



New Models for Rating Asset Backed Securities

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Abstract

The securitization of financial assets is a form of structured finance, developed by the U.S. banking world in the early 1980's (in Mortgage-Backed-Securities format) in order to reduce regulatory capital requirements by removing and transferring risk from the balance sheet to other parties. Today, virtually any form of debt obligations and receivables has been securitized, resulting in an approximately \$2.5 trillion ABS outstanding in the U.S. alone¹, a market which is rapidly spreading to Europe, Latin-America and Southeast Asia.

Though no two ABS contracts are the same and therefore each deal requires its very own model, there are three important features which appear in virtually any securitization deal: default risk, Loss-Given-Default and prepayment risk. In this paper we will only be concerned with default and prepayment and discuss a number of traditional (continuous) and Lévy-based (pure jump) methods for modeling the latter risks. After briefly explaining the methods and their underlying intuition, the models are applied to a simple ABS deal in order to determine the rating of the notes. It turns out that the pure jump models produce lower (i.e. more conservative) ratings than the traditional methods (e.g. Vasicek), which are clearly incapable of capturing the shock-driven nature of losses and prepayments.

Keywords: Lévy processes; Default probability; Prepayment probability; Rating; Asset-Backed securities.

Classification: 60G35, 62P05, 91B28, 91B70.

¹Source: SIFMA, Q2 2008.

1 Introduction

Securitization is the process whereby an institution packs and sells a number of financial assets to a special entity, created specifically for this purpose and therefore termed the Special Purpose Entity (SPE) or Special Purpose Vehicle (SPV), which funds this purchase by issuing notes secured by the revenues from the underlying pool of assets. In general, we can say that securitization is the transformation of illiquid assets (for instance, mortgages, auto loans, credit card receivables and home equity loans) into liquid assets (marketable securities that can be sold in securities markets).

This form of structured finance was initially developed by the U.S. banking world in the early 1980's (in Mortgage-Backed-Securities format) in order to reduce regulatory capital requirements by removing and transferring risk from the balance sheet to other parties. Over the years, however, the technique has spread to many other industries (also outside the U.S.) and the goal shifted from reducing capital requirements to funding and hedging. Today, virtually any form of debt obligations and receivables has been securitized, with companies showing a seemingly infinite creativity in allocating the revenues from the pool to the noteholders (respecting their seniority). This results in an approximately \$2.5 trillion ABS market in the U.S. alone, which is rapidly spreading to Europe, Latin-America and Southeast Asia.

Unlike the nowadays very popular Credit Default Swap, ABS contracts are not yet standardized. This lack of uniformity implies that each deal requires a new model. However, there are certain features that emerge in virtually any ABS deal, the most important ones of which are default risk, amortization of principal value (and thus prepayment risk) and Loss-Given-Default (LGD). Since defaults, losses and accelerated principal repayments can substantially alter the projected cashflows and therefore the planned investment horizon, it is of key importance to adequately describe and model these phenomena when pricing securitization deals.

In the current ABS practice, the probability of default is generally modeled by means of a sigmoid function, such as the Logistic function, or by Vasicek's one-factor model, whereas the prepayment rate and the LGD rate are assumed to be constant (or at least deterministic) over time and independent of default. However, it is intuitively clear that each of these events is coming unexpectedly and is generally driven by the overall economy, hence infecting many borrowers at the same time, causing jumps in the default and prepayment term structures. Therefore it is essential to model the latter by stochastic processes that include jumps. Furthermore, it is unrealistic to assume that prepayment rates and loss rates are time-independent and uncorrelated, neither with each other, nor with default rates. For instance, a huge economic downturn will most likely result in a large number of defaults and a significant increase of the interest rates, causing huge losses and a decrease in prepayments. Reality indeed shows a negative correlation between default and prepayment.

In this paper, we propose a number of alternative techniques that can be applied to stochastically model default, prepayment and Loss-Given-Default, introducing dependence between the latter as well. The models we propose are based on Lévy processes, a well know family of jump-diffusion processes that have already proven their modeling abilities in other settings like equity and fixed income (cf. Schoutens (2003)). The text is organized as follows. In the following section we present four models for the default term structure. In Section 3 we discuss three models for the prepayment term structure. Numerical results are presented in Section 4, where the default and prepayment models are built into a cashflow model in order to determine the cumulative expected loss rate, the Weighted Average Life (WAL) and the corresponding rating of two subordinated notes of a simple ABS deal. Section 5 concludes the paper.

2 Default models

In this section we will briefly discuss four models for the default term structure, respectively based on

1. the generalized Logistic function;
2. a strictly increasing Lévy process;
3. Vasicek's Normal one-factor model;
4. the generic one-factor Lévy model Albrecher et al (2006), with an underlying shifted Gamma process.

We will focus on the time interval between the issue ($t = 0$) of the ABS notes and the weighted average time to maturity ($t = T$) of the underlying assets. In the sequel we will use the term *default curve* to refer to the default term structure. By default *distribution*, we mean the distribution of the cumulative default rate at time T . Hence, the endpoint of the default curve is a random draw from the default distribution.

2.1 Generalized Logistic default model

Traditional methods typically use a sigmoid (S-shaped) function to model the term structure of defaults. One famous example of such sigmoid functions is the (generalized) Logistic function (Richards (1959)), defined as

$$F(t) = \frac{a}{1 + be^{-c(t-t_0)}}, \quad (1)$$

where $F(t)$ satisfies the following ODE

$$\frac{dF(t)}{dt} = c \left(1 - \frac{F(t)}{a} \right) F(t), \quad (2)$$

with $a, b, c, t_0 > 0$ being constants and $t \in [0, T]$.

In the context of default curve modeling, $P_d(t) := F(t)$ is the cumulative default rate at time t . Note that when $b = 1$, t_0 corresponds to the inflection point in the loss buildup, i.e. P_d grows at an increasing rate before time t_0 and at a decreasing rate afterwards. Furthermore, $\lim_{t \rightarrow +\infty} F(t) = a$, thus a controls the right endpoint of the default curve. For sufficiently large T we can therefore approximate the cumulative default rate at maturity by a , i.e. $P_d(T) \approx a$. Hence, a is a random draw from a predetermined default distribution (e.g. the Log-Normal distribution) and each different value for a will give rise to a new default curve. This makes the Logistic function suitable for scenario analysis. Finally, the parameter c controls the spread of the Logistic curve around t_0 . In fact, c determines the growth rate of the Logistic curve, i.e. the proportional increase in one unit of time, as can be seen from equation (2). Values of c between 0.10 and 0.20 produce realistic default curves.

