A Markovian Growth-Collapse Model

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Abstract

We consider growth-collapse processes (GCPs) that grow linearly between random partial collapse times at which they jump down according to some distribution depending on their current level. The jump occurrences are governed by a state-dependent rate function $r(x)$. We deal with the stationary distribution of such a GCP $X_t$, $t \geq 0$, and the distributions of the hitting times $T_a = \inf\{t \geq 0 \mid X_t = a\}$, $a > 0$. After presenting the general theory of these GCPs, several important special cases are studied. We also take a short look at the Markov-modulated case. In particular, we present a method to compute the distribution of $\min\{T_a, \sigma\}$ in this case (where $\sigma$ is the time of the first jump) and apply it to determine the long-run average cost of running a certain Markov-modulated disaster-ridden system.

Keywords: Growth-collapse process; piecewise deterministic Markov process; stationary distribution; hitting time; uniform cut-off; duality; Markov modulation.

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