Minimax and adaptive estimation of the Wigner function in quantum homodyne tomography with noisy data

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Abstract: We estimate the quantum state of a light beam from results of quantum homodyne measurements performed on identically prepared quantum systems. The state is represented through the Wigner function, a generalized probability density on $\mathbb{R}^2$ which may take negative values and must respect intrinsic positivity constraints imposed by quantum physics. The effect of the losses due to detection inefficiencies which are always present in a real experiment, is the addition to the tomographic data of independent Gaussian noise.

We construct a kernel estimator for the Wigner function, prove that it is minimax efficient for the pointwise risk over a class of infinitely differentiable functions, and implement it for numerical results. We construct adaptive estimators, i.e. which do not depend on the smoothness parameters, and prove that in some set-ups they attain the minimax rates for the corresponding smoothness class.

Keywords: Adaptive estimation, deconvolution, nonparametric estimation, infinitely differentiable functions, exact constants in nonparametric smoothing, minimax risk, quantum state, quantum homodyne tomography, Radon transform, Wigner function.

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