moment generating function (sometimes called the cumulant generating function) of $(x-h)r_m$. The aforementioned papers assume multivariate normal returns, in which case -1 times the log moment generation function is:

$$\max_x -[-\gamma(x-h)E(r_m) + \frac{(-\gamma)^2}{2}(x-h)^2 Var(r_m)] \quad (4)$$

which is a concave maximization problem with a unique solution given by the following first order condition:

$$x = h + \frac{1}{\gamma} \frac{E(r_m)}{Var(r_m)} \quad (5)$$

which is the manager's optimal portfolio derived in Becker, et.al (op.cit, eqn.5).¹

3 The Extant Model's Predictions and Problems

The papers quite reasonably assume that the tangency portfolio m has a positive expected return in excess of the riskless rate (i.e. the market risk premium) $E(r_m) > 0$, in which case (5) shows that $x > h$, i.e. the manager will choose to place a higher weight on the risky asset portfolio. To see how much higher, Becker, et.al (op.cit, p. 123) claim that $E(r_m)/Var(r_m)$ has "a typical magnitude of approximately equal to two." Substituting this value into (5), a manager with a degree of risk aversion γ equal to, say, 4 will choose $x = h + 2/4$, i.e. the manager will commit a much higher proportion of managed funds (50 percentage points!) to the tangency portfolio m . Unless the manager's degree of risk aversion γ is extremely high, the manager will choose a substantially riskier portfolio than the benchmark used to evaluate managerial performance. Only in the limiting case of infinite risk aversion will the two portfolios be the same. While this may

seem obvious to readers of this paper, unfortunately Becker, et.al (op.cit, p.123) make the following misleading claim:

Thus, benchmark investors are highly risk averse to deviations from the benchmark, and we expect to find estimates of γ for benchmark investors to be large relative to the conventional standards of risk aversion.

Correcting this statement ² requires the following changes, in italics:

Thus, benchmark investors *who have a high degree of risk aversion* γ are highly averse to deviations from the benchmark, and we *have no reason, a priori, to* expect to find estimates of γ for benchmark investors to be large relative to the conventional standards of risk aversion.

In summary, the extant model of portfolio choice does not restrict the manager's degree of risk aversion. But the alternative model in the following section will both justify the manager's use of exponential utility, and *will* restrict the manager's degree of risk aversion.

4 A Rationale For The Manager's Utility Function and Degree of Risk Aversion

Stutzer [7] provides a rationale for the manager's use of the exponential utility function in (1), and also for the specific degree of risk aversion γ used by the manager, *even when the returns are not normally distributed*. This last generalization of the extant model is important, because without normally distributed returns, there is no motivation for restricting the portfolio choices to the form in (2).

In brief, Stutzer (op.cit) posits that the manager seeks to maximize the probability of outperforming the time-averaged return of the benchmark portfolio, i.e. the manager seeks to minimize the probability of realizing a nonpositive time-averaged portfolio return in excess of the benchmark portfolio return.

The development there will now be applied, for the first time, to the extant model. I follow the extant model in assuming that it is possible to find a portfolio p with expected return higher than the benchmark portfolio's expected return (i.e. $E(R_p - R_b) > 0$). Under the extant model's restriction (2) and reasonable assumption that $E(r_m) > 0$, this implies that the manager will choose $x > h$. Then, the probability that portfolio p will realize a finite time averaged portfolio return less than or equal to the benchmark's decays to zero, at a positive exponential rate, as time progresses to infinity.³ Stutzer (op.cit) argued that a manager who is worried about earning a time averaged portfolio that is less than or equal to the benchmark's, over the uncertain length of time that managers are typically under contract with the principal, should choose a portfolio that makes this probability decay rate as large as possible. Of course, doing so will *maximize* the probability of realizing a time averaged portfolio return that will *exceed* the benchmark portfolio's.

One might guess that this apparent stress on the *probability* of time-averaged outperformance, rather than on both the probability and *size* of the outperformance, will lead to implausible portfolio choice rules. But Stutzer (op.cit) showed that this fear is groundless. Using a simple result called Cramer's Theorem [3, pp.7-14] in a very straightforward fashion, Stutzer (op.cit) reported that this *decay rate maximization hypothesis* of manager behavior is equivalent to maximizing (1) over *both* the space of portfolios p and *-1 times the degree of risk aversion* γ ! With the notation used in (1), use the third expression there to express the decay rate maximization hypothesis as :

$$\max_p \max_{-\gamma} -\log E[e^{-\gamma(R_p - R_b)}] \quad (6)$$

where the inner maximization over $-\gamma$ determines the aforementioned probability decay rate for the portfolio p . Of course, the second expression in (1) shows that the decay rate maximizing portfolio p that solves (6) may also be found by the same joint maximization of the expected exponential utility of $R_p - R_b$.

