On an Argument of David Deutsch

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Abstract: We analyse an argument of Deutsch, which purports to show that the deterministic part of classical quantum theory together with deterministic axioms of classical decision theory, together imply that a rational decision maker behaves as if the probabilistic part of quantum theory (Born’s law) is true. We uncover two missing assumptions in the argument, and show that the argument also works for an instrumentalist who is prepared to accept that the outcome of a quantum measurement is random in the frequentist sense: Born’s law is a consequence of functional and unitary invariance principles belonging to the deterministic part of quantum mechanics. Unfortunately, it turns out that after the necessary corrections we have done no more than give an easier proof of Gleason’s theorem under stronger assumptions. However, for some special cases the proof method gives positive results while using different assumptions to Gleason. This leads to the conjecture that the proof could be improved to give the same conclusion as Gleason under unitary invariance together with a much weaker functional invariance condition.

The first draft of this paper dates back to early 1999, was posted on my webpage, but never completed. It has since been partly overtaken by Barnum et al. (2000), Saunders (2002), and Wallace (2002). However there remain new points of view, new results, and most importantly, a still open conjecture.

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