Convergence of Coalescing Nonsimple Random Walks to the Brownian Web

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Abstract: The Brownian Web (BW) is a family of coalescing Brownian motions starting from every point in space and time $\mathbb{R} \times \mathbb{R}$. It was first introduced by Arratia, and later analyzed in detail by Tóth and Werner. More recently, Fontes, Isopi, Newman and Ravishankar (FINR) gave a characterization of the BW, and general convergence criteria allowing in principle either crossing or noncrossing paths, which they verified for coalescing simple random walks. Later Ferrari, Fontes, and Wu verified these criteria for a two dimensional Poisson Tree. In both cases, the paths are noncrossing. To date, the general convergence criteria of FINR have not been verified for any case with crossing paths, which appears to be significantly more difficult than the noncrossing paths case. Accordingly, in this paper, we formulate new convergence criteria for the crossing paths case, and verify them for non-simple coalescing random walks satisfying a finite fifth moment condition. This is the first time that convergence to the BW has been proved for models with crossing paths. Several corollaries are presented, including an analysis of the scaling limit of voter model interfaces that extends a result of Cox and Durrett.

Keywords: Brownian Web, Invariance Principle, Coalescing Random Walks, Brownian Networks, Continuum Limit.

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