1. Introduction

The discrepancy between the maximum willingness to pay for a good (WTP) and the minimum compensation demanded to part from the good (WTA) is a robust empirical observation in economics [Kahneman, Knetsch, and Thaler, 1990, 1991]. The measures of WTA exceeding the measures of WTP, however, cannot be explained by the neo-classical framework with smooth utility function. The leading alternative explanations for the difference between WTP and WTA are the "endowment effect" [Thaler, 1980] which captures the overvaluation of a good due to possession of it; the "status quo bias" [Samuelson and Zeckhauser, 1988] which is the preference to remain at a current state; and "prospect theory" [Kahneman and Tversky, 1979] where losses impact the agent's utility more than gains of the same magnitude. According to Kahneman, Knetsch and Thaler (1991, pg. 205), "after more than a decade of research on this topic we have become convinced that the endowment effect, status quo bias, and aversion to losses are both robust and important." A large empirical literature, drawing on surveys and psychological experiments, confirms the existence of those effects in many different fields.[1]

The purpose of this paper is not to dispute the fact that the disparity between WTP and WTA exits but to provide an alternative explanation for these observations within a rational expectation framework. Although both status quo and prospect theory could be explained by our approach, we focus on endowment effect. Specifically, the endowment effect that we have in mind is in respect to initial positions in the risk probability value (Viscusi, Magat and Huber, 1987) and not in respect to income level.[2] We show that a

[1] See among many other Olsen (1997) for confirming prospect theory in the case of investment managers' risk perception, Benartzi and Thaler (1995) for addressing the equity premium, Thaler (1980), Thaler et al (1997) in compensating risk, Bowman, Minehart, and Rabin (1999), Bateman et al (1997) in household consumption pattern, Hoorens et al (1999) for wage differentials by training, van Kijk and van Knippenberg (1996, 1998), Franciosi et al (1996) for the offer/ask price disperity in exchange goods.

[2] We thank Kip Viscusi for this clarification.

framework where rational agents face asymmetric information can also explain the wedge between ask and bid prices without invoking psychology.[3]

This paper is motivated by Glosten and Milgrom (1985), who study the pricing strategy of an uninformed market maker facing potentially better informed traders. Their approach provides a good way of framing compensation issues within a rational-expectations setting. In their framework, asymmetric information leads to a gap between the market maker's bid and ask prices. Our approach, however, differs from that of Glosten and Milgrom (1985) in several ways. First, they assume the existence of an equilibrium whereas we characterize the conditions for the existence of an equilibrium in our framework. Second, the informed trader's valuation of the asset in their model incorporates a stochastic time discount factor, θ. Necessary conditions for the existence of an equilibrium are fairly restrictive. For example, the equilibrium ask price fails to exist if the informed trader observes θ and $\theta \leq 1$ (see Appendix).

To relate the WTP/WTA to the bid/ask literature, we use an endowment economy where one asset is traded. One agent posts prices at which he is ready to buy or sell the asset, the other decides on which side he wants to trade after observing the posted prices. We first posit that the agent posting the prices is uninformed while the other agent possesses private information on the value of the traded asset. Later, we generalize this framework and let both agents have private information. By mapping the market maker's bid/ask framework into the WTP/WTA discrepancy literature, the paper is able to focus on the compensation issues using asymmetric information without using psychology-based explanations.

[3] "It is also quite possible that the rational disparity between WTP and WTA gets learned by us as a heuristic, and that we apply it in other areas where it is not appropriate. This is what Kahneman and Tversky (1974) call a heuristic, which is a decision tool that is fine for some areas but often extrapolated to others where it does not belong." (anonymous referee). We thank the referee for this remark.

Our results can be summarized as follows. With asymmetric information, the uninformed trader thinks that the informed trader with whom he might trade has private information about the true risks. When communicating the prices at which he is willing to trade, the agent takes into account that the informed trader will trade assets only at a profit. This leads to a wedge between the uninformed trader's buying and selling prices. This is analogous to the discrepancy between the WTP and WTA documented by Thaler (1980). When Thaler (1980) asks people directly how much they would be willing to pay to eliminate a one in a thousand risk of immediate death, and how much they would have to be paid to willingly accept an extra one chance in a thousand of immediate death, he reports that a typical answer was "I wouldn't pay more than 200 dollars, but I wouldn't accept an extra risk for $50,000$ dollars."[4] The difference between the prices at which individuals are willing to take on more risk or reduce risk can be interpreted as a particular case of WTP/WTA discrepancy.

The next section introduces the model. Section 3 analyses the valuation gap when only the informed trader has private information. Also in section 3, we use numerical examples as well as an insurance example to illustrate our results. Section 4 looks at the case where both the informed trader and uninformed trader have private information to explain the valuation gap. Section 5 concludes the paper.

[4] Jones-Lee et al (1985) has a similar survery results. Using a sample of more than a thousand individuals, people are told to imagine they have to travel, and are presented three options; they can use the company-paid coach firm, in that case the risk of death would be 8 in $100,000$. They can choose a safer coach firm, at an extra cost for them, or they can keep some of their company's travel expenses for themselves while using a less safe coach firm. The safer alternative would entail a risk of death of 2 to 4 in $100,000$, the riskier alternatives a risk of death of 16 to 32 in $100,000$. All of the people polled were willing to pay something to increase safety, but only 19 percent were willing to increase risk to save money.

