Abstract: We prove asymptotic normality for $L_k$-functionals $\int |\hat{F}_n - F_n|^k \, g(t) \, dt$, where $F_n$ is the empirical distribution function of a sample from a decreasing density and $\hat{F}_n$ is the least concave majorant of $F_n$. From this we derive two test statistics for the null hypothesis that a probability density is monotone. These tests are compared with existing proposals such as the supremum distance between $\hat{F}_n$ and $F_n$. 