Consumption, the Persistence of Shocks,

And Asset Pricing Puzzles*

Juan Carlos Rodriguez

EURANDOM

April, 2002

Abstract: This paper studies an equilibrium asset pricing model in which the endowment is the sum of two components: one permanent, the other transitory. Permanent shocks are assumed to be rare events and are modeled as realizations of a Poisson process. The transitory component is modeled as a diffusion. Empirical support is offered to justify this decomposition. It is shown analytically that there is a direct relation between the predictability of dividends and the volatility of returns, and that excess volatility is essentially an intertemporal substitution phenomenon. Moreover, in the presence of transitory shocks, intertemporal substitution explains most of the volatility of returns. With the permanent component modeled as a pure jump process, jump risk contributes to increase the equity premium, and fundamentally to reduce the risk-free interest rate, but it has no role in explaining returns volatility beyond the volatility of consumption growth. Numerical results show that the model developed in this paper is able to generate equity premium, volatility of returns and volatility of the risk-free rate in line with the data for levels of risk aversion lower than nine, and rate of intertemporal preference higher than zero.

JEL Classification: G12,E21,C22

* This paper is an essay from my dissertation, written when I was a graduate student at the Department of Economics of the University of Maryland. I thank Michael Binder, Fernando Broner, and Dilip Madan for insightful comments and suggestions. I am also grateful to John Haltiwanger, and seminar participants at the University of Maryland and Eurandom. All remaining errors are mine. Address for correspondence: Eurandom, P.O Box 513, 5600 MB, Eindhoven, The Netherlands. Email: rodriguez@eurandom.tue.nl.

1. Introduction

In the consumption-based asset pricing literature, the endowment has been modeled as being an integrated process[1] driven by permanent innovations. This important assumption has been virtually unchallenged by researchers. However, Watson (1986), Clark (1987, 1989), Cochrane (1988), and Blanchard and Quah (1989), in studies that explicitly consider decompositions of common macroeconomic series into a stochastic trend and a stationary component, found evidence that transitory shocks explain a large part of the variance of these series[2]. Moreover, on the theoretical level, an important paper by Quah (1992) has shown that any integrated time series can be decomposed into a permanent and a transitory component in such a way that the permanent component can be made arbitrarily smooth, so that the variability of the series is dominated by the transitory component at all finite horizons. But in spite of these theoretical and empirical findings, the role that transitory innovations might play in explaining asset pricing puzzles[3] has never been explored in the literature.

[1]"Integrated" refers here to integrated of order one. A process is integrated of order one if its first difference is a stationary process.

[2] With respect to the variance of 1-quarter ahead changes in the log of GDP, Watson (1986) found that 64% of it could be attributed to transitory shocks, while Clark (1987) found that proportion around of 50%, Cochrane (1988) between 67 and 82%, and Blanchard and Quah (1989), between 60 and 95%. Watson also found that 95% of 1-quarter ahead changes in Consumption of Nondurables could be due to transitory shocks.

[3] These are: the "equity premium puzzle" (Mehra and Prescott, 1985), which is the fact that the excess return of the risky asset over the risk-free rate observed in the data is too high to be explained by a standard asset pricing model with constant relative risk aversion preferences and exogenous endowment. The "excess volatility puzzle" (Shiller, 1981), or the fact that the volatility of dividends is too low to explain the observed volatility of returns. Related to this two puzzles, there is the "risk-free rate" puzzle, introduced in Weil (1989), which amounts to the observation that any intend to explain the equity premium by increasing the coefficient of risk aversion in standard models with CRRA preferences generates implausible high level of the endogenous risk-free rate.

To study this problem, in this paper I present an equilibrium model in which the endowment is the sum of two components: one transitory, the other permanent[4]. The presence of transitory innovations enhances the significance of the elasticity of intertemporal substitution relative to a model in which all innovations are permanent. An important result in this paper is that there exists a direct relation between the predictability of dividends and the volatility of returns, and that the excess volatility of returns is essentially an intertemporal substitution phenomenon[5].

These issues are studied in a continuous-time version of Lucas (1978)[6 7], in which the representative consumer has a separable utility function exhibiting constant relative risk aversion. Although it is assumed to have a transitory component[8], the endowment is made difference stationary by the presence of the permanent component. Difference-stationarity guarantees that the endowment's variance is infinite at the infinite horizon.

My modeling of the permanent component is based on an idea that can be traced back to Perron (1989), who suggests that different type of shocks should be distinguished by their frequency: permanent innovations must be modeled as rare events relative to transitory innovations. Perron (1989) and, in an extension of Perron's paper, Zivot and Andrews (1992), find evidence that many of the macroeconomics series studied by Nelson and Plosser (1982) can be described as trend-stationary, if structural change in the form of segmented trends is included into the analysis[9]. Balke and Fomby (1991), using

[4] Empirical support for the decomposition chosen in this paper can be found in Section 2.
[5] The role of intertemporal substitution in accounting for excess volatility was pointed out for the first time by Kandel and Stambaugh (1991). However, this issue did not receive further attention in the literature.
[6]Continuous time versions of Lucas (1978) are widely used in the Finance literature. See, for example, Naik and Lee (1990), and Bakshi and Chen (1996).
[7] I use a continuous time model for expository reasons, and to facilitate the derivation of analytical results.
[8] This assumption is empirically justified in section 2.
[9] See also Rappoport and Reichlin (1989).

intervention analysis[10], detect the presence of infrequent permanent shocks in the GNP deflator and real GNP series.

Therefore, to capture the idea of permanent innovations having low frequency relative to transitory innovations, I model the transitory component as a diffusion, and the permanent component as a pure jump process. Conditional on the realization of the jump process, the endowment is trend-stationary. As I explain below, an asset pricing model based on this description of the consumption-endowment process is well suited to explain puzzles: infrequent jumps with random size (the permanent innovations) generate a precautionary demand for bonds that keeps the interest rate low, while the stationary component produces high variability of stock returns. In fact, numerical results in section 5 show that the model developed in this paper is able to generate equity premium, volatility of returns and volatility of the risk-free rate in line with the data for levels of risk aversion lower than nine, and rate of intertemporal preference higher than zero.

The reason why a temporary component in the endowment's dynamics helps to explain asset pricing anomalies is the following. In a standard asset pricing model, the supply of shares is assumed constant; therefore, the variability of the asset price must reflect changes in demand in response to random innovations. The demand of financial assets reflects both attitudes towards risk and attitudes towards intertemporal substitution. Assuming rational expectations and a concave utility function, the desire of investors to save depends on the permanent or transitory character of the random innovations.

Trivially, the price of the asset can be expressed as the product of the current dividend and the price-dividend ratio, as:

[10] For a text book treatment of intervention analysis, see Pankratz (1991).

$$S_t \equiv q_t \left(\frac{S_t}{q_t} \right). \tag{1}$$

It will be shown in Section 4 that the price-dividend ratio can be written, in terms of the parameters of the utility function and the dividend process, as:

$$\frac{S_t}{q_t} = E_t \int_t^{\infty} e^{-\rho(s-t)} \frac{u'(q_s)}{u'(q_t)} \frac{q_s}{q_t} ds, \tag{2}$$

where *u'* is the marginal utility of consumption[11], and E_t is the expectations operator, conditional on information up to time t[12].