2. Model

The model features a static endowment economy where an informed agent and an uniformed agent trade an asset whose value z depends on a state variable X taking values 0 and 1. The informed trader observes a signal G correlated with X. The uniformed agent posts prices at which he commits to buy or sell units of the traded asset. The agents have rational expectations in the sense that they know the joint distribution of X and G and optimally use this information. The goal is to determine the equilibrium price. The absence of trade arises as an equilibrium outcome if the informed trader is not willing to trade at the prices proposed by the uninformed trader. The agents' objective functions are as follows.

- *Informed trader :*

  The informed trader observes a private signal G and is willing to trade with the uninformed trader if his conditional expected profit is positive. Let π be the informed trader profits, p the asset price, and Q^s the amount of assets supplied (Q^s can be negative). The informed trader's expected profit is

$$E[\pi|G] = (p - v)\, Q^s. \tag{2.1}$$

  where, $v = E\left[z\left(X\right)|G\right]$. If $p > v$, the informed trader would be willing to supply an infinite positive amount of assets. If $p < v$, the informed trader would be willing to supply an infinite negative amount of assets. If $p = v$, the supply of assets is undetermined.

- *Uninformed trader:*

The uninformed trader takes into account that he will be able to buy (resp., sell) only if the informed trader is willing to sell (resp., buy). It may seem natural to make the informed trader the one that initiates the trade but it is in no way necessary. As long as the uninformed trader believes that his possible counterparty is better informed than he is, the results of the paper hold. Let $F = sign(p - v)$. The uninformed trader wants to maximize his expected utility function

$$E(U) = E\left[u\left(e + (z - p)\, Q^d\right) |F\right], \tag{2.2}$$

where u is his utility function and Q^d is his demand. The uninformed trader takes into account that $F \geq 0$, if and only if $p \geq v$.

3. Private Information for one trader only

We begin our analysis of the valuation gap with the case when only the informed trader has some private information. The objective function of a risk-neutral uninformed trader is;

$$E(U) = E[e|F] + (E[z|F] - p)\; Q^d \tag{3.1}$$

Let's analyze the price at which the uninformed trader is willing to purchase assets. First, he takes into account that the informed trader is willing to sell only if $v \leq p$. Hence the uninformed trader's valuation in this case is $E[z|v \leq p]$, which is inferior or equal to p. If p is strictly greater than $E[z|v \leq p]$, the uninformed trader will not buy assets. Hence the only possible price at which the uninformed trader might be willing to purchase assets is $E[z|v \leq p]$. Symmetrically, the only possible price at which the uninformed trader might be willing to sell is $E[z|v \geq p]$. Such a price might not always exist. Conditions for existence are given below.

Proposition 1 *Let Y and G be two random variables and I be the indicator function. Write $v = E[Y|G]$, $\underline{v} = inf(v)$, and $\overline{v} = sup(v)$. Define H on $(-\infty, \overline{v})$ and L on $(\underline{v}, +\infty)$ by $H(p) = E[Y|v \geq p]$ and $L(p) = E[Y|v \leq p]$.*[5] *Then,*

1. $\underline{v} \leq L(p) \leq p \leq H(p) \leq \overline{v}$,
2. *$H(p)$ and $L(p)$ are non-decreasing functions of p,*
3. $H \to \overline{v}$, *when* $v \to \overline{v}$, $L \to \underline{v}$, *when* $v \to \underline{v}$.

Proof of Proposition 1: See Appendix

Proposition 2 *If $\overline{v}$ is finite, the function H on $(-\infty, \overline{v}]$ has a fixed point if and only if v reaches $\overline{v}$ with positive probability. If $\underline{v}$ is finite, the function L on $[\underline{v}, +\infty)$ has a fixed point if and only if v reaches $\underline{v}$ with positive probability. If $\overline{v}$ (resp., $\underline{v}$) is not finite H (resp., L) has no fixed point.*

Proof of Proposition 2: See Appendix

Proposition 1 and 2 can be summarized as follows. If v attains its minimum $\underline{v}$ with positive probability then $\underline{v}$ is the unique fixed point of L. At $\underline{v}$, the uninformed trader is willing to buy some assets. If v attains its maximum $\overline{v}$ with positive probability, then $\overline{v}$ is the unique fixed point of H. At $\overline{v}$, the uninformed trader is willing to sell. If v attains its boundaries with positive probability, then there are two prices (and two prices only), at which the uninformed trader is willing to transact; namely, $\overline{v}$ and $\underline{v}$. The uninformed trader's selling price is always higher than his buying price. In fact, the uninformed trader is willing to buy at the minimum possible price the informed trader would accept ($\underline{v}$), and

[5] with $\overline{v}$ included in the H's domain if $\overline{v}$ is finite, and $\underline{v}$ included in L's domain if $\underline{v}$ is finite.

he is willing to sell at the maximum possible price the informed trader would agree ($\overline{v}$). Even if H and L have no fixed points, $\overline{v}$ and $\underline{v}$ are close to being the uninformed trader's buying and selling prices in the sense that $H(p)$ converges to $\overline{v}$ as p approaches $\overline{v}$ and $L(p)$ converges to $\underline{v}$ as p approaches $\underline{v}$.

