Optimal Portfolio Allocation Under a Probabilistic Risk Constraint and the Incentives for Financial Innovation∗

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Abstract

We derive, in a complete markets environment, an investor’s optimal portfolio allocation subject to both a budget constraint and a probabilistic risk constraint. We demonstrate that the set of feasible portfolios need not be connected or convex, while the number of local optima increases exponentially with the number of securities implying that finding the optimal portfolio is computationally complex (NP hard). The resulting optimal portfolio allocation may not be monotonic in the state–price density. A novel type of financial innovation, which splits states of nature, is shown to weakly enhance welfare, restore monotonicity in the state–price density, and may reduce complexity.

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