Fluid limits for networks with bandwidth sharing and general document size distributions

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Abstract: We consider a stochastic model of Internet congestion control, introduced by Massoulié and Roberts [18], that represents the randomly varying number of flows in a network where bandwidth is shared amongst document transfers. In contrast to an earlier work by Kelly and Williams [12], the present paper allows interarrival times and document sizes to be generally distributed, rather than exponentially distributed. Furthermore, we allow a fairly general class of bandwidth sharing policies that includes the weighted α-fair policies of Mo and Walrand [2], as well as certain other utility based scheduling policies. To describe the evolution of the system, measure valued processes are used to keep track of the residual document sizes of all flows through the network. We propose a fluid model (or formal functional law of large numbers approximation) associated with the stochastic flow level model. Under mild conditions, we show that the appropriately rescaled measure valued processes corresponding to a sequence of such models (with fixed network structure) are tight, and that any weak limit point of the sequence is almost surely a fluid model solution. For the special case of weighted α-fair policies, we also characterize the invariant states of the fluid model.

Keywords and phrases: Bandwidth sharing, α-fair, flow level Internet model, congestion control, simultaneous resource possession, fluid model, workload, measure valued process, invariant manifold

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