The informed trader will transact only if his marginal profit is non-negative. Trade is possible only when the realization of v hits the upper or lower bounds of its distribution, then, both uninformed trader and informed trader are indifferent to the amount of assets exchanged. This result is akin to the non-speculation theorem of Kreps (1977) which shows that rational, risk-neutral agents, each having private information on the real value of the traded assets but sharing common priors about the distribution of uncertainty cannot be worse off if they refused to trade.[6] For exposition purpose, we present insurance and numerical examples for the valuation gap analysis in the following three subsections.[7]

3.1. Example 1: Insurance

To model insurance in this framework, let e_1(resp., e_0) be the endowment at "low" (resp., "high") state, where $e_1 < e_0$. Let $X = 1$ (resp., $X = 0$) denotes the "low" or "accident" state and let z pay \$1 in that state and \$0 in the other. The insurance premium p is the price of the contract and is paid in all states of the world. The informed trader acts as the insurer and the uninformed trader as the insured. Past literature has often considered that the insured enjoys some informational advantage over the insurer. This may be the case for some insured's idiosyncratic risk factors hidden from the insurer. However, insurance companies have the resources and the incentives to obtain precise estimates of the risk

[6] This result is also known in the asymmetry information literature as the "no-trade theorem" (see, Chatterjee and Samuelson, 2001).

[7] We take the insurance example as this is most natural and direct extension to our theoretical model. One could, however, also apply our framework to other exchanged goods; for example, wine auction (see, van-Dijk and van-Knippenberg, 1996, 1998).

they take. Indeed, in the long run, only insurance companies that do so will survive in a competitive market place.

Our paper features an endowment economy where the insured can increase or decrease his coverage. Moreover, the insurer does not ask the agent how much he would pay to reduce the risk of death by a constant probability. Rather, the agent is asked at which prices he is willing to change his coverage. The insured does not trade a higher or a lower risk of death or accident, instead, he trades amount of assets, which he can increase or decrease. The insured communicates the price at which he is willing to increase or decrease his coverage.

With some minor changes presented above, the general set up can be modified so that the objective functions for the insurer and insured are as follows.

- *Insurer:*

In the low state, the insurer pays out Q^s, while in both states, he gets $p\,Q^s$. The insurer's expected profit is

$$E[\pi|G] = (p - P(X = 1|G))\,Q^s. \tag{3.2}$$

where Q^s is the amount of insurance supplied. For the insurance example, the informed trader's valuation is $v = P(X = 1|G)$. If $p > P(X = 1|G)$, the insurer would be willing to supply an infinite positive amount of insurance. If $p < P(X = 1|G)$, the insurer would be willing to supply an infinite negative amount of insurance. Finally, if $p = P(X = 1|G)$, the supply of insurance is undetermined.

- *Insured:*

Let $F = sign(p - P(X = 1|G))$. The consumer wants to maximize his expected utility function

$$E\,(U) = P(X=0|F)\ \ u(e_0 - p\ Q) + P(X=1|F)\ \ u(e_1 - p\ Q + Q) \tag{3.3}$$

where u is the insured utility function. The insured takes into account that $F \geq 0$, if and only if $p \geq P(X=1|G)$.

When only the insurer has some private information then the objective function of a risk-neutral insured is

$$E\,(U) = E[e|F] + [P(X=1|F)\ (1-p) - (1 - P(X=1|F))p]\ Q^d \tag{3.4}$$

The insured takes into account that the insurer is willing to sell only if $P(X=1|G) \leq p$. Hence, the conditional probability of accident that the insured will use in this case is $P(X=1|P(X=1|G) \leq p)$, which is inferior or equal to p. If p is strictly greater than $P(X=1|P(X=1|G) \leq p)$, the insured will not buy insurance. Hence the only possible price at which the insured might be willing to purchase insurance is $p = P(X=1|P(X=1|G) \leq p)$. Symmetrically, the only possible price at which the insured might be willing to decrease his coverage is $p = P(X=1|P(X=1|G) \geq p)$.

3.2. Example 2: Bernoulli signal

Suppose that the insurer's signal G takes the values 0 and 1.

Proposition 3 *The gap between the price at which the insured accepts to lower his coverage and the price at which he is willing to increase his coverage is decreasing in the variance of G, increasing in the variance of X, and increasing in the correlation between G and X.*