Equation (1) suggests that the effect of a shock to the dividend level on the price of the asset can be decomposed into two parts: an effect on the dividend, and an effect on the price-dividend ratio. The dividend effect reflects the direct impact of the random innovation on the endowment's level. From equation (2), the effect on the price-dividend ratio reflects the degree to which the observation of the current shock changes the agents' expectations about future dividend growth and discount rates. Therefore, the effect of a shock on the price-dividend ratio depends on how the new dividend signals future values of the ratios $\frac{u'(q_s)}{u'(q_t)}$ and $\frac{q_s}{q_t}$. In the particular case of additive utility of the CRRA type[13], which is the case studied in this paper, the quotient of marginal utilities is also a function of $\frac{q_s}{q_t}$.

[11] Equation (2) is an equilibrium condition, and in equilibrium in the simple economy I am considering, consumption and the endowment-dividend are equal. See Lucas (1978) for an early example of a model constructed on these lines.

[12] A transversality condition is assumed. See Section 4 for details.

[13] Utility functions of the CRRA type restrict the risk aversion coefficient and the intertemporal substitution parameter to be mutually reciprocal. The model developed in this paper shows that it is possible to separate the effects of risk aversion from those intertemporal substitution in the endogenously generated volatility of returns, and still keep the tractability of the CRRA model of preferences through the distinction between permanent and transitory innovations.

Therefore, computing asset prices requires forecasts of future values of the rate of growth of the economy.

Consider, for example, equation (1) when the representative consumer has CRRA utility function, and assume that every shock to the dividend is permanent. In this case, innovations have no effect on the expected future ratio $\frac{q_s}{q_t}$, and the price-dividend ratio remains constant after a shock. The change in the asset price reflects only the impact of the shock on the current dividend level. On the other hand, suppose that innovations are transitory, and that there is a positive shock to the dividend. The difference in this case is that, besides the positive impact on the current dividend, the expected future values of $\frac{q_s}{q_t}$ are also affected as the representative agent expects that future dividends will revert to their long-run levels. Therefore, when innovations are transitory, the change in the asset price reflects not only the impact of the unanticipated shock on the dividend level, but also the change in all forecasted future rates of growth, which appears as a change of the price-dividend ratio. When the coefficient of relative risk aversion is higher than one, the dividend level and the price-dividend ratio move in the same direction after an unanticipated random shock, increasing the variability of the asset price.

A positive transitory innovation increases the endowment's rate of growth and the rate of return of the risky asset more than proportionally. This raises the covariance between both rates and results in an increase in the equity premium relative to a situation in which all innovations are permanent.

In the recent literature, Cechetti, Lam and Mark (1993) also studied an asset pricing model with structural breaks, but they modeled the endowment process as a random walk with two state Markov drift[14]. Therefore, they did not address the issue of transitory innovations, or the role of intertemporal substitution in generating volatility of returns. Rietz (1988), argued that the small probability of a catastrophe (a major structural change) affecting the consumption process can help explain both the size of the equity premium and the level of the risk-free interest rate. However, there are three main differences between the model developed in this paper and the model in Rietz (1989). First, to get his results, Rietz needs a fall in consumption of at least 25% in one year. The most dramatic fall in consumption in one year, in the period 1899-1985, was about 10%. This corresponds to year 1930. Consumption fell about 25%, but along the 4 years of the Great Depression. In this paper I estimate the time and the size of the structural change of the consumption series in the framework of the segmented-trend hypothesis. The time of the structural break corresponds to the year 1930, with estimated size of the jump of –11.6%. Second, in Rietz (1989) the size of the jump is deterministic. Agents have uncertainty about the timing, but are sure about the size (-25% or less, depending on the parameterization chosen). In this paper the size of the jump is a random variable, whose variance plays a key role in determining the endogenous moments of returns. Third, Rietz (1989) does not study second moments of returns, as this paper does

Results in this paper reveal a surprising connection between the predictability of dividends and the volatility of returns, and shed light on the role that intertemporal substitution plays in explaining asset pricing anomalies. They also show that, although jump risk contributes to increase the equity premium, and fundamentally to reduce the risk-free interest rate, it has no role in explaining returns volatility beyond the volatility of

[14] They separate consumption from the dividend process, and model both as a bivariate random walk with Markov drift.

consumption growth. When all innovations are permanent there is no excess volatility, no matter the size or the variability of the jump. Intertemporal substitution helps to raise the equity premium through the increase in the volatility of returns. Finally, numerical results in section 5 show that the model developed in this paper is able to generate equity premium, volatility of returns and volatility of the risk-free rate in line with the data for levels of risk aversion lower than nine, and rate of intertemporal preference higher than zero.

The organization of the paper is the following. Section 2 shows empirical results on per capita consumption, which support the decomposition of the endowment process presented in Section 3. Section 4 solves the asset pricing model using the endowment process developed in Section 3. Section 5 shows a calibration exercise using estimates from Section 2. Section 6 concludes.

2. Empirical Results on Per Capita Consumption

Data of per capita consumption is taken from the Shiller[15] database. The data set consists of 97 annual observations covering the period 1889-1985.

Perron (1989) and Rappoport and Reichlin (1989) were the first to argue that if structural change in the form of breaking trends is included into the analysis, most macroeconomic time series can be described as trend-stationary, rejecting the unit root hypothesis suggested by Nelson and Plosser (1982). In the breaking trends framework,

[15] This database can be found at http://www.econ.yale.edu/~shiller/data.htm. See also Appendix 2 for additional information about the data.

there are permanent shocks, but they are seen as rare events, and transitory shocks play a key role in explaining the variability of the series.

In both Perron's and Rappoport and Reichlin's analysis, structural changes are treated as exogenous events, whose time of occurrence is known by the researcher. This methodology can be criticized in terms of pre-testing and data mining[16]. To avoid this criticism, Zivot and Andrews (1992) proposed an extension of Perron's analysis that endogenizes the shocks, interpreting them as realizations from the tail of the distribution of the underlying data generating process, instead of exogenous events.

Following Zivot and Andrews (1992) I consider as a null hypothesis that the log of consumption is I(1), without structural breaks. That is:

$$lcons_t = \mu + lcons_{t-1} + e_t \,. \tag{3}$$

The alternative hypothesis is that the log of consumption can be represented by a trend-stationary process with a single break in trend occurring at an unknown date. The alternative models considered are:

A) A level shift in the trend:

$$lcons_t = \mu_1 + \beta t + (\mu_2 - \mu_1) DU_t + u_t \,, \tag{4}$$

where DU = 1 if $t > T_B$, the time of the break, and 0 otherwise.

B) A shift in the slope of the trend:

$$lcons_t = \mu + \beta_1 t + (\beta_2 - \beta_1) DT_t + u_t \,, \tag{5}$$

where $DT = t - T_B$ if $t > T_B$, and 0 otherwise.

[16] See, for example, Christiano (1992).