When $R_p - R_b$ has the normal distribution that motivated Becker, et.al (op.cit) to adopt the two-fund special formulation (2) and hence the extant model (4), the special case of (6) is just to maximize (4) over both x and $-\gamma$, i.e. the manager solves:

$$\max_x \max_{-\gamma} [-\gamma(x - h)E(r_m) + \frac{(-\gamma)^2}{2}(x - h)^2 Var(r_m)] \quad (7)$$

The first order condition for the inner maximization over $-\gamma$ yields:

$$\gamma = \frac{1}{x - h} \frac{E(r_m)}{Var(r_m)} > 0 \quad (8)$$

which is positive because $x > h$. Substituting (8) into (7) and simplifying yields the decay rate for the probability that the portfolio with weight x will realize a time averaged normally distributed return less than the benchmark portfolio with weight h . Hence the manager maximizes this decay rate, yielding:

$$\max_x \frac{1}{2} \frac{E(r_m)^2}{Var(r_m)} = \frac{1}{2} \left(\frac{E(R_m - R_f)}{\sqrt{Var(R_m)}} \right)^2 \equiv \frac{1}{2} \lambda_m^2 \quad (9)$$

From (9), we see that in the extant model (4), the aforementioned decay rate for *any* portfolio x is half the squared Sharpe Ratio (λ_m) of the market portfolio with excess return r_m . To understand this result, note that the argument in the extant model's utility function (3) is the excess return

$(x-h)r_m$. The ratio of its expected value $(x-h)E(r_m)$ to its standard deviation $(x-h)\sqrt{Var(r_m)}$, i.e. its Sharpe Ratio, is independent of the portfolio weight x . A manager who wants to ensure that her portfolio will outperform the average return of the benchmark over the contract period should maximize this Sharpe Ratio, because a high numerator obviously helps increase the probability of a high average return in excess of the benchmark, while a low denominator helps prevent the possibility of volatility-induced low returns that drag the average return below the benchmark's.

So, in accord with the extant model, the decay rate maximization hypothesis *does* predict that the manager will restrict risky asset investments to the tangency portfolio with return R_m and maximum Sharpe Ratio λ_m , and that the fraction of managed funds x devoted to the tangency portfolio is greater than the benchmark portfolio's fraction h .⁴ But the hypothesis does not predict that the manager will *necessarily* choose the allocation x in (5), although in this special case where returns are normally distributed, the manager would not be averse to choosing the x given by (5).⁵ The latter prediction of the extant model does not take account of the possibility that the principal's choice of benchmark could change the manager's degree of risk aversion, as the deeper model does.

In summary, the extant model assumed (i) normally distributed returns, (ii) that both the principal's designated benchmark portfolio *and* the manager's portfolio could be restricted to the forms in (2), and (iii) that the manager evaluates returns in excess of the benchmark using an exponential (i.e. CARA) utility function. Adopting just the assumptions (i) and (ii), the decay rate maximization hypothesis implied (iii), and is thus a deeper model. But it also implied a Lucas Critique of the extant model: the manager reacts to the principal's choice of benchmark (under (2), this is determined by h) by changing the degree of risk aversion γ used to evaluate portfolio returns in excess of the benchmark's. In the extant model, principals do not take account of this

possibility, and hence do not realize that the manager's only concern will be the Sharpe Ratio of the managerial utility function's argument, which won't depend on the particular $x > h$. Principals in the extant model are unwarrantly optimistic about their ability to induce the manager to choose the specific x given by (5).

The critique will be more specific in more realistic cases where returns are nonnormally distributed, and/or where portfolios are not restricted as in (2). To understand why, let us examine the Taylor expansion of the log moment generating function in (1), producing:

$$-\log E[e^{-\gamma(R_p - R_b)}] = \kappa_1\gamma - \kappa_2\gamma^2/2 + \kappa_3\gamma^3/3! - \kappa_4\gamma^4/4!\dots \quad (10)$$

The coefficient κ_i in (10) is the i -th cumulant of $R_p - R_b$. In the extant model, returns are normally distributed, and using (2) we derived $\kappa_1 = (x - h)E(r_m)$, $\kappa_2 = (x - h)^2 Var(r_m)$, and all the higher order cumulants of the normal distribution are zero, resulting in the problem (7). But when returns are *not* normally distributed, higher order cumulants will appear in (10), changing the solution of (6). Because of the alternating signs in (10), decay rate maximizers, who maximize (10), will exhibit a preference for choosing portfolios p so that $R_p - R_b$ will have higher skewness (contributing to a higher value of κ_3) as well as higher values of the other odd-order cumulants. Of course, the opposite is true for the even-order cumulants (like κ_4). Ceteris paribus, such portfolios lower the probability that the manager will realize a time averaged portfolio return that does not exceed the benchmark's.