The left panel of Figure 1 shows five default curves, generated by the Logistic function with parameters $b = 1, c = 0.1, t_0 = 55, T = 120$ and decreasing values of a , drawn from a Log-Normal distribution with mean 0.20 and standard deviation 0.10. Notice the apparent inflection in the default curve at $t = 55$. The probability density function (p.d.f.) of the cumulative default rate at time T is shown on the right.

It has to be mentioned that the Logistic function (1) has several drawbacks when it comes to modeling a default curve. First of all, assuming real values for the parameters, the Logistic function does not start at 0, i.e. $P_d(0) > 0$. Moreover, a is only an approximation of the cumulative default rate at maturity, but in general we have that $P_d(T) < a$. Hence P_d has to be rescaled, in order to guarantee that a is indeed the cumulative default rate in the interval $[0, T]$. Secondly, the Logistic function is a deterministic function of time (the only source of randomness is in the choice of the endpoint), whereas defaults generally come

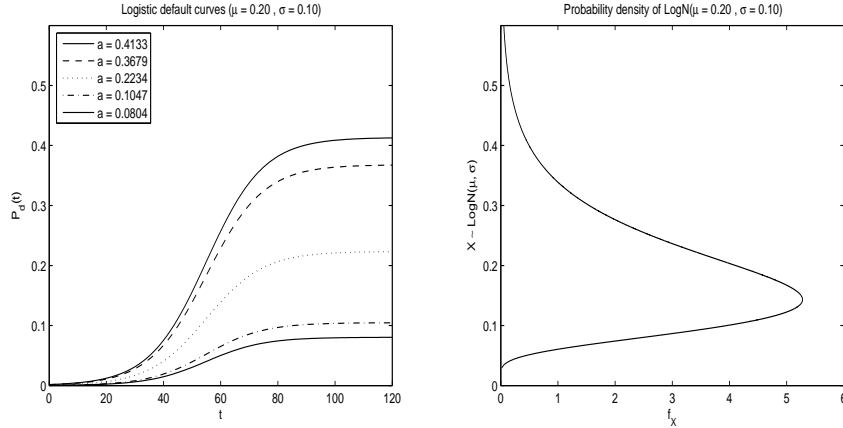


Figure 1: Logistic default curve (left) and Lognormal default distribution (right).

as a surprise. And finally, the Logistic function is continuous and hence unable to deal with the shock-driven behavior of defaults.

In the next sections we will describe three default models that (partly) solve the above mentioned problems. The first two problems will be solved by using a stochastic (instead of deterministic) process that starts at 0, whereas the shocks will be captured by introducing jumps in the model.

2.2 Lévy portfolio default model

In order to tackle the shortcomings of the Logistic model, we propose to model the default term structure by the process²

$$P_d = \left\{ P_d(t) = 1 - e^{-\lambda_t^d}, \quad t \geq 0 \right\}, \quad (3)$$

with $\lambda^d = \{\lambda_t^d : t \geq 0\}$ a strictly increasing Lévy process. The latter introduces both jump dynamics and stochasticity, i.e. $P_d(t)$ is a random variable, for all $t > 0$. Therefore, in order to simulate a default curve, we must first draw a realization of the process λ^d . Moreover, $P_d(0) = 0$, since by the properties of a Lévy process $\lambda_0^d = 0$.

In this paper we assume that λ^d is a Gamma process $G = \{G_t : t \geq 0\}$ with shape parameter a and scale parameter b , hence $\lambda_t^d \sim \text{Gamma}(at, b)$, for $t > 0$. Hence, the cumulative default rate at maturity follows the law $1 - e^{-\lambda_T^d}$, where $\lambda_T^d \sim \text{Gamma}(aT, b)$. Using this result, the parameters a and b can be found as the solution to the following system of equations

$$\begin{cases} \mathbb{E} \left[1 - e^{-\lambda_T^d} \right] &= \mu_d; \\ \text{Var} \left[1 - e^{-\lambda_T^d} \right] &= \sigma_d^2, \end{cases} \quad (4)$$

for predetermined values of the mean μ_d and standard deviation σ_d of the default distribution. Explicit expressions for the left hand sides of (4) can be found, by noting that the expected value and the variance can be written in terms of the characteristic function of the Gamma distribution.

²This can be linked to the world of intensity-based default modeling. See Lando (1994) and Schönbucher (2003) for a more detailed exposition. Cariboni and Schoutens (2008) incorporate jump dynamics into intensity models.

The left panel of Figure 2 shows five default curves, generated by the process (3) with parameters $a \approx 0.024914$, $b \approx 12.904475$ and $T = 120$, such that the mean and standard deviation of the default distribution are 0.20 and 0.10. Note that all curves start at zero, include jumps and are fully stochastic functions of time, in the sense that in order to construct a new default curve, one has to rebuild the whole intensity process over $[0, T]$, instead of just changing its endpoint. The corresponding default p.d.f. is again shown on the right. Recall, in this case, that $P_d(T)$ follows the law $1 - e^{-\lambda_T^d}$, with $\lambda_T^d \sim \text{Gamma}(aT, b)$.

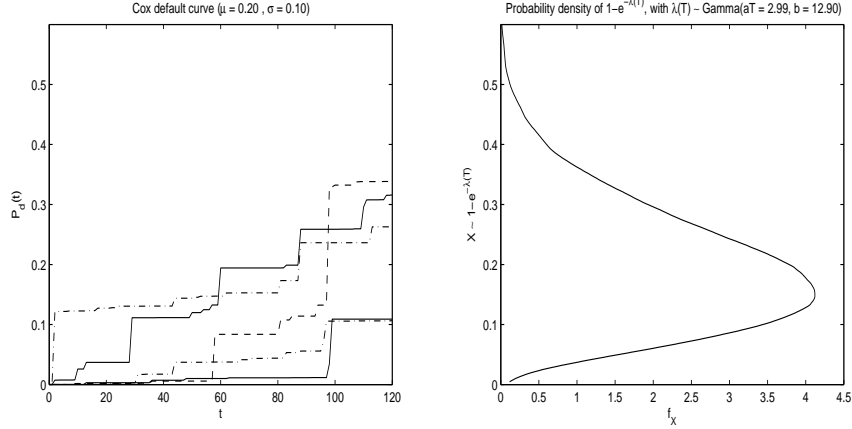


Figure 2: Lévy portfolio default curves (left) and corresponding default distribution (right).