C) A model with shifts in the level and in the slope of the trend:

$$lcons_t = \mu_1 + \beta_1 t + (\mu_2 - \mu_1) DU_t + (\beta_2 - \beta_1) DT_t + u_t . \tag{6}$$

The break point $\lambda = \frac{T_B}{T}$, where T is the number of observations, is chosen to minimize the unit root test statistics[17] computed from the regressions:

A')

$$lcons_t = \hat{\mu} + \hat{\beta}_1^A t + \hat{\beta}_2^A DU(\hat{\lambda})_t + a^A lcons_{t-1} + \sum_{i=1}^{k} \hat{c}_i^A \Delta lcons_{t-i} + \hat{u}_t . \tag{7}$$

B')

$$lcons_t = \hat{\mu} + \hat{\beta}_1^B t + \hat{\beta}_2^B DT(\hat{\lambda})_t + a^B lcons_{t-1} + \sum_{i=1}^{k} \hat{c}_i^B \Delta lcons_{t-i} + \hat{u}_t . \tag{8}$$

C')

$$lcons_t = \hat{\mu} + \hat{\beta}_1^C t + \hat{\beta}_2^C DU(\hat{\lambda})_t + \hat{\beta}_3^C DT(\hat{\lambda})_t + a^C lcons_{t-1} + \sum_{i=1}^{k} \hat{c}_i^C \Delta lcons_{t-i} + \hat{u}_t . \tag{9}$$

A'), B') and C') were estimated by ordinary least squares. The break fraction λ ranged from $\frac{2}{T}$ to $\frac{T-1}{T}$. The number of extra regressors, *k*, was determined as in Perron (1989), although Table 1 in Appendix 1 shows that the results obtained are robust to the number of additional regressors selected.

[17] See Zivot and Andrews (1992) for details.

For the log of consumption, the break year that minimizes the test statistic for testing $a^i = 1$ corresponds to 1929, the year of the Great Depression, and the model favored is A), the level shift in the trend.

The critical values for testing $a^i = 1$ calculated by Zivot and Andrews (1992) are higher in absolute value than the critical values used in standard tests of unit roots. They are the following:

Table 1:

Critical Values (Zivot and Andrews, 1992)

t_a	p value
-5.57	1%
-5.3	2.50%
-5.08	5%
-4.82	10%

Results from regression A') are shown in the following table:

Table 2:

Estimation results (Equation 7)

Parameter	Estimate
μ	-0.18719* (.03586)
β_1	0.009983* (.00165)
β_2	-0.09483* (.0171)
a	0.455246*
a/t stat.**	-5.85008
K	10

Standard deviation in parenthesis
* Significant at the 1% level
** This t statistics tests the hypothesis that $\alpha = 1$.

Based on Table 1, these results show that the unit root hypothesis can be rejected at the 1% level. Therefore, the implied model is trend-stationary with a level shift corresponding to the year 1930:

$$\begin{aligned} lcons_t &= \mu + \beta_1 T_t + \beta_2 DU_t + u_t, \\ \Phi(L)u_t &= \Theta(L)\varepsilon_t. \end{aligned} \tag{10}$$

This model was estimated through nonlinear least squares. Results are shown in table 3:

Table 3:
Estimation results (model 10)

Parameter	Estimate
μ	-0.316502* (.03397)
β_1	0.017819* (.000727)
β_2	-0.116414* (.03139)
AR(1)	0.787532* (.06485)
SE Reg.	0.032275

Standard deviation in parenthesis.
* Significant at the 1% level

These results show that per capita consumption grew at a trend rate of 1.78% over the past century. They also show that that the log of per capita consumption can be described as a fairly persistent autoregressive process, with a major break in trend corresponding to the year 1930. This break in trend is a level shift, that is, a permanent fall in the level of the series. The estimated shift is –11.64 %. The estimated autoregressive coefficient is equal to .79[18].

[18] More general autoregressive schemes were tested and rejected.

These estimates give support to the contention that the consumption series can be modeled as trend-stationary, subject to infrequent permanent breaks in the trend. They also suggest that the modeling of the consumption-endowment process as the sum of two components: one transitory, the other permanent, is a sensible one. In the next section I develop a model of the endowment in which the permanent component is interpreted as a Poisson process with random size of the jumps, and deviations from trend are interpreted as an Ornstein-Uhlenbeck process. Finally, estimates in table 3 will be used in Section 5 to calibrate the model.

3. A Stochastic Process for the Endowment

In this section I develop a model for the log of the endowment, which is defined as:

$$log(q_t) = x_t + y_t, \tag{11}$$

where x_t and y_t are the permanent and transitory components, respectively.

To capture the idea of infrequent permanent shocks affecting the level of $\log(q_t)$, the permanent component is modeled as a pure jump process. Its dynamics is described by the following equation:

$$dx_t = udN_t, \tag{12}$$

where N_t is a Poisson process with parameter λ, and u, the random size of the jump, is assumed normally distributed, with mean θ and variance σ^2 [19]. N_t and u are assumed independent[20].

[19] Other models describing discontinuities in financial time series as the outcome of a Poisson process with normally distributed jumps are Naik and Lee (1990) and Bates (1996).

Conditional on the level of the series, the log endowment is trend- stationary. In particular, departures from the trend are modeled as an Ornstein-Uhlenbeck process[21], with dynamics:

$$d\delta_t = -\alpha\delta_t dt + \omega dZ_t , \tag{13}$$

where:

$$\delta_t = y_t - \mu t , \tag{14}$$

In (13) and (14) μ is the long run rate of growth of the log endowment, α is the rate of reversion to the long run mean. It is assumed higher that zero, to make the process stationary. ω is the instantaneous volatility. Finally, *Z* is a standard Wiener process.

Using (14), equation (13) can be written as:

$$d(y_t - \mu t) = -\alpha(y_t - \mu t)dt + \omega dZ_t . \tag{15}$$

To show more explicitly the dynamics of the transitory component, equation (15) can also be written as:

$$dy_t = [\mu - \alpha(y_t - \mu t)]dt + \omega dZ_t . \tag{16}$$

Equation (16) shows that, if $\alpha = 0$, y_t is a random walk with drift. Finally, note that from (13) δ_t is a normal random variable, with unconditional mean equal to zero, and unconditional variance equal to $\frac{\omega^2}{2\alpha}$. All processes, *Z*, *u* and *N*, are assumed to be independent.

The change in the log endowment can be expressed, in terms of permanent and transitory components, as:

---

[20] Jumps with random size make the market incomplete. For an analysis of market incompleteness in a jump-diffusion model of security prices, see Naik and Lee (1990).

$$d\,log(q_t) = \left[\mu - \alpha\left(y_t - \mu t\right)\right]dt + \omega dZ_t + u dN_t\,. \tag{17}$$

The log of the endowment's rate of growth between to arbitrary points in time *s* and *t* is obtained as a solution of equation (17):

$$log\left(\frac{q_s}{q_t}\right) = \mu(s-t) + \left(\mu t - y_t\right)\left(1 - e^{-\alpha(s-t)}\right) + \omega\int_t^s e^{-\alpha(v-t)}dZ_v + \sum_{i=N_{t+1}}^{N_s} u_i\,. \tag{18}$$

Conditional on information at *t*, $log\left(\frac{q_s}{q_t}\right)$ has mean:

$$E_t\left[log\left(\frac{q_s}{q_t}\right)\right] = \left(\mu + \lambda\theta\right)(s-t) + \left(\mu t - y_t\right)\left(1 - e^{-\alpha(s-t)}\right). \tag{19}$$

Equation (19) shows that the expected rate of growth of the log endowment reflects both the constant rate of growth of the transitory component and the expected size of the jump, weighted by its probability of occurrence. The last term is an adjustment to reflect information up to time t.

The conditional variance of the rate of growth is:

$$\left(\theta^2 + \sigma^2\right)\lambda(s-t) + \omega^2\,\frac{1 - e^{-\alpha(s-t)}}{2\alpha}. \tag{20}$$

Equation (20) is the sum of the conditional variances of the permanent and transitory components. The variance of the permanent component depends not only on the variance of the jump, but also on its expected size and probability of occurrence.

