Random graphs with arbitrary i.i.d. degrees

Abstract: In this paper we study distances and connectivity properties of random graphs with an arbitrary i.i.d. degree sequence. When the tail of the degree distribution is regularly varying with exponent $1 - \tau$ there are three distinct cases: (i) $\tau > 3$, where the degrees have finite variance, (ii) $\tau \in (2, 3)$, where the degrees have infinite variance, but finite mean, and (iii) $\tau \in (1, 2)$, where the degrees have infinite mean. These random graphs can serve as models for complex networks where degree power laws are observed. The distances between pairs of nodes in the three cases mentioned above have been studied in three previous publications, and we survey the results obtained there. Apart from the critical cases $\tau = 1$, $\tau = 2$ and $\tau = 3$, this completes the scaling picture. We explain the results heuristically and describe related work and open problems. We also compare the behavior in this model to Internet data, where a degree power law with exponent $\tau \approx 2.2$ is observed.

Furthermore, in this paper we derive results concerning the connected components and the diameter. We give a criterion when there exists a unique largest connected component of size proportional to the size of the graph, and study sizes of the other connected components. Also, we show that for $\tau \in (2, 3)$, which is most often observed in real networks, the diameter in this model grows much faster than the typical distance between two arbitrary nodes.