2.3 Normal one-factor default model

The Normal one-factor (structural) model (Vasicek Vasicek (1987), Li Li (1995)) models the cash position $V^{(i)}$ of a borrower, where $V^{(i)}$ is described by a geometric Brownian motion,

$$\begin{aligned} V_T^{(i)} &= V_0^{(i)} \exp \left[a \left(\mu_T^{(i)}, \sigma_T^{(i)} \right) + b \left(\mu_T^{(i)}, \sigma_T^{(i)} \right) W_T^{(i)} \right] \\ &\stackrel{d}{=} V_0^{(i)} \exp \left[a \left(\mu_T^{(i)}, \sigma_T^{(i)} \right) + b \left(\mu_T^{(i)}, \sigma_T^{(i)} \right) Z_i \right], \end{aligned} \quad (5)$$

for $i = 1, 2, \dots, N$, with N the number of loans in the asset pool. Here $\stackrel{d}{=}$ denotes equality in distribution and $Z_i \sim N(0, 1)$. Furthermore, Z_i satisfies

$$Z_i = \sqrt{\rho}X + \sqrt{1 - \rho}X_i, \quad (6)$$

with $X, X_1, X_2, \dots, X_N \stackrel{\text{i.i.d.}}{\sim} N(0, 1)$. It is easy to verify that $\rho = \text{Corr}(Z_i, Z_j)$, for all $i \neq j$. The latter parameter is calibrated to match a predetermined value for the standard deviation σ of the default distribution.

A borrower is said to default at time t , if his financial situation has deteriorated so dramatically that $V_T^{(i)}$ hits a predetermined lower bound B_t^d , which (as can be seen from (5)) is equivalent to saying that Z_i hits some barrier H_t^d . The latter barrier is chosen such that the expected probability of default before time t matches the default probabilities observed in the market, where it is assumed that the latter follow a homogeneous Poisson process with intensity λ , i.e. H_t^d satisfies

$$\Pr [Z_i \leq H_t^d] = \Phi [H_t^d] = \Pr [N_t > 0] = 1 - e^{-\lambda t}, \quad (7)$$

where λ is set such that $\Pr [Z_i \leq H_T^d] = \mu_d$, with μ_d the predetermined value for the mean of the default distribution. From (7) it then follows that

$$\lambda = -\log \left([1 - \mu_d]^{\frac{1}{T}} \right) \quad (8)$$

and hence

$$H_t^d = \Phi^{-1} \left[1 - (1 - \mu_d)^{\frac{t}{T}} \right], \quad (9)$$

with Φ the standard Normal cumulative distribution function.

Given a sample of (correlated) standard Normal random variables $\mathbf{Z} = (Z_1, Z_2, \dots, Z_N)$, the default curve is then given by

$$P_d(t; \mathbf{Z}) = \frac{\#\{Z_i \leq H_t^d; i = 1, 2, \dots, N\}}{N}, \quad t \geq 0. \quad (10)$$

In order to simulate default curves, one must thus first generate a sample of standard Normal random variables Z_i satisfying (6), and then, at each (discrete) time t , count the number of Z_i 's that are less than or equal to the value of the default barrier H_t^d at that time.

The left panel of Figure 3 shows five default curves, generated by the Normal one-factor model (6) with $\rho \approx 0.121353$, such that the mean and standard deviation of the default distribution are 0.20 and 0.10. All curves start at zero and are fully stochastic, but unlike the Lévy portfolio model the Normal one-factor default model does not include any jump dynamics. Therefore, as will be seen later, this model is unable to deal with the shock-driven nature of defaults and as such generates ratings that are too optimistic (high). The corresponding default p.d.f. is again shown in the right panel.

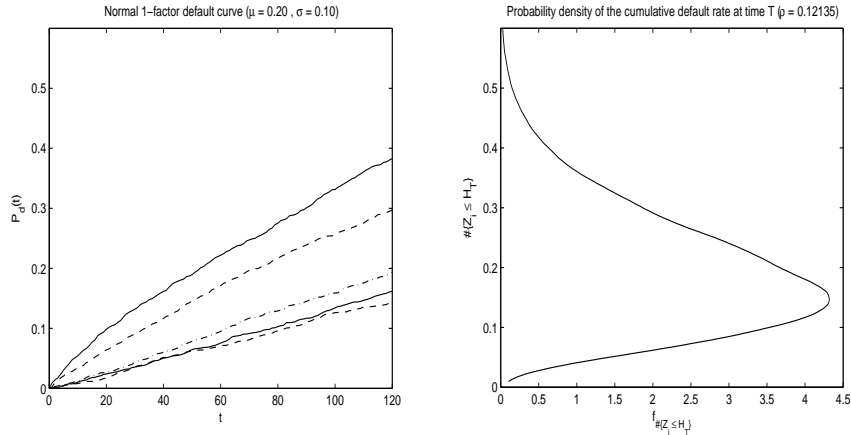


Figure 3: Normal one-factor default curves (left) and corresponding default distribution (right).

2.4 Generic one-factor Lévy default model

The generic one-factor Lévy model Albrecher et al (2006) is comparable to and in fact is a generalization of the Normal one-factor model. Instead of describing a borrower's cash position by a geometric Brownian motion, $V^{(i)}$ is now modeled with a geometric Lévy model, i.e.

$$V_T^{(i)} = V_0^{(i)} \exp \left[A_T^{(i)} \right], \quad (11)$$

for $i = 1, 2, \dots, N$. The process $A^{(i)} = \{A_t^{(i)} : t \geq 0\}$ is a Lévy process and satisfies

$$A_T^{(i)} = Y_\rho + Y_{1-\rho}^{(i)}, \quad (12)$$

with $Y, Y^{(1)}, Y^{(2)}, \dots, Y^{(N)}$ i.i.d. Lévy processes, based on the same mother infinitely divisible distribution L , such that $E[Y_1] = 0$ and $\text{Var}[Y_1] = 1$, which implies that $\text{Var}[Y_t] = t$. From this it is clear that $E[A_T^{(i)}] = 0$ and $\text{Var}[A_T^{(i)}] = 1$, such that $\text{Corr}(A_T^{(i)}, A_T^{(j)}) = \rho$, for all

$i \neq j$. As with the Normal one-factor model, the cross-correlation ρ will be calibrated to match a predetermined standard deviation for the default distribution.

