Abstract: We study the thermodynamic formalism for generalized Gibbs measures, such as renormalization group transformations of Gibbs measures or joint measures of disordered spin systems. We first show existence of the relative entropy density and obtain a familiar expression in terms of entropy and relative energy for "almost Gibbsian measures" (almost sure continuity of conditional probabilities). We also describe these measures as equilibrium states and establish an extension of the usual variational principle. As a corollary, we obtain a full variational principle for quasilocal measures. For the joint measures of the random field Ising model, we show that the weak Gibbs property holds, with an almost surely rapidly decaying translation invariant potential. For these measures we show that the variational principle fails as soon as the measures loses the almost Gibbs property. These examples suggest that the class of weakly Gibbsian measures is too broad from the perspective of a reasonable thermodynamic formalism.

Keywords: Gibbs vs non-Gibbs, generalized Gibbs measures, variational principle, renormalization group, disordered systems, random field Ising model.

AMS Subject Classification: 60G60 (Primary), 82B20, 82B30 (Secondary).