It is important to note that the conditional expectation of $log\left(\frac{q_s}{q_t}\right)$ depends only on y_t, the transitory component. The expression $e^{-\alpha(s-t)}$ can be interpreted as the impulse-

[21] This model was introduced in the financial literature by Lo and Wang (1995), to study the effect of mean reversion in asset prices on the valuation of derivative securities.

response function of a transitory innovation: the fraction of a unitary shock that remains after *s-t* periods have passed.

The change of the conditional expectation (19) over the next instant, is:

$$dE_t\left[log\left(\frac{q_s}{q_t}\right)\right] = m_t dt - \left(1 - e^{-\alpha(s-t)}\right)\omega dZ_t\,, \qquad (21)$$

where:

$$m_t = -\left(\lambda\theta + E_t dy_t\right). \qquad (22)$$

Note that, in the extreme case in which all shocks are permanent ($\alpha = 0$), (19) has no random component. On the other hand, when $\alpha > 0$, the conditional expectation of the rate of growth has y_t as its random component. Therefore, the conditional expectation of the rate of growth changes after an unexpected transitory innovation with mean 0 and variance $\omega^2 dt$, and the term $-\left(1 - e^{-\alpha(s-t)}\right)$ measures this reaction. This term reflects that $\left(1 - e^{-\alpha(s-t)}\right)$ of a unitary shock is expected to vanish after *s-t* periods.

As the endowment's rate of growth is the main variable to compute asset prices in an economy populated by agents having isoelastic utility, expectation revision after a transitory shock will prove central to explain the ability of the asset pricing model in section 4 to generate excess volatility.

4. An Equilibrium Model of Asset Prices

In this section I develop an asset pricing model on the lines of Lucas (1978). In order to study how transitory innovations affect prices, I present analytical solutions for the term structure of interest rates and for the price of the risky asset.

Let's assume an infinite horizon, pure exchange economy, populated by a representative agent. The agent trades a single risky asset, which can be viewed as the market portfolio, and a pure-discount bond.

There is one share of the risky asset outstanding. Bonds are in zero net supply. Lets denote the agent's portfolio at time t as $\pi_t = \{z_t, b_t\}$, where z_t and b_t represent the number of shares invested in the risky asset and in the zero-coupon bond, respectively. The agent finances her consumption per period, c_t, by the trading strategy π_t.

Preferences of the representative agent are described by the utility functional:

$$V = E_t \int_t^{\infty} u(c_t) e^{-\rho s} ds . \tag{23}$$

where $u(c_t)$, the instantaneous utility index, satisfies $u'(c_t) > 0$, and $u''(c_t) < 0$. ρ is the rate of intertemporal preference.

The endowment is defined as:

$$q_t = e^{x_t + y_t} , \tag{24}$$

where x_t and y_t are, as in section 3, the permanent and transitory components of log q_t, respectively. x_t and y_t are exogenously given stochastic processes defined on a probability space $(\Omega, \Im, P)$. They generate the uncertainty in the model. The information structure of the agent is given by the filtration $\Im_t = F(x_s, y_s \,;\, 0 \leq s \leq t)$.The agent knows all the parameters of the model, and, in particular, she can distinguish permanent from transitory innovations[22].

The problem of the consumer is to maximize (23) subject to:

$$\frac{dq_t}{q_t} = \left[\mu + \alpha(\mu t - y_t) + \frac{\omega^2}{2}\right]dt + \omega dZ_t + \left(e^u - 1\right)dN_t\,, \tag{25}$$

and the budget constraint:

$$\left(S_t + q_t\right)z_t + b_t = c_t dt + S_t z_{t+dt} + b_{t+dt}\frac{1}{1 + r_t dt}\,. \tag{26}$$

The first order conditions of the consumer's program are:

$$u'\left(c_t\right) = e^{-\rho dt} E_t\left[u'\left(c_{t+dt}\right)\left(1 + r_t dt\right)\right], \tag{27}$$

$$u'\left(c_t\right) = e^{-\rho dt} E_t\left[u'\left(c_{t+dt}\right)\left(\frac{S_{t+dt} + q_{t+dt}}{S_t}\right)\right]. \tag{28}$$

Equations (27) and (28) show that at an optimum, the agent is indifferent between one unit of consumption at *t* and investing that unit in either of the financial assets. If the agent is optimizing, both strategies give her the same expected utility at the margin.

The markets of the good and securities must clear in general equilibrium. So, for all *t*:

$$c_t = q_t\,. \tag{29}$$

$$z_t = 1\,. \tag{30}$$

$$b_t = 0\,. \tag{31}$$

In what follows, in order to get closed form solutions for the model, it will be assumed that the consumer has an isoelastic utility function, which is represented as:

$$u\left(c_t\right) = \frac{c_t^{1-\gamma}}{1-\gamma}\,. \tag{32}$$

---

[22] This amounts to say that the representative agent knows the structure of the economy, a common assumption in rational expectations models. In a related paper (Rodriguez, 2001), I explore some consequences of relaxing this assumption.

As has been widely noted in the literature, the isoelastic utility function restricts the coefficient of relative risk aversion γ to be the reciprocal of the coefficient of intertemporal substitution. This paper shows that decomposing random innovations into permanent and transitory components provides a natural way to distinguish the effects of risk aversion from those of intertemporal substitution in the endogenously generated volatility of returns within the separable utility framework.

4.1. The Term Structure of Interest Rates

In this part, in order to study the role of transitory innovations to determine the dynamics of the discount rate, I derive closed form expressions for the price of a riskless zero-coupon bond, and the term structure of the interest rates.

Let $b(t, \tau)$ be the price of a pure discount bond that promises to pay one unit of consumption τ periods ahead. From the first order conditions of utility maximization, $b(t, \tau)$ must satisfy:

$$b(t,\tau) = e^{-\rho\tau} E_t\left[\frac{u'(c_{t+\tau})}{u'(c_t)}\right]. \tag{33}$$

Replacing the utility function in (33), we get

$$b(t,\tau) = e^{-\rho\tau} E_t\left(\frac{q_{t+\tau}}{q_t}\right)^{-\gamma}. \tag{34}$$

The price of a pure discount bond is the expected present value of one unit of consumption to be received τ periods ahead. Therefore, it can be understood as the expectation of the stochastic discount factor of the economy: $e^{-\rho\tau}\frac{u'(q_{t+\tau})}{u'(q_t)}$, conditional on the information up to time t. Writing $log\left(\frac{q_{t+\tau}}{q_t}\right) = L_\tau$, (34) becomes:

$$b(t,\tau) = E_t\left(e^{-(\gamma L_\tau + \rho\tau)}\right). \tag{35}$$

Lets $R(t,\tau) = \gamma\frac{L_\tau}{\tau} + \rho$, be the implied τ-period yield to maturity. Then, recalling equation (18), equation (35) can be written as the moment generating function of the sum of a normal plus a compound Poisson process:

$$\begin{aligned} b(t,\tau) &= e^{-\tau R(t,\tau)} \\ &= exp\left[-\gamma\left[\mu\tau + (\mu t - y_t)(1-e^{-\alpha\tau})\right] - \rho\tau + \lambda\tau(e^M - 1) + \frac{\gamma^2}{4\alpha}\omega^2(1-e^{-2\alpha\tau})\right] \end{aligned} \tag{36}$$

where:

$$M = -\gamma\theta + \frac{\gamma^2}{2}\sigma^2. \tag{37}$$

Note that (36) implies that the real price of the bond can be written as a function of t, τ and y_t:

$$b(t,\tau,y_t). \tag{38}$$

Applying Ito's lemma to (36), the price change of the bond over the next instant is:

$$db = \left(r_t + \gamma\omega^2\left(1-e^{-\alpha(s-t)}\right)\right)b(\tau,y_t)dt + \gamma\left(1-e^{-\alpha\tau}\right)b(\tau,y_t)\sigma dZ_t, \tag{39}$$

where r_t is the risk-free instantaneous interest rate (see equation (42))[23].

