Abstract: We consider a discrete-time multi-server queue for which the moments of the stationary queue length can be expressed in terms of series over the zeros in the closed unit disk of a queue-specific characteristic function. In many important cases these zeros can be considered to be located on a queue-specific curve, called generalized Szegö curve. By adopting a special parametrization of these Szegö curves, the relevant zeros occur as equidistant samples of a $2\pi$-periodic function whose Fourier coefficients can be determined analytically. Thus the series occurring in the expressions for the moments can be written as Fourier aliasing series with terms given in analytic form. This gives rise to formulas for e.g. the mean and variance of the queue length that are reminiscent of Spitzer’s identity for the moment generating function of the steady-state waiting time for a $G/G/1$ queue. Indeed, by considering the queue under investigation as a $G/G/1$ queue, the new formulas for the mean and variance also follow from Spitzer’s identity. The approach in this paper can also be used to compute the probability distribution function of the queue length in analytic form.

Keywords: Discrete-time queue, multi-server, Szegö curve, Spitzer’s identity, Fourier sampling.

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