Back to the roots of the M/D/s queue and the works of Erlang, Crommelin, and Pollaczek

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Abstract

A.K. Erlang introduced the M/D/s queue in 1917, while F. Pollaczek and C.D. Crommelin formalized the theory using complex analysis and transforms. Let \( D(s, \lambda) \) denote the stationary probability of experiencing no waiting time in the M/D/s queue with arrival rate \( \lambda \) and service requirement 1. We use \( D(s, \lambda) \) as a vehicle to give an overview of some of the results we obtained over the last years, including explicit characterizations of the roots, the derivation of infinite series from expressions in terms of roots using Fourier sampling, and heavy-traffic limits obtained from square-root staffing. We propose to call \( D(s, \lambda) \) the Erlang D formula, for which several new results are presented and compared to the results of Pollaczek.

Key Words and Phrases: M/D/s queue, Erlang D formula, queueing theory, roots, square-root staffing.

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