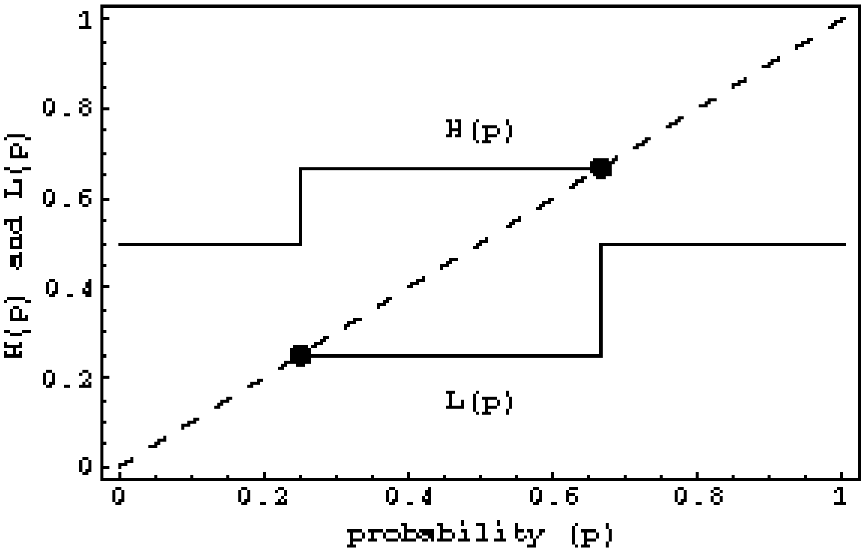

Figure 3.1: $L(p)$ and $H(p)$ when X and G are jointly Bernoulli with $P(X=1, G=1) = .4$, $P(X=1, G=0) = .1$, $P(X=0, G=1) = .2$, and $P(X=0, G=0) = .3$.

Proof of Proposition 3: See Appendix

Put another way, the gap between the insured's selling and buying prices is an increasing function of the precision of the insurer's private information, measured by the correlation between X and G. This makes sense, the better informed the insurer is, the worse the adverse selection problem is for the insured. The latter wants to protect himself by increasing the gap between buying and selling prices. Figure 3.1 graphs the function $H(p)$, and $L(p)$, when $P(X=1, G=1) = .4$, $P(X=1, G=0) = .1$, $P(X=0, G=1) = .2$, and $P(X=0, G=0) = .3$. In that case $P(X=1|G)$ can take two values, $1/4$ and $2/3$. That is, there are two fixed points with the upper bound at $2/3$ and the lower bound at $1/4$: the valuation gap then being $5/12$.

3.3. Example 3: Normally Distributed Signal

To illustrate that the valuation gap is robust to various distributions, we show our analysis using the Normal distribution. Suppose that the state-determining random variable X is generated by a random variable ξ , which is observed with error by the insurer. Specifically, $\xi \sim N(0, \sigma_\xi), X = I[\xi \geq 0]$, and $G = \xi + \eta$, with $E[\eta|\xi] = 0$. G, ξ and η are jointly normally distributed. The random variable $P(X = 1|G)$ does not attain its upper and lower boundaries with positive probability.

This implies that there is no equilibrium if the insured is risk neutral or if he is perfectly diversified across states (i.e. $e_0 = e_1$) although the function $L(p)$ (resp., $H(p)$) converges to 0 (resp., towards 1), when $p \to 0$ (resp., $p \to 1$). If the insured is risk averse, and $e_0 > e_1$, there is some price at which the insured might buy assets, but there is no price at which he would sell. That is, at the posted prices, the insurer is indifferent between trading or not. We conjecture that agents must have other motives to transact than private information for non-zero trade to exist in equilibrium. For exposition purpose, the functions H and L are graphed in Figure 3.2 with appropriate parameters.

4. Both insurer and insured have private information; Bernoulli and Normal cases

In contrast with most of the current literature on asymmetric information in insurance, we assumed so far that only the insurer possessed private information about the likelihood of accident. A less extreme and more realistic approach is to assume that both sides of the market have some private information. Observing a private signal should make the insured more willing to trade insurance even though, as before, his eagerness to participate in the insurance market should be dampened by the willingness of the insurer to be his

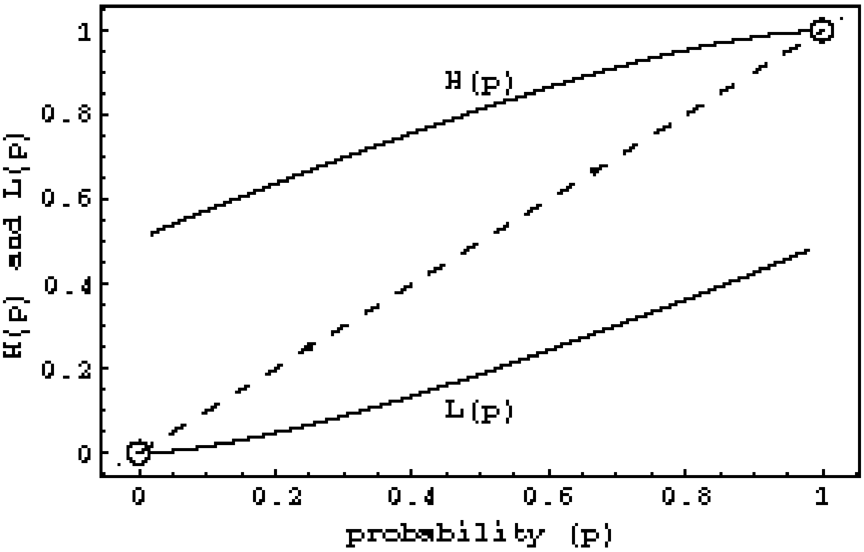

Figure 3.2: $P(X=1|P(X=1|G)\leq p)$, and $P(X=1|P(X=1|G)\geq p)$, with $\sigma_z = 1/2$, $\sigma_\eta = 1/3$.

counterparty.

Endowing each trader with private information, however, does not guarantee trade. For example, Milgrom and Stockey (1982) use a rational-expectations model where each agent has some private information about the value of the traded asset. If the structure of the economy is common knowledge, the allocation before each trader observes his private signal is Pareto optimal, and the traders are strictly risk averse, no trade occurs after the traders observe their private signals.[8] The information revealed by the market price swamps the private information of each trader, that is, observing the market price alone gives as much information to a trader as jointly observing the market price and his private signal. However, the equilibrium market price becomes a more informative proxy of the true value of the asset, so that an outside observer would benefit from the traders having private information even if they do not.