Note that:

[23] Risk premia in bond markets will not be discussed in this paper, but it is interesting to note that in the model bonds command a risk premium only if $\alpha > 0$.

$$\gamma\left(1-e^{-\alpha\tau}\right)=\frac{1}{b}\frac{\partial b}{\partial y_t}, \tag{40}$$

is the semi-elasticity of the price of the bond with respect to changes in the stationary component. This semi-elasticity measures the percentage change of the expected stochastic discount factor after an unanticipated transitory shock. It can also be understood as the change in the stochastic discount rate induced by an unanticipated transitory shock. The expectation revision is directly related to α: the higher α, the faster the transitory component is expected to return to its long run value after having been shocked by a random innovation. And it is also directly related to γ: the higher γ, the more valuable becomes one unit of future consumption if the economy has experienced an unanticipated positive transitory shock.

The implied τ-period yield to maturity is:

(41)

$$\begin{aligned}R(t,\tau) &= -\frac{log\left[b(t,\tau)\right]}{\tau}\\ &=\rho+\gamma\left[\mu+\left(\mu t-y_t\right)\frac{\left(1-e^{-\alpha\tau}\right)}{\tau}\right]-\lambda\left(e^{M}-1\right)-\gamma^2\omega^2\frac{\left(1-e^{-2\alpha\tau}\right)}{4\alpha\tau}.\end{aligned}$$

As τ goes to 0, equation (32) collapses to the instantaneous interest rate:

$$r_t=\rho+\gamma\left[\mu+\alpha\left(\mu t-y_t\right)\right]-\lambda\left(e^{M}-1\right)-\frac{\gamma^2}{2}\omega^2. \tag{42}$$ [24]

Equation (28) shows that the variability of the instantaneous risk-free interest rate depends only on the transitory component of the endowment process. Moreover, its

[24] Different version of this equation have been derived in the literature. See, for example, Breeden (1979), Naik and Lee (1990), and Bakshi and Chen (1996).

variance is proportional to the variance of y_t, the constant of proportionality being the square of the product of γ times the reversion parameter in y_t:

$$Var(r_t) = (\gamma\alpha)^2 Var(y_t). \tag{43}$$

Permanent innovations reduce the level of the instantaneous risk-free interest rate through a precautionary effect (the term $-\lambda(e^M - 1)$), but do not affect its variance. Finally, when $\alpha = 0$, the instantaneous risk-free rate is constant.

4.2. The Price-Dividend Ratio

In this part I derive analytical expressions for the volatility of stock returns and for the equity premium. First I need to calculate the price-dividend ratio. In order to do that, let's rewrite first order condition (23) as:

$$S_t = e^{-\rho dt} E_t\left[\frac{u'(c_{t+dt})}{u'(c_t)}\left(S_{t+dt} + q_{t+dt}\right)\right]. \tag{44}$$

Then, after repeated forward substitution, and assuming that the following transversality condition is satisfied:

$$\lim_{s\to\infty} e^{-\rho(s-t)} E_t\left[\frac{u'(q_s)}{u'(q_t)} S_s\right] = 0, \tag{45}$$

equation (44) can be written as:

$$S_t = E_t \int_t^\infty e^{-\rho(s-t)} \frac{u'(q_s)}{u'(q_t)} q_s ds. \tag{46}$$

Equation (46) expresses the price of the risky asset as the expected value of future dividends, which are discounted by the stochastic discount factor:

$$e^{-\rho(s-t)}\frac{u'(q_s)}{u'(q_t)}. \tag{47}$$

Dividing both sides of (46) by q_t, the price-dividend ratio can be written as:

$$\frac{S_t}{q_t}=E_t\int_t^{\infty}e^{-\rho(s-t)}\frac{u'(q_s)}{u'(q_t)}\frac{q_s}{q_t}ds. \tag{48}$$

Equation (48) shows that the price-dividend ratio embodies consumer's forecast, given the information generated by x_t and y_t, of future dividend growth and discount rates. Replacing the utility function, (48) reads:

$$\frac{S_t}{q_t}=E_t\int_t^{\infty}e^{-\rho(s-t)}\left(\frac{q_s}{q_t}\right)^{1-\gamma}ds. \tag{49}$$

Passing the expectation operator through the integral, it follows that:

$$\frac{S_t}{q_t}=\int_t^{\infty}g(y_t,s)ds, \tag{50}$$

where:

(51)

$$g(t,y_t)=exp\left[(1-\gamma)\left[\mu(s-t)+(\mu t-y_t)\left(1-e^{-\alpha(s-t)}\right)\right]-\rho(s-t)+\lambda(s-t)\left(e^{M_1}-1\right)+\frac{(\gamma-1)^2}{4\alpha}\omega^2\left(1-e^{-2\alpha(s-t)}\right)\right]$$

In (51) M_1 is:

$$M_1=(1-\gamma)\theta+\frac{(1-\gamma)^2}{2}\sigma^2. \tag{52}$$

$g(y_t,s)$ is the expected present value of $\frac{q_s}{q_t}$ units of consumption to be received *s-t* periods ahead, discounted by the stochastic discount factor.

The finiteness of the integral in (50) is guaranteed by the condition:

$$(\gamma-1)\mu+\rho-\lambda\left(e^{M_1}-1\right)>0\,. \tag{53}$$

Equation (50) shows that the price-dividend ratio is a function of y_t:

$$\frac{S_t}{q_t}=h(y_t)\,. \tag{54}$$

The rate of return of the risky asset is obtained after multiplying equation (54) by q_t, and applying Ito's lemma:

$$\frac{dS_t}{S_t}=k_t dt+\left(1+\frac{h_y}{h}\right)\omega dZ_t+\left(e^{u}-1\right)dN_t\,, \tag{55}$$

where k_t is the instantaneous expected rate of return (see equation 61). Note that when α = 0 (all shocks are permanent), h_y = 0, and the volatility of the rate of return is equal to the volatility of the consumption rate of growth (equation 25). On the other hand, h_y will be shown to be positive, when γ is higher than one, only if α is positive. In this way, transitory innovations have generated excess volatility.

To further investigate the excess volatility generated by the model, let's analyze the expression $\frac{h_y}{h}$. Applying Leibniz's rule of derivation under the integral sign to equation (50), we get:

$$h_y=\int_t^{\infty}(\gamma-1)\left(1-e^{-\alpha(s-t)}\right)g(y_t,s)ds\,, \tag{56}$$

The expression under the integral sign can be expressed as:

$$\left(\gamma-1\right)\left(1-e^{-\alpha(s-t)}\right)g\left(y_t,s\right)=\frac{\partial g\left(y_t,s\right)}{\partial y_t}, \tag{57}$$

where $\frac{\partial g\left(y_t,s\right)}{\partial y_t}$ is the revision of the expected present value of $\frac{q_s}{q_t}$ units of consumption to be received *s-t* periods ahead, after a transitory shock has occurred. Equation (57) can be decomposed in:

$$g\left(y_t,s\right)\left[\gamma\left(1-e^{-\alpha(s-t)}\right)-\left(1-e^{-\alpha(s-t)}\right)\right]. \tag{58}$$

The negative term in (58) reflects that $\left(1-e^{-\alpha(s-t)}\right)$ of a unitary positive shock to the endowment is expected to vanish after *s-t* periods. As the value of one unit of consumption to be received in future periods increases (recall that the representative agent has a concave utility function), the stochastic discount rate decreases. This explains the positive term in (58). If $\gamma > 1$, the overall expression is positive.

