Symmetric Measures via Moments
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Abstract: A finite $G \leq GL(m, \mathbb{R})$ fixes $\Omega \subset \mathbb{R}^m$ and induces its action on $\mathcal{P}$, the set of probability distributions on $\Omega$. $\mathcal{P}^G$ is the set of distributions invariant under this action. We consider models based on $\mathcal{P}^G$. Ignoring the invariance, a common approach to modeling $P \in \mathcal{P}$ is to progressively match its moments. Among all distributions with a requested match, one reasonable choice is $P^0$ that maximizes the entropy $H(P^0)$. Matching in the limit all the moments guarantees convergence to $P$ if $P$ is uniquely determined by its moments. We thereby generalize ordinary determinacy to determinacy within $\mathcal{P}^G$ and prove sufficiency of $G$-invariant moments for the latter. Using generators of $G$-invariant polynomials, we also give several sufficient conditions for the generalized property to hold. For applications, we propose a sequential procedure with adaptive convergence toward $P$. The procedure combines with one’s favorite statistical model selection principle, and we present two such examples. We also describe a distribution of small subimages extracted from a large database of natural images, and compute generators for the relevant invariance. We discuss computations of $G$-invariant probability distributions. For example, concerned with computational efficiency, we lift the invariantsly constrained entropy maximization problem to an appropriate quotient space of “lower dimension”.

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