[8] However, as the referee points out in different setups "even if all value is common value, if the underlying ex ante distribution of its value is tight enough these problems diminish or disappear".

The framework in the present paper is somewhat different. For example, the insurer and the insured face each other in a situation more similar to a bargaining negotiation than to a competitive market. A more fundamental difference is that the players do not move simultaneously. The insured announces his selling and buying prices first: This gives a strategic advantage to the insurer, who uses this information to decide whether he is going to buy or sell insurance, or stay out of the market. Naturally, the insured takes this advantage into account when deciding which prices he should quote.

Let's introduce our framework in more details. Call H the signal observed by the insured. As before, X and G are the random variables determining accident occurrence and the insurer's private signal, respectively, and a and b are the prices at which the insured agent is willing to decrease or increase his coverage. We experiment with two setups. In both, X and G are Bernoulli random variables which take values 1 if the corresponding latent random variables $\tilde{X}$ and $\tilde{G}$ are positive and 0 otherwise. In the first setup, H is a similarly defined Bernoulli random variable associated with the laten variable $\tilde{H}$ and $\tilde{X}$, $\tilde{G}$, $\tilde{H}$ are jointly normally distributed with zero means and unit variances. In the second setup, H is a random variable jointly normally distribute with $\tilde{X}$ and $\tilde{G}$.

The insurer is willing to buy back some insurance if his estimation of the likelihood of accident, v, is higher than a and wants to provide more insurance when v is lower than b. The random variable v depends on G and possibly on H itself if the insurer uses the prices quoted by the insured to extract information about H. We assume that G and H are positively correlated with X and conjecture that the insurer buys ($v \geq a$) only if $G = 1$ and sells ($v \leq b$) only if $G = 0$. We check that this conjecture is correct. This conjecture flows quite naturally from the assumption that X and G are positively correlated.

In our framework, the prices posted by the insured fully reveal his private signal because they are strictly increasing in the realization of this signal. Since the signal of the insurer is still private, the informational advantage he enjoys is not fundamentally altered. However, endowing the insured with some private information yields a richer equilibrium for the game.

When X, G, and H take values 0 and 1, the insured agent's selling and buying prices a and b are such that

$$E[X|v \geq a, H] = P(X=1|G=1,H) = \begin{cases} \frac{P(X=1,G=1,H=1)}{P(G=1,H=1)} & \text{if } H=1 \\ \frac{P(X=1,G=1,H=0)}{P(G=1,H=0)} & \text{if } H=0 \end{cases}$$

$$E[X|v \leq b, H] = P(X=1|G=0,H) = \begin{cases} \frac{P(X=1,G=0,H=1)}{P(G=0,H=1)} & \text{if } H=1 \\ \frac{P(X=1,G=0,H=0)}{P(G=0,H=0)} & \text{if } H=0. \end{cases}$$

Hence, if $a = E[X|v \geq a, H]$ and $b = E[X|v \leq b, H]$, a and b depend on H. If the insured agent's quotes differ when $H = 0$ and when $H = 1$, they reveal the realization of the random variable H.

If the insurer's expectations are rational, that is, if he uses all the information available, his valuation is $v = E[X|a(H), b(H), G] = E[X|H, G]$. Call $\bar{v}(H)$ the value of v when $G = 1$ and $\underline{v}(H)$ its value when $G = 0$. Because X is more likely to equal 1 when G or H equals 1, we have $\underline{v}(H) = b(H) < a(H) = \bar{v}(H)$ when the insurer is rational, which is consistent with his buying when $G = 1$ and selling when $G = 0$.

Like in the earlier case, at the posted prices, the rational insurer is indifferent between trading or not. However, in contrast with Milgrom and Stokey (1982), selling price and buying price here are different. The prices reveal the private signal of the insured but not that of the insurer. The latter reveals the value of his signal by trading on the sell- or buy-sides.

If the insurer is not rational, in the sense that he does not exploit the information about

H contained in the bid and ask quotes, his valuation is $v = E[X|G]$. In that case, the insurer refuses to buy back insurance when $H = 1$ and to sell more insurance when $H = 0$ while he thinks he makes a profit when he transacts because $a(0) < \bar{v} < a(1)$ and $b(0) < \underline{v} < b(1)$.[9]

As an example, we fix the correlation between the insured agent's private signal and $\tilde{X}$ to 0.2, assume that the insured agent's private signal and the insurer's private signal are independent, and vary the correlation between the insurer's signal and $\tilde{X}$. This correlation, call it ρ, measures the precision of the information of the insurer. We graph the insured agent's buying and selling prices as functions of ρ when the insured agent's signal equals 1 and when it equals 0 (Figure 4.1). For both realizations of H, the selling price increases in ρ while the buying price is decreasing in this variable. The two prices, which coincide when the insurer has no private information ($\rho = 0$) spread farther apart when ρ rises. This spread protects the insured from the growing information asymmetry. The whole pricing schedule shifts up or down (in an almost parallel fashion) according to the realization of H. When $H = 1$, the likelihood of accident is higher, and the insured increases accordingly the prices at which he is willing to modify his risk exposure.