Therefore, h_y *is the sum (the integral) of revisions in expectations of future consumption growth and stochastic discount rates*.

Now, let's write equation (56) as

$$h_y=\int_t^{\infty} g\left(y_t,s\right)\left[\gamma\left(1-e^{-\alpha(s-t)}\right)-\left(1-e^{-\alpha(s-t)}\right)\right]ds\ . \tag{59}$$

Dividing equation (59 by h, $\frac{h_y}{h}$ can be expressed as:

$$\frac{h_y}{h}=\int_t^{\infty}\frac{g\left(y_t,s\right)}{\int_t^{\infty} g\left(y_t,s\right)}\left[\gamma\left(1-e^{-\alpha(s-t)}\right)-\left(1-e^{-\alpha(s-t)}\right)\right]ds\ . \tag{60}$$

The term $\frac{h_y}{h}$ is a weighted average of expected changes in the rate of growth of the economy and expected changes in the stochastic discount rate[25]. Each weight is the expected present value of one unit of consumption to be received *s-t* periods ahead, relative to the entire stream of discounted consumption. These weights come from the desire of the representative consumer to smooth her consumption path. Equation (60) is surprising, because $\frac{h_y}{h}$ determines the part of the volatility of returns that cannot be explained by the volatility of consumption. What it says is that this component of the volatility of returns is essentially an intertemporal substitution phenomenon.

Note that, assuming $\gamma > 1$, $\frac{h_y}{h} > 0$ only if $\alpha > 0$, and also that $\frac{h_y}{h}$ is an increasing function of α.

Transitory innovations affect the equity premium through the increase in the volatility of returns:

$$\frac{1}{dt}E_t\left(\frac{dS_t}{S_t}\right)+\frac{q_t}{S_t}-r_t=\gamma\omega^2\left(1+\frac{h_y}{h}\right)-\lambda E_t\left[\left(e^{-\gamma u}-1\right)\left(e^u-1\right)\right]. \qquad (61)$$

In the extreme case in which shocks are permanent, $h_y = 0$. The term $\gamma\omega^2\frac{h_y}{h}$ is the increase in the equity premium due to the presence of a stationary component, relative to the case in which all innovations are permanent. It will be shown in part 5 that, depending on the model parameters, the proportion of the equity premium due to the stationary component is substantial.

---

[25] See equations (21) and (40).

5. Numerical Results

The process for the log endowment presented in equation (11) was calibrated for the moments of the distribution of consumption using data from the Shiller database (see appendix 2 for details).

The model was simulated by generating 1000 realizations of the processes N_t and Z_t. The transitory component was calculated using an exact discretization[26] of (13):

$$log(y_k) = \mu t_k + \left[log(y_{k-1}) - \mu t_{k-1}\right]e^{-\alpha} + \omega\sqrt{\frac{1-e^{-2\alpha}}{2\alpha}}\varepsilon_k, \tag{62}$$

where ε_k is a sequence of random variables iid, normally distributed with 0 mean and variance 1.

The permanent component was generated from:

$$x_k = x_{k-1} + u\Delta N_k, \tag{63}$$

where $u \sim N(\theta^2, \sigma^2)$, and N is a Poisson counter with intensity λ.

Once the series for y was generated, the price dividend ratio was calculated integrating numerically equation (50) using a 12-node, Gauss-Laguerre quadrature scheme. The scheme selects quadrature nodes, t_i, and weights, w_i, to approximate the integral in (50) as:

$$\int_0^\infty e^{-ru} f(u)du \cong \frac{1}{r}\sum_1^{12} w_i f\left(\frac{t_i}{r}\right), \tag{64}$$

where:

[26] For the use of exact discretizations in the simulation and estimation of stochastic differential equations, see Gourieroux and Monfort (1997).

$$r = \rho + (\gamma - 1)(\mu + \lambda\theta) - \lambda\left(e^{M_1} - 1\right), \tag{65}$$

and:

$$f(u) = exp\left[(1-\gamma)\left[(\mu t - y_t)\left(1 - e^{-\alpha u}\right)\right] + \frac{(1-\gamma)^2}{2}\frac{\omega^2}{2\alpha}\left(1 - e^{-2\alpha u}\right)\right]. \tag{66}$$

Results from estimations in part 4 were used to calibrate the model. μ, the unconditional mean of $log\left(\frac{c_k}{c_{k-1}}\right)$, was selected equal to .0178. The formula for the unconditional variance is:

$$v^2 = \lambda\left(\theta^2 + \sigma^2\right) + \frac{\omega^2\left(1 - e^{-\alpha}\right)}{\alpha}, \tag{67}$$

and the value chosen for v^2 was .0345, to match the estimated value in the series from the Shiller database. The other parameters selected using estimates from part 2 were: θ = -.1164, and λ = .01. The parameter α was chosen as –ln(.79) = .24.

Because it is not possible to estimate separately σ and ω, they were left as free parameters, and calibration results are shown under different values of the ratio $\omega^* = \frac{\omega^2\left(1 - e^{-\alpha}\right)/\alpha}{v^2}$. ω^* is the proportion of the variance of the one year consumption rate of growth explained by the transitory component.

The intertemporal preference parameter ρ is assumed equal to .01. Finally, results are shown under different values of the parameter γ.

The calibration exercise intends to meet the challenge posed by Mehra and Prescott (1984): to show an artificial economy that generates an equity premium of the order of 6%, and a risk free rate lower than 4%, under the restrictions that $\gamma < 10$, and $\rho > 0$.

Additionally, it is explored if the artificial economy is able to match simultaneously second moments of the distribution of returns.

Table (5) shows calibration results for values of γ varying from 5 to 10. Results are presented for different values of the ratio ω^*. The range of the ratio varies in the different tables to meet the transversality condition (53).

Table (5) shows that for all values of γ, the standard deviation of returns increases monotonically with the ratio ω^*: the higher the proportion of the variance of the change in the log endowment explained by the transitory component, the higher the variance of returns. Interestingly, for $\gamma < 9$, in all tables can be found combinations of γ and the ratio ω^* producing high variability of returns without excessive variation in the risk-free interest rate.

Consider now the case in which $\gamma = 7$. For $\omega^* = .4$, the model gives volatility of returns of 13.24%, risk-free rate equal to 3.73%, volatility of the risk-free rate equal to 5.43%, and an equity premium of 5.74%. . For $\gamma = 8$ and $\omega^* = .5$, the model gives volatility of returns of 16.72%, risk-free rate equal to 2.93%, volatility of the risk-free rate equal to 6.94%, and an equity premium of 7.46%. These values must be compared to those obtained from data: 17.76%, 1.9%, 5.82%, and 5.38%, respectively.

The ability of the model to generate high volatility of returns does not rely on strong negative autocorrelation of consumption growth. Table (7) shows first autocorrelations of consumption growth generated endogenously by the model for different values of ω^*. Obviously, the autocorrelation is higher (in absolute value) the higher is the proportion of the variance of consumption growth explained by the transitory component, but it is always kept lower (in absolute value) than .1. In particular, for the values of ω^* mentioned above, the autocorrelations are -.0427, and -.0533. This shows that a model with a persistent stationary component is capable to generate high volatility of returns without imposing implausible negative values to the first autocorrelation of consumption. It also shows that a model of this kind can generate simultaneously high volatility of returns and low volatility of the real interest rate[27].

