Local asymptotics for the cycle maximum of a heavy-tailed random walk

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Abstract: Let $\xi, \xi_1, \xi_2, \ldots$ be a sequence of independent and identically distributed random variables. We assume that $\xi$ has a heavy-tailed distribution and negative finite mean $E\xi < 0$. Consider a random walk $S_n = \xi_1 + \cdots + \xi_n$. Let $M_n = \max_{k \leq n} S_k$ and let $\tau = \min\{n \geq 1 : S_n \leq 0\}$. We find asymptotics for $P(M_\tau \in (x, x + T])$ as $x \to \infty$ for a fixed positive constant $T \leq \infty$.

Keywords: random walk, cycle maximum, heavy-tailed distribution, stopping time, subexponential distribution.

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