In a second experiment, we let H take a continuum of values. The random variables $\tilde{X}$, $\tilde{G}$, and H are jointly normally distributed with zero means and the correlation between any two of these random variables equal 0.5. Figure 4.2 plots the insured agent's buying and selling prices as functions of the realization of his private signal. The selling price (which is always higher than the buying price) both increase in H. Because H and $\tilde{X}$ are positively correlated, a higher realization of H reveals to the insured an increase in the odds of an accident (case $X = 1$). Accordingly, the insured is willing to pay more for insurance and

[9]
$$\begin{aligned}\bar{v} &= P(X=1|G=1)\\ &= P(H=0|G=1)\,P(X=1|G=1,H=0) + P(H=1|G=1)\,P(X=1|G=1,H=1),\\ &= P(H=0|G=1)\,a(0) + P(H=1|G=1)\,a(1).\end{aligned}$$

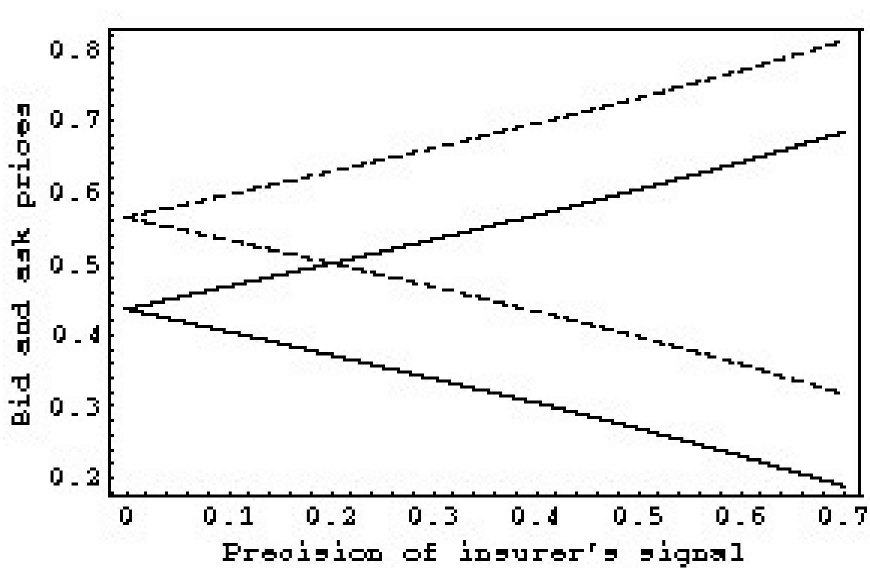

Figure 4.1: **Bid and ask prices when the insured agent's signal takes values 0 and 1.** Insured agent's buying and selling prices as functions of the precision of the insurer' private signal when the insured agent's signal equals 1 (the dashed lines) and when it equals 0 (the solid lines). The precision of the insurer's signal is measured by ρ, its correlation with $\tilde{X}$ (which determines the occurrence of accident). The correlation between the insured agent's private signal and $\tilde{X}$ is 0.2. The insured agent's private signal and the insurer's private signal are independent.

demands a higher price to decrease his coverage.

5. Conclusion

This paper studies whether the large discrepancy between the price that individuals demand as a compensation to increase risk and the price they are willing to pay to reduce risk can be explained within a rational expectation framework with asymmetric information. We base our model on Glosten and Milgrom (1985), who study the market makers' bid/ask spread using asymmetric information, to address the compensation issues.

In the present set-up, we use an endowment economy where agents trade assets in order to relate to the bid/ask (or WTP/WTA) literature. We let the insurer play the role of the informed trader, and the insured take the role of the market-maker. We have to assume

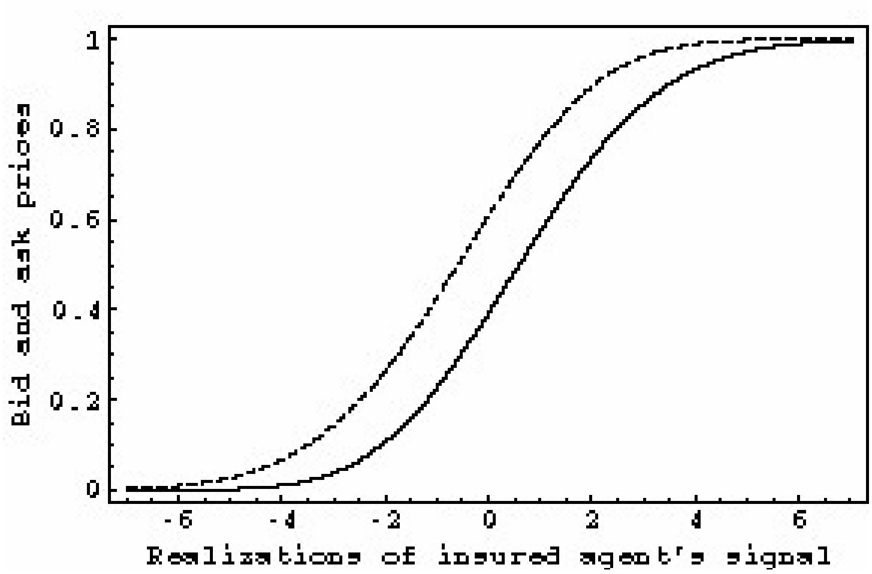

Figure 4.2: **Bid price and ask prices when the insured agent's signal is normally distributed.** Insured agent's buying price (solid line) and selling price (dashed line) as functions of the realization of his private signal. The correlation between $\tilde{X}$ (which determines the occurrence of accident) and the insured agent's and the insurer's private signals and between these two signals all equal 0.5.

