Adaptive regression on the real line 
in classes of smooth functions

L.M. Artiles Martínez †
B.Y. Levit ‡

May 14, 2002

Abstract: Adaptive pointwise estimation of an unknown regression function $f(x), x \in \mathbb{R}$ corrupted by additive Gaussian noise is considered in the equidistant design setting. The function $f$ is assumed to belong to the class $A(\alpha)$ of functions whose Fourier transform are rapidly decreasing in the weighted $L^2$-sense. The rate of decrease is described by a weight function that depends on the vector of parameters $\alpha$ which, in the adaptive setting, is typically unknown.

First, for any of the classes $A(\alpha)$, $\alpha$ fixed, we describe minimax estimators up to a constant as the bin-width goes to zero. Then we allow $\alpha$ to vary freely inside large scales $K$. Conditions under which an adaptive study is suitable are presented and a notion of adaptive asymptotic optimality is introduced based on distinguishing, among all possible functional scales, between the so-called non-parametric (NP) and pseudo-parametric (PP) scales.

Finally we propose adaptive estimators which ‘tune up’ point-wisely to the unknown smoothness of $f$. We prove them to be asymptotically adaptively minimax for large collections of NP functional scales, subject to being rate efficient for any of the PP functional scales.

Keywords: Non-parametric statistics, minimax estimation, adaptive estimation, Fourier transformations.

AMS Subject Classification: Primary 62G05, 62G20; secondary 62C20.

†EURANDOM, P.O. Box 513 - 5600 MB Eindhoven, The Netherlands
‡Department of Statistics, Queen’s University, Kingston, Ontario, Canada, K7L 3N6