Because these results follow from the use of exponential utility, neither the principal's benchmark portfolio nor the manager's portfolio should be restricted by (2) in the presence of non-normally distributed returns.

The following illustrative example shows that the decay rate maximization hypothesis gives

sensible answers, and is easy to implement nonparametrically.

5 Empirical Comparison

Following both Brennan (op.cit) and Becker, et.al, we assume that the equity portfolio is an index of large stocks, i.e. the S&P 500 index portfolio. In addition, we allow a fixed income investment by obtaining a corresponding series of returns for long-term government bonds. For the sole purpose of fostering comparison with Becker, et.al (op.cit), the portfolio of risky assets used to form the benchmark (i.e. the “market” portfolio) is the tangency portfolio of the stocks and bonds. But due to the possible presence of non-normalities, the manager will be allowed to choose a portfolio of risky assets that differs from the tangency portfolio.

Following Kroll, Levy and Markowitz [5] and general econometric practice, the required expected exponential utilities are estimated by replacing the expectation operator with its sample average, using Ibbotson Associates’ returns measured annually from 1926-1996 ($T = 71$ years). Accordingly, the riskless return is chosen to be the average annual Treasury Bill return over the same period, reported by Ibbotson Associates to be $R_f = .038$. Formally, let R_{st} denote the large stock return in year $t = 1, \dots, 71$, while R_{gt} denotes the long-term government bond return. Then an estimate of the decay rate maximizing portfolio (6) is:

$$\max_{w_s, w_g} \max_{-\gamma} -\log \frac{1}{T} \sum_{t=1}^{T=71} e^{-\gamma(w_s R_{st} + w_g R_{gt} + (1-w_s-w_g)R_f - (h R_{mt} + (1-h)R_f))} \quad (11)$$

where $R_f = .038$ and R_{mt} is the return from the estimated tangency portfolio of stocks and bonds.

In Table 1, this decay rate maximizing portfolio is contrasted with its corresponding benchmark portfolio for each h , in order to re-examine the misleading claim made by Becker, et.al (op.cit,

p.123). The first line in Table 1 is the benchmark portfolio when the fraction $h = 1$ is invested in the risky assets, i.e. the tangency portfolio of stocks and bonds that maximizes the Sharpe Ratio. The tangency portfolio invests 64% of funds in stocks and 36% in bonds, similar to many investment advisors' recommendations. Using this annual data set, the stocks appear to be close to normally distributed, with a slight negative skewness of $-.31$ and almost no kurtosis, while the bonds have a desirable positive skewness of 1.49 but an undesirable positive kurtosis of 2.98 . While the slight degree of undesirable negative stock skewness is inherited in the (tangency) benchmark portfolio, the conflicting effects of the bonds' modest desirable skewness and undesirable kurtosis will help keep the allocation of stocks to bonds, *relative to the total investment in the two*, close to that of the tangency portfolio. But the actual allocation weights for stocks and bonds will be substantially different from the tangency portfolio's, due to the presence of the riskless asset. For example, with $h = 1$, the decay rate maximizing portfolio shorts the riskless asset to invest $86 + 51 = 137\%$ of its own funds in the risky assets. But while this is $x - h = 37$ percentage points more than the benchmark invests in the risky assets, the decay rate maximizing portfolio of the *risky* assets has a stock weight of $86/137 = 62.7\%$ with the rest (37.3%) invested in bonds. Relative to the tangency portfolio, the slightly higher relative allocation to bonds is caused by the dominant effect of the slightly negative skewness of stocks ($-.31$) and positive skewness of bonds (1.48), despite the latter's undesirably higher kurtosis.

Examining Table 1 from top to bottom, we see that as the benchmark fraction h allocated to the tangency portfolio decreases, column 3 shows that the riskless asset position changes from short to long, the weight reaching 25% when the benchmark is the riskless asset return ($h = 0$).

Column 4 shows that $x - h$, which is the fraction of funds devoted to the risky assets in excess of the benchmark's fraction, successively increases, reaching 75 percentage points when $h = 0$.

Column 6 shows that the endogenous degree of risk aversion successively decreases, from 9.05 when $h = 1$, down to 4.4 when $h = 0$. So like the extant model's prediction (5), $x - h$ is still inversely related to (the now endogenous) γ . But unlike the extant model, if the principal chooses a lower h , the manager acts as-if she had a higher degree of risk aversion γ , and will hence will choose a risky asset allocation weight x closer to h than one would predict using the lower degree of risk aversion.

Column 5 shows that the relative under allocation to stocks in the *risky* asset portfolio of stocks and bonds becomes more pronounced, dropping to 58.7% when $h = 0$. So in this data set, the relative allocation of stocks in the manager's risky asset portfolio does not stray more than about 5 percentage points from the allocation in the principals' tangency portfolio. But differences would be more pronounced in data sets where some assets' returns are more heavily skewed. For example, the manager might want to purchase.(positively skewed) protective put options on some of the risky asset holdings (portfolio insurance) when the benchmark does not include them.

