Limiting shapes for deterministic internal growth models

Anne Fey-den Boer*     Frank Redig†

March 23, 2007

Abstract

We study the rotor router model and two deterministic sandpile models. For the rotor router model in $\mathbb{Z}^d$, Levine and Peres proved that the limiting shape of the growth cluster is a sphere. For the other two models, only bounds in dimension 2 are known. A unified approach for these models with a new parameter $h$ (the initial number of particles at each site), allows to prove a number of new limiting shape results in any dimension $d \geq 1$.

For the rotor router model, the limiting shape is a sphere for all values of $h$. For one of the sandpile models, and $h = 2d - 2$ (the maximal value), the limiting shape is a cube. For both sandpile models, the limiting shape is a sphere in the limit $h \to -\infty$. Finally, we prove that the rotor router shape contains a diamond, which is a new result even in the case studied by Levine and Peres.

*Vrije Universiteit, De Boelelaan 1081a, 1081 HV Amsterdam, The Netherlands, fey@eurandom.tue.nl
†Mathematisch Instituut Universiteit Leiden, Niels Bohrweg 1, 2333 CA Leiden, The Netherlands, redig@math.leidenuniv.nl