As for the Normal one-factor model, we again say that a borrower defaults at time t , if $A_T^{(i)}$ hits a predetermined barrier H_t^d at that time, where H_t^d satisfies

$$\Pr \left[A_T^{(i)} \leq H_t^d \right] = 1 - e^{-\lambda t}, \quad (13)$$

with λ given by (8).

In this paper we assume that $Y, Y^{(1)}, Y^{(2)}, \dots, Y^{(N)}$ are i.i.d. shifted Gamma processes, i.e. $Y = \{Y_t = t\tilde{\mu} - G_t : t \geq 0\}$, where G is a Gamma process, with shape parameter a and scale parameter b . From (12) and the fact that a Gamma distribution is infinitely divisible it then follows that

$$A_T^{(i)} \stackrel{d}{=} \tilde{\mu} - \tilde{X} \stackrel{d}{=} \tilde{\mu} - [X + X_i], \quad (14)$$

with $X \sim \text{Gamma}(a\rho, b)$ and $X_i \sim \text{Gamma}(a(1-\rho), b)$ mutually independent and $X_i \sim \text{Gamma}(a, b)$. If we take $\tilde{\mu} = \frac{a}{b}$ and $b = \sqrt{a}$, we ensure that $E[A_T^{(i)}] = 0$, $\text{Var}[A_T^{(i)}] = 1$ and $\text{Corr}(A_T^{(i)}, A_T^{(j)}) = \rho$, for all $i \neq j$.

Furthermore, from (13), (14) and the expression for λ it follows that

$$H_t^d = \tilde{\mu} - \Gamma_{a,b}^{-1} \left[(1 - \mu_d)^{\frac{t}{T}} \right], \quad (15)$$

where $\Gamma_{a,b}$ denotes the cumulative distribution function of a $\text{Gamma}(a, b)$ distribution.

In order to simulate default curves, we first have to generate a sample of random variables $\mathbf{A}_T = (A_T^{(1)}, A_T^{(2)}, \dots, A_T^{(N)})$ satisfying (12), with $Y, Y^{(1)}, Y^{(2)}, \dots, Y^{(N)}$ i.i.d. Shifted-Gamma processes and then, at each (discrete) time t , count the number of $A_T^{(i)}$'s that are less than or equal to the value of the default barrier H_t^d at that time. Hence, the default curve is given by

$$P_d(t; \mathbf{A}_T) = \frac{\#\left\{ A_T^{(i)} \leq H_t^d; i = 1, 2, \dots, N \right\}}{N}, \quad t \geq 0. \quad (16)$$

The left panel of Figure 4 shows five default curves, generated by the Gamma one-factor model (12) with $(\tilde{\mu}, a, b) = (1, 1, 1)$, and $\rho \approx 0.095408$, such that the mean and standard deviation of the default distribution are 0.20 and 0.10. Again, all curves start at zero and are fully stochastic. Furthermore, when comparing the curves of the one-factor shifted Gamma-Lévy model (hereafter termed the shifted Gamma-Lévy model or Gamma one-factor model) to the ones generated by the Lévy portfolio default model, one might be tempted to conclude that the former model does not include jumps. However, it does, but the jumps are embedded in the underlying dynamics of the asset return A_T . The corresponding default p.d.f. is shown in the right panel. Compared to the previous three default models, the default p.d.f. generated by the shifted Gamma-Lévy model seems to be squeezed around μ_d and has a significantly larger kurtosis.

It should also be mentioned that the latter default distribution has a rather heavy right tail, with a substantial probability mass at the 100 % default rate. This can be explained by looking at the right-hand side of equation (14). Since both terms between brackets are strictly positive and hence cannot compensate each other (unlike the Normal one-factor model), $A_T^{(i)}$ is bounded from above by $\tilde{\mu}$. Hence, starting with a large systematic risk factor X , things can only get worse, i.e. the term between brackets can only increase and therefore $A_{i,T}$ can only decrease, when adding the idiosyncratic risk factor X_i . This implies that when we have a substantially large common factor (close to $\Gamma_{a,b}^{-1}[1 - \mu_d]$, cf. (15)), it is very likely that all borrowers will default, i.e. that $A_T^{(i)} \leq H_T^d$ for all $i = 1, 2, \dots, N$.

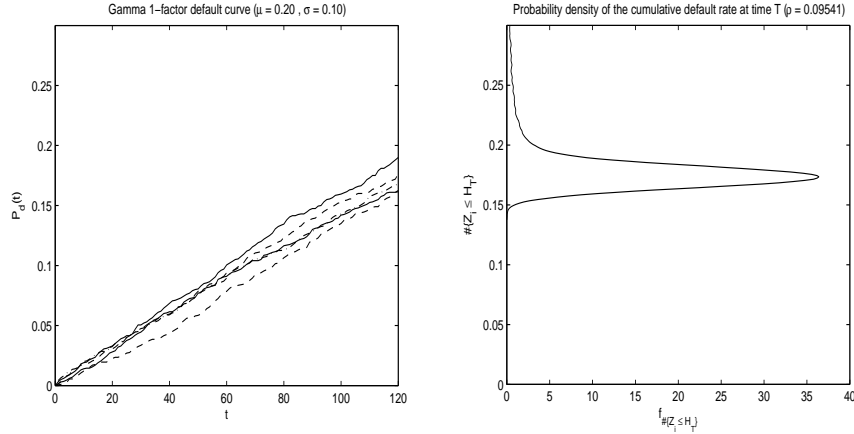


Figure 4: Gamma 1-factor default curves (left) and corresponding default distribution (right).

3 Prepayment models

In this section we will briefly discuss three models for the prepayment term structure, respectively based on

1. constant prepayment;
2. a strictly increasing Lévy process;
3. Vasicek's Normal one-factor model.