Finally, it is useful to note that the decay rate maximization hypothesis can be easily extended to cover the case where the return process is stationary and ergodic, as long as the process satisfies regularity conditions sufficient to prove Ellis' [4] Theorem (also see Bucklew [3, pp.20-22]). This result, used for alternative purposes in Stutzer [6, Appendix], substitutes a different function for the log moment generating function. But under the not unreasonable restriction that returns are not identically distributed but still are independent, the estimator (11) is still sensible.

Portfolio R_b vs. R_p	Stock %	Bond %	Riskfree %	$x - h$ %	Stocks \div Stocks + Bonds	Endogenous Risk Aversion
$h = 1$	64	36	0		64	
Rate Max	86	51	-37	37	62.7	9.05
$h = .8$	51	29	20		64	
Rate Max	75	45	-20	40	62.5	8.27
$h = .6$	39	21	40		64	
Rate Max	65	49	-5	45	61.9	7.24
$h = .4$	26	14	60		64	
Rate Max	57	36	7	53	61.3	6.25
$h = .2$	13	7	80		64	
Rate Max	49	33	18	62	59.8	5.27
$h = 0$	0	0	100			
Rate Max	44	31	25	75	58.7	4.40

TABLE 1: Comparison of Benchmark Portfolio, with Fraction h in Riskless Asset, to the Corresponding Decay Rate Maximizing Portfolio. The Investment Opportunity Set and h Restrict the Endogenous Degree of Risk Aversion.

6 Conclusions

Recent models of a fund manager's portfolio choice have posited that investors as principal will designate a benchmark portfolio, and that the manager will evaluate the expected exponential (i.e. CARA) utility of portfolio returns in excess of this benchmark's return. The exponential utility function can be *derived* from the deeper alternative hypothesis that a fund manager strives to maximize (minimize) the probability that the chosen portfolio return will (not) outperform the designated benchmark return *on average* over the years the contract is in place. But in this deeper model, the manager's degree of risk aversion is not independent of the principal's choice of benchmark. It is determined by *jointly* maximizing the expected exponential utility over *both* the space of portfolios *and* -1 times the degree of risk aversion, and hence depends on the investment

opportunity set and the designated benchmark's returns.

Contrary to the contention of Becker, et.al (op.cit, p.123), the extant model does not restrict the degree of risk aversion in any way. But this alternative model does. Using illustrative empirical data and a range of benchmarks considered by Becker, et.al (op.cit), the alternative hypothesis restricted the degree of risk aversion to lie between 4 and 9, depending on the specific benchmark used.

Notes

¹In addition, Becker, et.al developed an estimable model that permits the manager to make use of conditioning information. But the Lucas Critique developed herein does not depend on the manager's use of conditioning information. So I follow Brennan (op.cit) in assuming a simple IID returns environment.

²The problem with their argument is their assumption that the manager will always choose a portfolio p so that $E(R_p)/Var(R_p) \approx E(R_m)/Var(R_m)$. In the extant model, this will only occur when the manager's degree of risk aversion is unconventionally high.

³Regularity conditions on the return distribution, needed to ensure exponential decay of that probability, are given in Bucklew [3, pp.7-14].

⁴Recall that $x > h$ ensures that the manager's portfolio has a positive expected excess return over the benchmark, enabling the manager to find a portfolio that outperforms the benchmark on average, so that both the underperformance probability decay rate and the endogenous coefficient of risk aversion are positive.

⁵We will see in section 5 that when returns aren't normally distributed, the decay rate maximization hypothesis will make a sharp prediction about the specific value of x chosen by the manager.

References

- [1] Connie Becker, Wayne Ferson, David H. Myers, and Michael J. Schill. Conditional market timing with benchmark investors. *Journal of Financial Economics*, 52(1):119–148, 1999.
- [2] Michael Brennan. Agency and asset pricing. Finance Working Paper no.6-93, Anderson Graduate School of Management, University of California, Los Angeles, 1993.
- [3] James A. Bucklew. *Large Deviation Techniques in Decision, Simulation, and Estimation*. Wiley, 1990.
- [4] Richard S. Ellis. Large deviations for a general class of random vectors. *Annals of Probability*, 12(1):1–12, 1984.
- [5] Yoram Kroll, Haim Levy, and Harry Markowitz. Mean-variance versus direct utility maximization. *Journal of Finance*, 39:47–61, 1984.
- [6] Michael Stutzer. A Bayesian approach to diagnosis of asset pricing models. *Journal of Econometrics*, 68:367–397, 1995.
- [7] Michael Stutzer. A portfolio performance index. *Financial Analysts Journal*, 56(3):52–61, 2000.