that the uninformed agent is risk neutral for him to communicate two prices. Then, his selling price is the highest possible price the insurer could accept while his buying price is the lowest possible price the insurer could accept. If the agent is risk averse, no price is high enough to entice him to take more risk. Therefore, the valuation gap is replaced by a refusal to trade on one side. We analyze the cases where (i) only the insurer and (ii) both the insurer and the insured have private information with the random variable following Bernoulli and Normal distributions. For both cases, the equilibrium is of the no-trade variety when all agents are rational. This may be consistent with the Thaler's (1980) results and Jones-Lee et al (1985) study, where the majority of the people questioned in surveys failed to give a price that would compensate them for taking on more risk. We conclude that the asymmetric information framework used in this paper can explain the valuation gap without invoking psychology.

Appendix

Proof of Proposition 1:

The conditional expectation of Y on $I\,[v \geq p]$ is

$$E\,[Y|I\,[v \geq p]] = \left\{ \begin{array}{ll} \frac{E[I[v\geq p]Y]}{P(v\geq p)} & \text{if } v \geq p \\ \frac{E[I[v<p]Y]}{P(v<p)} & \text{if } v < p \end{array} \right\}$$

1. To prove that $\overline{v} \geq H \geq p$, we have by definition,

$$H(p) = \frac{1}{P(v \geq p)} E[I[v \geq p]Y] \tag{5.1}$$

and

$$\begin{aligned} E[I[v \geq p]Y] &= E[I[v \geq p]v] \\ &\geq E[I[v \geq p]p] \\ &= p(v \geq p)p. \end{aligned} \tag{5.2}$$

To obtain the first line, write $Y = v + \eta$, with $E[\eta|G] = 0$. Then note that $E[I[v \geq p]\eta] = E[E[I[v \geq p]\eta|G]] = E[I[v \geq p]E[\eta|G]] = 0$, using the law of iterated expectations, and the fact that $I[v \geq p]$ is measurable with respect to G. To obtain the second line, note that $I[v \geq p]v \geq I[v \geq p]p$. For the third line, use the fact that p is a constant. Equation (5.1), and (5.2) imply that $H(p) \geq p$. As $v \leq \overline{v}$, $E[I[v \geq p]v] \leq P(v \geq p)\overline{v}$, and hence $H(p) \leq \overline{v}$. Likewise, $\underline{v} \leq L(p) \leq p$.

2. Let p_1, and p_2 in H's domain with $p_1 \leq p_2$.

$$\begin{aligned} H(p_1) &= \frac{E[v\ I[v\geq p_1]]}{P(v\geq p_1)}, \\ &= \theta\ \frac{E[v\ I[v\geq p_2]]}{P(v\geq p_2)} + (1-\theta)\ \frac{E[vI[p_1\leq v<p_2]]}{P(p_1\leq v<p_2)}, \end{aligned} \tag{5.3}$$

with $\theta = \frac{P(v \geq p_2)}{P(v \geq p_1)}$, $0 \leq \theta \leq 1$. As $\frac{E[v\ I[p_1 \leq v < p_2]]}{P(p_1 \leq v < p_2)} \leq p_2 \leq H(p_2)$, $H(p_1) \leq H(p_2)$. Now, let p in H's domain, $p \leq \underline{v}$, $I[v \geq p] = 1$, hence $H(p) = \frac{E[v\ I[v \geq p]]}{P(v \geq p)} = E[v]$. For L, proceed as follows.

$$L(p_2) = \lambda\ L(p_1) + (1 - \lambda)\ \frac{E[v\ I[p_1 \leq v \leq p_2]]}{P(p_1 \leq v \leq p_2)}, \tag{5.4}$$

with $\lambda = \frac{P(v \leq p_1)}{P(v \leq p_2)}$, $0 \leq \lambda \leq 1$. As $\frac{E[v\ I[p_1 \leq v \leq p_2]]}{P(p_1 \leq v \leq p_2)} \geq p_1 \geq L(p_1)$, $L(p_2) \geq L(p_1)$. Let p in L's domain, $p \geq \overline{v}$, $I[v \leq p] = 1$, hence $L(p) = \frac{E[v\ I[v \leq p]]}{P(v \leq p)} = E[v]$.

3. H is defined on $D_H = (-\infty, \overline{v})$, with the upper bound included if H is defined at that point. For all $p \in D_H$, $H(p) \leq \overline{v}$. The function H is an increasing and bounded on D_H, therefore H admits a limit, call it $limH$. $limH \leq \overline{v}$, and for all $p \in D_H$, $H(p) \leq limH$. Now suppose $limH \neq \overline{v}$, that is $limH < \overline{v}$. Then there is $\alpha \in D_H$, $limH < \alpha < \overline{v}$. As $H(p) \geq p$, for all $p \in D_H$, we have $H(\alpha) \geq \alpha > limH$. But this contradicts the property that, as H in non-decreasing and converges to $limH$, $H \leq limH$. One concludes that $limH$ cannot be different from $\overline{v}$, the same holds for L and $\underline{v}$. Q.E.D.