As before, we will use the terms *prepayment curve* and *prepayment distribution* to refer to the prepayment term structure and the distribution of the cumulative prepayment rate at maturity T .

3.1 Constant prepayment model

The idea of constant prepayment stems from the former Public Securities Association³ (PSA). The basic assumption is that the (monthly) amount of prepayment begins at 0 and rises at a constant rate of increase α until reaching its characteristic steady state rate at time t_{00} , after which the prepayment rate remains constant until maturity T . Note that t_{00} is generally not the same as the inflection point t_0 of the default curve.

The corresponding marginal (e.g. monthly) and cumulative prepayment curves are given by

$$\text{cpr}(t) = \begin{cases} \alpha t & ; 0 \leq t \leq t_{00} \\ \alpha t_{00} & ; t_{00} \leq t \leq T \end{cases} \quad (17)$$

and

$$\text{CPR}(t) = \begin{cases} \frac{\alpha t^2}{2} & ; 0 \leq t \leq t_{00} \\ -\frac{\alpha t_{00}^2}{2} + \alpha t_{00} t & ; t_{00} \leq t \leq T \end{cases} \quad (18)$$

From (17) it is obvious that the marginal prepayment rate increases at a speed of α per period before time t_{00} and remains constant afterwards. Consequently, the cumulative prepayment curve (18) increases quadratically on the interval $[0, t_{00}]$ and linearly on $[t_{00}, T]$. Given t_{00}

³In 1997 the PSA changed its name to The Bond Market Association (TBMA), which merged with the Securities Industry Association on November 1, 2006, to form the Securities Industry and Financial Markets Association (SIFMA).

and $\text{CPR}(T)$, i.e. the cumulative prepayment rate at maturity, the constant rate of increase α equals

$$\alpha = \frac{\text{CPR}(T)}{Tt_{00} - \frac{t_{00}^2}{2}}. \quad (19)$$

Hence, once t_{00} and $\text{CPR}(T)$ are fixed, the marginal and cumulative prepayment curves are completely deterministic. Moreover, the CPR model does not include jumps. Due to these features, the CPR model is an unrealistic representation of real-life prepayments, which are shock-driven and typically show some random effects. In the next sections we will describe two models that (partially) solve these problems.

Figure 5 shows the marginal and cumulative prepayment curve, in case the steady state t_{00} is reached after 48 months and the cumulative prepayment rate at maturity equals $\text{CPR}(T) = 0.20$. The corresponding constant rate of increase is $\alpha = 0.434\text{bps}$.

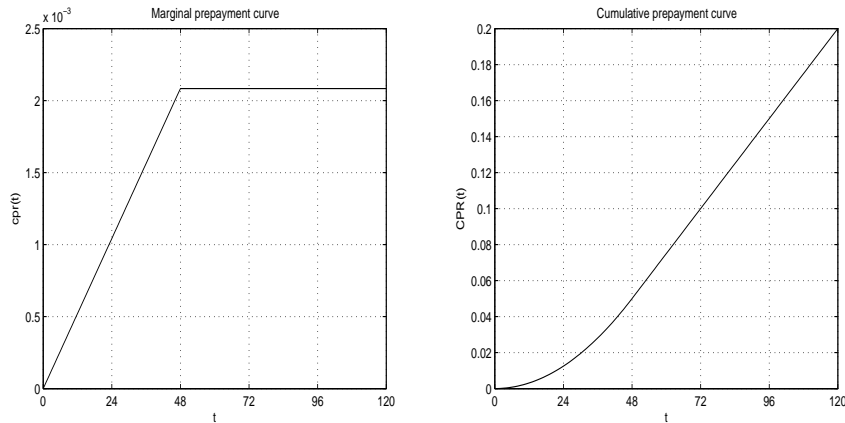


Figure 5: Marginal (left) and cumulative (right) constant prepayment curve.

3.2 Lévy portfolio prepayment model

The Lévy portfolio prepayment model is completely analogous to the Lévy portfolio default model described in Section 2.2, with λ_t^d replaced λ_t^p . Although there is empirical evidence that defaults and prepayments are negatively correlated, in the simulation study in Section 4 we assumed the above mentioned processes to be mutually independent.

Evidently, also the Lévy portfolio prepayment curves start at zero, are fully stochastic and include jumps, solving the above mentioned problems of the CPR model.

3.3 Normal one-factor prepayment model

The Normal one-factor prepayment model starts from the same underlying philosophy as its default equivalent of Section 2.3. We again model the cash position $V^{(i)}$ of a borrower. Just as a borrower is said to default if his financial situation has deteriorated so dramatically that $V^{(i)}$ hits some predetermined lower bound B_t^d , we state that a borrower will decide to prepay if his financial health has improved sufficiently, so that $V^{(i)}$ (or equivalently Z_i) hits a prespecified upper bound B_t^p (H_t^p).

The barrier H_t^p is chosen such that the expected probability of prepayment before time t equals the (observed) cumulative prepayment rate $\text{CPR}(t)$, given by (18), i.e.

$$\Pr[Z_i \geq H_t^p] = 1 - \Phi[H_t^p] = \text{CPR}(t), \quad (20)$$

which implies,

$$H_t^p = \Phi^{-1} [1 - \text{CPR}(t)], \quad (21)$$

with Φ the standard Normal cumulative distribution function.

In order to simulate prepayment curves, we must thus draw a sample of standard Normal random variables $\mathbf{Z} = (Z_1, Z_2, \dots, Z_N)$ satisfying (6), and then, at each (discrete) time t , count the number of Z_i 's that are greater than or equal to the value of the prepayment barrier H_t^p at that time. The prepayment curve is then given by

$$P_p(t; \mathbf{Z}) = \frac{\#\{Z_i \geq H_t^p : i = 1, 2, \dots, N\}}{N}, \quad t \geq 0. \quad (22)$$

The left panel of Figure 6 shows five prepayment curves, generated by the Normal one-factor model (6) with $\rho \approx 0.121353$, such that the mean and standard deviation of the prepayment distribution are 0.20 and 0.10 (as for the default model). The fact that the cross-correlation coefficient ρ is the same as the one of the default model is a direct consequence of the symmetry of the Normal distribution. The curves start at zero and are fully stochastic, but the model lacks jump dynamics. As will be seen later on, ignoring prepayment shocks results in an overestimation of the weighted average life of an ABS, which in turn produces higher (unsafe) ratings. The corresponding prepayment p.d.f. is shown in the right panel.