Finally, table (6) compares the models discussed in the above paragraphs, to models in which all innovations are permanent ($\alpha = 0$). The results are striking: the volatility of returns is more than four times higher in the model in which there is a transitory component. Moreover, the equity premium is more then 40% higher, showing that the increase in the volatility of returns generated by the interplay between predictability and the elasticity of intertemporal substitution plays a quantitatively important role in explaining the equity premium.

6. Conclusions

Results in this paper reveal a surprising connection between the predictability of dividends and the volatility of returns, and shed light on the role that intertemporal substitution plays in explaining asset pricing anomalies. They also show that, although jump risk contributes to increase the equity premium, and fundamentally to reduce the risk-free interest rate, it has no role in explaining returns volatility beyond the volatility of consumption growth. When all innovations are permanent there is no excess volatility, no matter the size or the variability of the jump. Intertemporal substitution helps to raise the equity premium through the increase in the volatility of returns.

[27] Compare to Campbell (1996), referring to equilibrium asset pricing models: "…it is hard to produce

The ability of the model to match first and second moments depends on the relative importance of transitory versus permanent innovations in accounting for the volatility of consumption growth[28]. The model renders a good match of the data for values of ω* between .4 and .5. These values do not seem unreasonable regarding the results obtained in the empirical literature[29].

One limitation of the model is that it allows for only one break in the trend. This is a problem of the estimation method. One way to avoid this limitation is to relax the restriction that makes consumption and dividends equal. Perron (1989) found evidence of breaks both in the levels and in the slope of the dividends series. Separating dividends from consumption will enrich the model, allowing the incorporation of more jumps, and the study of the correlation among jump processes affecting simultaneously the dividend and the discount rate of the economy.

Another possible extension of the model is relaxing the assumption that agents are able to distinguish transitory from permanent innovations. The importance of the type of innovation in the generation of results by the model suggests that the study of the role of incomplete information (a signal extraction problem) can be a fruitful agenda of research in order to understand asset pricing anomalies, especially the excess volatility puzzle.

sufficient variation in stock prices without excessive variation in expected consumption growth and in riskless real interest rates".

[28] Christiano and Eichenbaum (1991) were the first to point out that what really matters in rational expectation models is the relative importance of transitory versus permanent shocks, and not if the processes should be described as difference or trend stationary, an issue that it is impossible to settle given the actual span of macroeconomic data.

[29] See footnote (2) for references.

Table 5

Moments of the distribution of returns for values of γ between 5 and 10, and ω* between .1 and .8. Values of ω* were selected so that the transversality condition in (43) is satisfied. Highlighted are combinations of first and/or second moments that satisfy the following constraints: standard deviation of returns (std ret) between 13% and 18%, risk-free rate (rikfree) lower than 4%, standard deviation of the risk-free rate (std rfree) lower that 7%, and equity premium (RP) higher than 5%.

γ = 5 ω*	Mean ret	Std ret	Riskfree	std rfree	RP	std RP
0.1	7.63	5.02	4.94	1.94	2.69	4.66
0.2	8.03	6.96	5.55	2.74	2.48	6.44
0.3	8.41	8.44	6.06	3.36	2.35	7.80
0.4	8.75	9.69	6.48	3.88	2.28	8.94
0.5	9.08	10.78	6.81	4.34	2.27	9.94
0.6	9.38	11.77	7.07	4.75	2.30	10.84
0.7	9.66	12.67	7.28	5.14	2.38	11.67
0.8	9.92	13.52	7.43	5.49	2.49	12.44

γ = 6 ω*	Mean ret	Std ret	Riskfree	std rfree	RP	std RP
0.1	7.05	5.99	1.57	2.33	5.48	5.56
0.2	7.99	8.26	3.40	3.29	4.59	7.64
0.3	8.81	9.96	4.83	4.03	3.97	9.18
0.4	9.52	11.37	5.94	4.66	3.58	10.45
0.5	10.15	12.60	6.79	5.21	3.36	11.56
0.6	10.70	13.70	7.42	5.71	3.27	12.56
0.7	11.18	14.72	7.89	6.16	3.29	13.47
0.8	11.61	15.66	8.22	6.59	3.39	14.32

γ = 7 ω*	Mean ret	Std ret	Riskfree	std rfree	RP	std RP
0.1	3.82	7.56	-8.27	2.72	12.08	7.10
0.2	6.11	10.00	-2.96	3.84	9.08	9.32
0.3	7.97	11.78	0.92	4.71	7.05	10.90
0.4	**9.47**	**13.24**	**3.73**	**5.43**	**5.74**	**12.17**
0.5	10.70	14.51	5.75	6.08	4.95	13.28
0.6	11.70	15.65	7.17	6.66	4.53	14.28
0.7	12.53	16.72	8.15	7.19	4.38	15.22
0.8	13.22	17.72	8.80	7.69	4.42	16.10

Table 5 (Cont.)

Moments of the distribution of returns for values of γ between 5 and 10, and ω* between .1 and .8. Values of ω* were selected so that the transversality condition in (43) is satisfied. Highlighted are combinations of first and second moments that satisfy the following constraints: standard deviation of returns (std ret) between 13% and 18%, risk-free rate (riskfree) lower than 4%, standard deviation of the risk-free rate (std rfree) lower that 7%, and equity premium (RP) higher than 5%.

γ = 8 ω*	Mean ret	Std ret	riskfree	std rfree	RP	std RP
0.3	4.69	14.87	-8.80	5.38	13.49	13.95
0.4	7.94	15.72	-1.75	6.21	9.68	14.56
0.5	**10.39**	**16.72**	**2.93**	**6.94**	**7.46**	**15.34**
0.6	12.23	17.73	5.99	7.61	6.25	16.15
0.7	13.64	18.73	7.94	8.22	5.70	16.98
0.8	14.71	19.73	9.13	8.78	5.58	17.82

γ = 9 ω*	Mean ret	std ret	riskfree	std rfree	RP	std RP
0.5	8.59	19.82	-3.17	7.81	11.77	18.37
0.6	12.02	20.14	3.29	8.56	8.73	18.40
0.7	14.40	20.84	7.06	9.24	7.33	18.85
0.8	6.06	21.71	9.19	9.88	6.87	19.52

γ =10 ω*	Mean ret	std ret	riskfree	std rfree	RP	std RP
0.6	10.64	23.44	-1.98	9.51	12.62	21.61
0.7	14.67	23.18	5.23	10.27	9.44	20.99
0.8	17.23	23.71	8.90	10.98	8.33	21.24

Table 6:

Compares the magnitudes of the volatility of returns and equity premium between two models that differ only in the presence of a transitory component. Column A shows the quotient std ret (α = .24)/ std ret (α = 0). Column B shows the quotient RP(α = .24)/ RP(α = 0).

γ = 7	Mean ret	std ret	A	Riskfree	std rfree	RP	B	std RP
ω*=.4, α =.24	9.47	13.24	4.01	3.73	5.43	5.74	1.45	12.17
ω* = .4, α = 0	7.94	3.30		3.97	0.00	3.96		3.30
γ = 8	**Mean ret**	**std ret**	**A**	**Riskfree**	**std rfree**	**RP**	**B**	**std RP**
ω*=.5, α=.24	10.38	16.72	4.90	2.93	6.94	7.46	1.64	15.34
ω*=.5, α = 0	7.85	3.41		3.29	0.00	4.56		3.41

Table 7:

First autocorrelation of the consumption rate of growth for different values of ω*.