Proof of Proposition 2*:*

The proposition is proven for H, results for L can be derived in the same fashion. Suppose that $\overline{v}$ is finite, and that v attains $\overline{v}$ with positive probability. Assume that H has a fixed point p, let's prove that $p = \overline{v}$. By definition, $p \leq \overline{v}$. Suppose now that $p < \overline{v}$. Let α a positive scalar so that $p < \alpha < \overline{v}$, and $P(v \geq \alpha) > 0$. As $I[v \geq p] = I[v \geq \alpha] + I[p \leq v < \alpha]$, one gets

$$
\begin{aligned}
E[I[v \geq p]v] &= E[I[v \geq \alpha]v] + E[I[p \leq v < \alpha]v], \\
&\geq \alpha P(v \geq \alpha) + pP(p \leq v < \alpha), \\
&> pP(v \geq \alpha) + pP(p \leq v < \alpha), \\
&> pP(v \geq p).
\end{aligned} \tag{5.5}
$$

Hence, $p < \overline{v}$ cannot be a fixed point of H. Hence, the only possible fixed point of H is $\overline{v}$. Let's show that $\overline{v}$ is indeed a fixed point of H, i.e., that $E[I[v \geq \overline{v}]v] = P(v \geq \overline{v})\overline{v}$. The last equation follows directly from the fact that $I[v \geq \overline{v}]v = I[v \geq \overline{v}]\overline{v}$.

Suppose $\overline{v}$ is not finite, or that v does not attain $\overline{v}$ with positive probability. Then, the domain of H is $(-\infty, \overline{v})$. Let p be a candidate fixed point, then $p < \overline{v}$, but then, equation (5.5) shows that p cannot be a fixed point. Q.E.D.

Proof of proposition 3:

The signal G takes the values 0 and 1, with $P(G = 1) \in (0,1)$. Let $p_{i,j} = P(X = i, G = j)$, $i \in \{0,1\}$, $j \in \{0,1\}$. $P(X = 1|G)$ takes two values $P(X = 1|G = 0)$, and $P(X = 1|G = 1)$. Suppose that $cov(X,G) \geq 0$ (this is harmless, if $cov(X,G) \leq 0$, one would substitute $1 - G$ for G and obtain a non-negative covariance), then $P(X = 1|G = 1) \geq P(X = 1|G = 0)$. Then, proposition (3) follows from the following lemma. Q.E.D.

Lemma 1. *$P(X = 1|G = 1) - P(X = 1|G = 0) = \frac{cov(X,G)}{var(G)} = \frac{\sigma_X}{\sigma_G}\rho_{X,G}$ where σ_X is the standard deviation of X, σ_G is the standard deviation of G, and $\rho_{X,G}$ is the correlation between X and G.*

Proof of Lemma *:*

$$\begin{aligned} P(X=1|G=1)-P(X=1|G=0) &= \tfrac{P(X=1,G=1)}{P(G=1)}-\tfrac{P(X=1,G=0)}{P(G=0)}, \\ &= \tfrac{P(G=0)P(X=1,G=1)-P(G=1)P(X=1,G=0)}{P(G=1)P(G=0)}. \end{aligned} \tag{5.6}$$

Now, note that $var(G)=P(G=1)P(G=0)$, and that

$$\begin{aligned} cov(X,G) &= E[XG]-E[X]E[G], \\ &= P(X=1,G=1)-P(X=1)P(G=1), \\ &= P(X=1,G=1)-[P(X=1,G=0)+P(X=1,G=1)]P(G=1), \\ &= P(G=0)P(X=1,G=1)-P(G=1)P(X=1,G=0). \end{aligned} \tag{5.7}$$

This shows that $P(X=1|G=1)-P(X=1|G=0)=\frac{cov(X,G)}{var(G)}$. Of course, as $P(G=1)\in(0,1)$, $var(G)\neq 0$. Q.E.D.

Conditions for existence of an equilibrium in Glosten and Milgrom (1985)

The informed trader has a stochastic time discount factor, noted by θ. If x is the true value of the asset and G is the informed trader's private signal, then his valuation is $v=\theta\,E[x|G]$. Glosten and Milgrom then assume the existence of prices a and b such that $a=E[x|v\geq a]$ and $b=E[x|v\leq b]$. However, if θ is measurable with respect to G (i.e. the informed trader observes θ), then, equilibrium values of a and b may not always exist. If θ is measurable and $\theta\leq 1$, then $E[x|v\geq a]\geq E[\theta x|E[\theta x|G]\geq a]$; $\sup v$ is the only a such that $a=E[\theta x|E[\theta x|G]\geq a]$ and consequently there may be no a such that $a=E[x|v\geq a]$. Hence, a necessary—and rather odd—condition for the existence of an equilibrium ask price is that θ take values above 1 with positive probabilities, i.e., that the informed trader in some states of the world is willing to consume less if it means consuming later. (Conditions for the existence of a bid price can be derived in a similar way.) Assuming that θ is not measurable with respect to G means that the informed trader does not know at which rate he discounts time, which seems quite implausible.

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