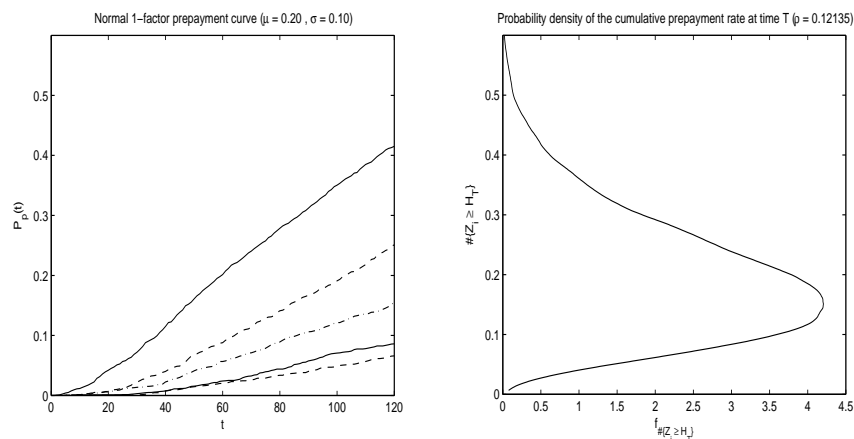


Figure 6: Normal one-factor prepayment curves (left) and corresponding prepayment distribution (right).

4 Numerical results

4.1 Introduction

One can now build these default and prepayment models into any scenario generator for rating and analyzing asset-backed securities. Any combination of the above described default and prepayment models is meaningful, except for the combination of the shifted Gamma(-Lévy) default model with the Normal one-factor prepayment model. In that case the borrower's cash position would be modeled by two different processes: one to obtain his default probability and another one for his prepayment probability, which is neither consistent nor realistic.

Hence, all together we can construct 11 different scenario generators. Table 1 summarizes the possible combinations of default and prepayment models.

		Prepayment models		
		CPR	Lévy portfolio	Normal one-factor
Default models	Logistic	ok	ok	ok
	Lévy portfolio	ok	ok	ok
	Normal one-factor	ok	ok	ok
	Gamma one-factor	ok	ok	nok

Table 1: Possible combinations of default and prepayment models.

We will now apply each of the above mentioned 11 default-prepayment combinations to derive the expected loss, the WAL and the corresponding rating of two (subordinated) notes backed by a homogeneous pool of commercial loans. Table 2 lists the specifications of the ABS deal under consideration (cf. Raynes and Rutledge (2003)).

ASSETS		
Initial balance of the asset pool	V_0	\$30,000,000
Number of loans in the asset pool	N_0	2,000
Weighted Average Maturity of the assets	WAM	10 years
Weighted Average Coupon of the assets	WAC	12% p.a.
Payment frequency		monthly
Reserve target		5%
Eligible reinvestment rate		3.92% p.a.
Loss-Given-Default	LGD	50%
Lag		5 months
LIABILITIES		
Initial balance of the senior note	A_0	\$24,000,000
Premium of the senior note	r_A	7% p.a.
Initial balance of the subordinated note	B_0	\$6,000,000
Premium of the subordinated note	r_B	9% p.a.
Servicing fee	r_{sf}	1% p.a.
Servicing fee shortfall rate	r_{sf-sh}	20% p.a.
Payment method		Pro-rata Sequential

Table 2: Specifications of the ABS deal.

Note that the cash collected (from the pool) and distributed (to the note holders) by the SPV, in a particular period, contains both principal and interest. Each period, principal (scheduled, prepaid and recoveries from default) and interest collections are combined into a pool, which is then used to pay the interest and principal (in this order) due to the investors. Whatever cash is left after fulfilling the interest obligations is used to pay the principal due (scheduled principal + prepaid principal + defaulted face value) on the notes, according to the priority rules. From this it is evident that default and prepayment will have a significant effect on the amortization of the notes and (consequently) on the interest received by the note holders.

Furthermore, as can be seen from Table 2, the ABS deal under consideration benefits from credit enhancement under the form of a reserve account, required to be equal to 5% of the balance of the asset pool at the end of each payment period. The funds available in this account are reinvested at the 10-year US Treasury rate (of May 22, 2008) and will be used to

fulfill the payment obligations, in case the collections in a specific period are insufficient to cover the expenses. In order to achieve the targeted reserve amount of 5% of the asset pool's balance at the end of each payment period, before being transferred to the owners of the SPV, any excess cash is first used to replenish the reserve account. Hence it is possible that the owners of the SPV are not compensated in certain periods, or in the worst case not at all. On the other hand, there may also be periods in which the SPV owners receive a substantial amount of cash. This especially happens in periods with a high number of defaults and/or prepayments, where the outstanding balance of the asset pool suddenly decreases very fast, requiring the reserve account to be reduced in order to match the targeted 5% of the asset pool at the end of the payment period.

Furthermore, unless explicitly stated otherwise, the parameter values mentioned in Table 3 will be used.

Mean of the default distribution	μ_d	20%
Standard deviation of the default distribution	σ_d	10%
Mean of the prepayment distribution	μ_p	20%
Standard deviation of the prepayment distribution	σ_p	10%
Parameters of the Logistic curve	b	1
	c	0.1
	t_0	55 months
Steady state of the prepayment curve	t_{00}	45 months

Table 3: Default parameter values for the default and prepayment models.

