Intermittency on catalysts: three-dimensional simple symmetric exclusion

J. Gärtner†
F. den Hollander‡
G. Maillard§

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Abstract: We continue our study of intermittency for the parabolic Anderson model $\partial u/\partial t = \kappa \Delta u + \xi u$ in a space-time random medium $\xi$, where $\kappa$ is a positive diffusion constant, $\Delta$ is the lattice Laplacian on $\mathbb{Z}^d$, $d \geq 1$, and $\xi$ is a simple symmetric exclusion process on $\mathbb{Z}^d$ in Bernoulli equilibrium. This model describes the evolution of a reactant $u$ under the influence of a catalyst $\xi$. In [?] we investigated the behavior of the annealed Lyapunov exponents, i.e., the exponential growth rates as $t \to \infty$ of the successive moments of the solution $u$. This led to an almost complete picture of intermittency as a function of $d$ and $\kappa$. In the present paper we finish our study by focussing on the asymptotics of the Lyapunov exponents as $\kappa \to \infty$ in the critical dimension $d = 3$, which was left open in [?] and which is the most challenging. We show that, interestingly, this asymptotics is characterized not only by a Green term, as in $d \geq 4$, but also by a polaron term. The presence of the latter implies intermittency of all orders above a finite threshold for $\kappa$.

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