ω*	corr
0.1	-0.0107
0.2	-0.0213
0.3	-0.0320
0.4	-0.0427
0.5	-0.0533
0.6	-0.0640
0.7	-0.0747
0.8	-0.0853

APPENDIX 1

Values of the t-statistic of the α estimator in equation (56) using different values of k.

K = 0	-5.11
K=1	-4.87
K=2	-5.32
K=3	-5.39
K = 4	-5.37
K = 5	-5.64
K = 6	-5.69
K = 7	-5.86
K = 8	-6.17
K = 9	-5.46
k = 10	-5.85

APPENDIX 2

Empirical moments of Consumption and Returns Annual Data for 1889-1985

Consumption growth rate	
Mean	1.69%
Standard deviation	3.45%
Real Returns	
Mean	7.28%
Standard deviation	17.76%
Real interest rate	
Mean	1.9%
Standard deviation	5.82%
Equity Premium	
Mean	5.38%
Standard deviation	19.00 %

Source: Shiller (http://www.econ.yale.edu/shiller)

References

Backus, D., Gregory, A., and Zin, S., "Risk Premiums in the Term Structure: Evidence from Artificial Economies", 1989, *Journal of Monetary Economics*, 24, pp. 371-399.

Bakshi, G., and Chen, Z., "Inflation, Asset Prices, and The term Structure of Interest Rates in Monetary Economies", *Review of Financial Studies*, 1996, v 9, pp. 241-275.

Bates, D., "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutschemark Options", *Review of Financial Studies*, 1996, v 9, pp. 69-108.

Balke, N., and Fomby, T., "Shifting Trend, Segmented Trends, and Infrequent Permanent Shocks", *Journal of Monetary Economics*, 1991, 28, pp. 61-85.

Blanchard, O., and Quah, D., "The Dynamic Effects of Aggregate Demand and Supply Disturbances", *American Economic Review*, 1989, 79, pp. 655-673.

Breeden, D., "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities", *Journal of Financial Economics*, 1979, 7, pp. 265-296.

Burnside, C., "Solving Asset Pricing Models with Gaussian Shocks", 1998, *Journal of Economic Dynamics and Control*, pp. 329-340.

Campbell, J., "Bond and Stock Returns in a Simple Exchange Model", *Quarterly Journal of Economics*, 1986, 101, pp. 785-803.

_______, "A Variance Decomposition of Stock Returns", *Economic Journal*, 1991, 101, pp. 157-69.

_______, "Consumption and the Stock Market: Interpreting International Experience", 1996, NBER Working Paper No. 5610.

_______, Lo, A. , and MacKinlay, A., *The Econometrics of Financial Markets*, 1997, New Jersey, Princeton University Press.

_______, and Mankiw, N., "Are Output Fluctuations Transitory?", *Quarterly Journal of Economics*, 1987, 102, pp. 857-880.

_______, and Shiller, R., "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors", *Review of Financial Studies*, 1988a, 1, pp. 195-227.

_______, "Stock Prices, Earnings, and Expected Dividends", *Journal of Finance*, 43, 661-676.

Cecchetti, M., Lam, P., and Mark, N., “Mean Reversion in Equilibrium Asset Prices”, *American Economic Review*, 1990, 80, pp. 398-418.

________, “The Equity Premium and the Risk Free Rate: Matching the Moments”, *Journal of Monetary Economics*, 1993, 31, pp. 21-45.

Christiano, L., “Searching for a Break in GNP”, 1992, *Journal of Business and Economic Statistics*, 1992, 10, pp. 237-250.

Christiano, L., and Eichenbaum, M., “Unit Roots in Real GNP: Do We Know, and Do We Care?, *Carnegie-Rochester Conference Series on Public Policy*, 1991, 32, pp. 7-62.

Clark, P., “The Cyclical Component of U.S. Economic Activity, *Quarterly Journal of Economics*, 1987, pp. 797’814.

Cochrane, J., “How Big is the Random Walk in GNP?”, *Journal of Political Economy*, 1988, 96, pp. 893-920.

Deaton, A., *“Understanding Consumption”*, 1992, Oxford, Clarendon Press.

Gourieroux, Ch., and Monfort, A., “*Simulation Based Econometrics Methods*”, 1997, Oxford University Press.

Grossman, S., and Shiller, R., “The Determinants of the Variability of Stock Market Prices”, *American Economic Review,* 1981, 71, 222-227.

Kandel, S., and Stambaugh, R., “Expectations and Volatility of Consumption and Asset Returns”, *Review of Financial Studies*, 1990, 3, pp. 207-32.

___________, Asset Returns and Intertemporal Preferences”, *Journal of Monetary Economics,* 1991, 27, pp. 39-71.

Kocherlakota, N., “On the ‘Discount’ Factor in Growth Economies”, *Journal of Monetary Economics*, 1990, 25, pp. 43-47.

_______, “On Tests of Representative Consumer Asset Pricing Models”, *Journal of Monetary Economics,* 1990, 26, pp. 285-304.

Leroy, S., and Porter, R., “The Present Value Relation: Tests Based on Variance Bounds”, 1981, *Econometrica*, 49, pp: 555-577.

Lo, A., and Wang, J., “Implementing Option Pricing Models when Asset Returns are Predictable”, Journal of Finance, 1995, 50, pp. 87-129.

Lucas, R., “Asset Prices in an Exchange Economy”, *Econometrica*, 1978, 46, pp. 1429-45.

Mehra, R., and Prescott, E., "The Equity Premium: A Puzzle", *Journal of Monetary Economics*, 1985, 15, pp.145-61.

Nelson, Ch., and Plosser, C., "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications", *Journal of Monetary Economics*, 1982, 10, pp. 139-162.

Pankratz, A., "*Forecasting with Dynamic Regression Models*", 1991, John Wiley & Sons.

Perron, P., "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis", 1989, *Econometrica*, 1989, 57, pp. 1361-1401.

Naik, V., Lee, M., AGeneral Equilibrium Pricing of Options on the Market Portfolio with Discontinuous Returns≅, *Review of Financial Studies*, 1990, 3, pp. 493, 521.

Quah, D., "The Relative Importance of Permanent and Transitory Components: Identification and some Theoretical Bounds", *Econometrica*, 1992, 60, pp. 107-18.

Rappoport, P., and Reichlin, L., "Segmented Trends and Non-Stationary Time Series*", The Economic Journal*, 1989, 99, pp. 168-177.

Rietz, T., "The Equity Premium: a Solution", *Journal of Monetary Economics*, 1988, 22, pp. 117-33.

Rodriguez, J., "Asset Prices with Incomplete Information: Can Learning Solve the Puzzles?", 2001, Working Paper, University of Maryland, College Park.

Shiller, R., "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?", *American Economic Review*, 1981, 71, 421-426.

Watson, M., "Univariate Detrending Methods with Stochastic Trends", *Journal of Monetary Economics*, 1986, 18, pp. 49-75.

Weil, P., "The Equity Premium Puzzle and the Risk-Free Rate Puzzle", *Journal of Monetary Economics*, 1989, 24, pp. 401-421.

Zivot, E., and Andrews, D., "Further Evidence on the Great Crash, the Oil Price Shock, and the Unit Root Hypothesis", *Journal of Business and Economic Statistics*, 1992, 10, pp. 251-270.