Finally, before moving on to the actual sensitivity analysis, we introduce two important concepts, i.e. the DIRR and the WAL of an ABS. By DIRR we mean the difference between the promised and the realized internal rate of return. The WAL is defined as

$$\text{WAL} = \frac{1}{P} \left(\sum_{t=1}^T t \cdot P_t + T \left[P - \sum_{t=1}^T P_t \right] \right), \quad (23)$$

where P_t is the total principal paid at time t and P is the initial balance of the note. The term between the square brackets accounts for principal shortfall, in the sense that if the note is not fully amortized after its legal maturity, we assume that the non-amortized amount is redeemed at the legal maturity date⁴. Clearly, both the DIRR and the WAL are non-negative. Furthermore, by inspecting the rating table mentioned Cifuentes and O'Connor (1996) and Cifuentes and Wilcox (1998), it is obvious that there is some interplay between the DIRR and the WAL: of two notes with the same DIRR, the one with the highest WAL will have the highest rating. For instance, consider two notes A_1 and A_2 with a DIRR of 0.03%, but with respective WALs of 4 and 5 years. Then note A_1 will get a Aa3 rating, whereas the A_2 note gets a Aa2 rating. Obviously, of two notes with the same WAL, the one with the highest DIRR will get the lowest rating.

4.2 Sensitivity analysis

Tables 4-6 contain ratings – based on the Moody's Idealized Cumulative Expected Loss Rates⁵– and DIRRs and WALs of the two ABS notes, obtained with each of the 11 default-prepayment combinations, for several choices of μ_d and μ_p . The figures mentioned in these tables are averages based on a Monte Carlo simulation with 1,000,000 scenarios.

⁴This method is proposed in Mazataud and Yomtov (2000). Moreover, in Moody's ABSROMTM application (v 1.0) the WAL of a note is calculated as $\frac{\sum_{t=0}^{T-1} F_t}{F_0}$, with F_t the note's outstanding balance at time t . Hence $F_0 = P$. It is left as an exercise to the reader to verify that this formula is equivalent to formula (23).

⁵See Cifuentes and O'Connor (1996) and Cifuentes and Wilcox (1998) for further details.

More specifically, in Table 4 we investigate what happens to the ratings if μ_d is changed, while holding μ_p and σ_p constant⁶, whereas Table 4 provides insight in the impact of a change in μ_p , while keeping μ_d and σ_d fixed.

Unless stated otherwise, the (principal) collections from the asset pool are distributed across the note holders according to a pro-rata payment method, i.e. proportionally with the note's outstanding balances. However, Table 6 presents the ratings using both pro rata and sequential payment method, where the subordinated B note starts amortizing only after the outstanding balance of the senior A Note is fully redeemed, in both cases assuming that there exists a reserve account. The effect of having no reserve account in the pro rata case is also shown in Table 6.

4.2.1 Influence of μ_d

From Table 4 we may conclude that when increasing the average cumulative default rate the credit rating of the notes stays the same or is lowered for all combinations of default and prepayment models.

For the model dependence we first analyse the rating columns for the A note. For $\mu_d = 10\%$ we can see that all but the two pairs with the Gamma one-factor default model give Aaa ratings, indicating that the rating is not so model-dependent for a relatively low cumulative default rate.

Increasing μ_d to 20%, the rating using the Normal one-factor default model stays at Aaa regardless of prepayment models. For the Logistic default model the rating is changed to Aa1 for all combination of prepayment models and for the Gamma one-factor model the rating is Aa3. It is only for the Lévy portfolio default model that we can see a small difference between the CPR model and the two other prepayment models.

Finally, assuming that $\mu_d = 40\%$, the Lévy portfolio prepayment model in combination with either the Logistic or the Normal one-factor default model gives lower ratings than the other two prepayment models. For the other default models no dependence on the prepayment model can be traced.

Analyzing the influence of the prepayment model, it is worth noticing that the Lévy portfolio model always gives the lowest WAL and the highest DIRR for any default model, compared to the other two prepayment models. This can be explained by looking at the typical path of a Lévy portfolio process (cf. Figure 2). Note that such a path does not increase continuously, but moves up with jumps, between which the curve remains rather flat. Translated to the prepayment phenomena, this means that there will be times when a large number of borrowers decide to prepay, followed by a period where there are virtually no prepayments, until the next time where a substantial amount of the remaining debtors prepays. Obviously, this results in a very irregular cash inflow, which will cause difficulties when trying to honour the payment obligations. Indeed, as previously explained, in payment periods with a jump in the prepayment rate, the outstanding balance of the asset pool and consequently the reserve account will be significantly decreased, which in turn increases the probability of future interest and principal shortfalls, leading to higher DIRRs. Moreover, since a shock-driven prepayment model increases the probability that a substantial number of borrowers will choose to prepay very early in the life of the loan, it is not surprising that the Lévy portfolio prepayment model produces lower WALs than the other two models. Finally, as explained before, higher DIRRs and lower WALs lead to lower ratings.

The Gamma one-factor model always gives the lowest rating, and a look at the DIRR and WAL columns gives the explanation for this, namely, the DIRRs for the Gamma one-factor model is always much higher than for any of the other default models but the WALs is almost the same leading to a lower rating. The Normal one-factor default model gives in general the highest rating, which can be explained by the fact that it produces the lowest DIRRs.

⁶In order to keep μ_p and σ_p fixed, also the cross-correlation ρ must remain fixed, since there is a unique parameter ρ for each pair (μ_p, σ_p) (or equivalently (μ_d, σ_d)). This explains why also σ_d changes if μ_d changes.

For the B note the general tendency is that the rating is lowered when the mean cumulative default rate is increased. It is worth mentioning that the Normal one-factor model gives the highest rating among the default models and that the Gamma one-factor model gives the lowest rating for $\mu_d = 10\%$ and the Lévy portfolio model gives the lowest for $\mu_d = 40\%$. Thus, the jump-driven default models produce the lowest ratings. The Lévy portfolio prepayment model combined with the Lévy portfolio, Normal one-factor or Gamma one-factor default model gives generally the lowest rating compared to the other prepayment models, for reasons explained in the previous paragraph.

4.2.2 Influence of μ_p

The influence of changing the mean cumulative prepayment rate is given in Table 5. A comparison to Table 4 learns that the ratings are less sensitive to changes in the mean prepayment rate than they are to changes in the expected default rate, as the rating transitions caused by the former are significantly smaller.

Furthermore, any of the above made observations concerning specific prepayment or default models remains valid also here. Especially it still holds that the Normal one-factor default model gives the same or higher rating of both notes than the other default models and that the jump-driven models give the lowest ratings, for each of the prepayment models.

