Sub-additivity re-examined: the case for Value-at-Risk*

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Abstract

This paper studies the issue of sub-additivity of Value-at-Risk (VaR) for heavy tailed asset returns. Using the notion of “regular variation” to define heavy tailed distribution, we establish that for heavy tailed asset return distributions with well defined mean, VaR is sub-additive in the tail region, the most relevant region for risk management. This is further demonstrated with the help of Monte Carlo simulation of 95% and 99% VaR for asset returns following three categories of bivariate distributions. Our results provide a new dimension into the ongoing debate over non sub-additivity of VaR.

KEY WORDS: Value-at-Risk, sub-additivity, regular variation, tail index, heavy tailed distribution.

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