4.2.3 Influence of the reserve account

Table 6 provides insight into the effect of incorporating a reserve account (credit enhancement) into the cash flow waterfall of an ABS deal. The results in this table show no surprises: since assuming there is no reserve account implies that there are less funds available for reimbursing the note investors (on the contrary, any excess cash is fully transferred to the SPV owners) it is evident that removing the reserve account will lead to higher DIRRs and WALs and lower ratings. This is indeed what we see, when comparing the above mentioned two tables. Notice that the effect is greater for the B note. This is of course due to its subordinated status.

4.2.4 Influence of the payment method

Table 6 shows the impact of choosing either the pro-rata or the sequential payment method, for allocating the (principal) collections to the different notes. What is clear from the definition of the two payment methods is that sequential payment will shorten the WAL of the A note and increase the WAL of the B note. Consulting Moody's Idealized Cumulative Expected Loss Rate table one can see that an increase in WAL, keeping the DIRR fixed, will result in a higher rating. The expected decrease and increase in WAL for the A note and B note, respectively, are evident. In fact, the WAL increases on average with a factor 1.72 (or 3.8 years) for the B note, going from pro rata to sequential payment. The decrease of the WAL for the A note is on average with a factor 0.82 (or 0.95 years). Thus the change in WAL is much more dramatic for the B note than for the A note. So based only on the change of the WALs, without taking the change in DIRR into account, we can directly assume that the rating would improve for the B note and for the A note we would expect the rating to stay the same or be lowered. However, taking the change in DIRR into account, we see that the actual rating of both the A note and the B note stays the same or improves going from pro rata to sequential payment. The improvement of the A note rating is due to the fact that the DIRR is smaller for the sequential case than for the pro rata case, compensating for the decrease in WAL. For the B note the changes of the DIRRs are not enough to influence the rating improvements due to the increases in WALs.

5 Conclusion

Traditional models for the rating and the analysis of ABSs are typically based on Normal distribution assumptions and Brownian motion driven dynamics. The Normal distribution belongs to the class of the so-called light tailed distributions. This means that extreme events, shock, jumps, crashes, etc. are not incorporated in the Normal distribution based models. However looking at empirical data and certainly in the light of the current financial crisis, it are these extreme events that can have a dramatical impact on the product. In order to do a better assessment, new models incorporating these features are needed. This paper has introduced a whole battery of new models based on more flexible distributions incorporating extreme events and jumps in the sample paths. We observe that the jump-driven models in general produce lower ratings than the traditional models.

Model pair	Note A								
	Rating			DIRR (bp)			WAL (year)		
	$\mu_d = 10\%$	$\mu_d = 20\%$	$\mu_d = 40\%$	$\mu_d = 10\%$	$\mu_d = 20\%$	$\mu_d = 40\%$	$\mu_d = 10\%$	$\mu_d = 20\%$	$\mu_d = 40\%$
Logistic – CPR	Aaa	Aa1	Aa3	0.026746	0.3466	5.3712	5.4867	5.2742	4.8642
Logistic – Lévy portfolio	Aaa	Aa1	A1	0.039664	0.48683	7.4258	5.343	5.1311	4.729
Logistic – Normal one-factor	Aaa	Aa1	Aa3	0.027104	0.3278	5.1148	5.4869	5.2745	4.8656
Lévy portfolio – CPR	Aaa	Aaa	A1	0.0017992	0.16105	9.0857	5.4799	5.2529	4.7895
Lévy portfolio – Lévy portfolio	Aaa	Aa1	A1	0.0067859	0.34616	12.044	5.3355	5.1101	4.6543
Lévy portfolio – Normal one-factor	Aaa	Aa1	A1	0.0032759	0.20977	9.0265	5.4795	5.2532	4.7912
Normal one-factor – CPR	Aaa	Aaa	Aa2	0.00036114	0.034631	2.9626	5.4775	5.2427	4.7309
Normal one-factor – Lévy portfolio	Aaa	Aaa	Aa3	0.00060627	0.055516	3.6883	5.3335	5.0986	4.5895
Normal one-factor – Normal one-factor	Aaa	Aaa	Aa2	0.00014211	0.017135	2.0175	5.4774	5.2427	4.7303
Gamma one-factor – CPR	Aa1	Aa3	A2	1.4443	4.6682	18.431	5.4828	5.2599	4.7939
Gamma one-factor – Lévy portfolio	Aa2	Aa3	A2	2.5931	4.9614	20.385	5.3427	5.1167	4.6503

Model pair	Note B								
	Rating			DIRR (bp)			WAL (year)		
	$\mu_d = 10\%$	$\mu_d = 20\%$	$\mu_d = 40\%$	$\mu_d = 10\%$	$\mu_d = 20\%$	$\mu_d = 40\%$	$\mu_d = 10\%$	$\mu_d = 20\%$	$\mu_d = 40\%$
Logistic – CPR	Aa1	A1	Baa3	0.93026	10.581	139.46	5.4901	5.3124	5.3358
Logistic – Lévy portfolio	Aa1	A1	Baa3	1.1996	13.624	164.07	5.3471	5.1771	5.2456
Logistic – Normal one-factor	Aa1	A1	Baa3	0.93764	10.906	140.55	5.4903	5.3135	5.3391
Lévy portfolio – CPR	Aa1	A2	Baa3	1.4051	17.801	175.75	5.4949	5.3525	5.4753
Lévy portfolio – Lévy portfolio	Aa2	A2	Ba1	1.9445	21.891	195.61	5.3526	5.2204	5.373
Lévy portfolio – Normal one-factor	Aa1	A2	Baa3	1.6019	18.35	175.49	5.4951	5.353	5.4738
Normal one-factor – CPR	Aaa	Aa1	Baa1	0.033692	1.5642	57.936	5.4777	5.2502	4.9709
Normal one-factor – Lévy portfolio	Aaa	Aa2	Baa2	0.041807	1.9829	65.669	5.3337	5.1071	4.8421
Normal one-factor – Normal one-factor	Aaa	Aa1	Baa1	0.023184	1.156	48.936	5.4776	5.2491	4.9498
Gamma one-factor – CPR	Aa3	A2	Baa2	6.288	20.736	85.662	5.4955	5.3022	4.9739
Gamma one-factor – Lévy portfolio	A1	A3	Baa3	15.293	28.406	120.76	5.3631	5.1588	4.8351

Table 4: Ratings, DIRR and WAL of the ABS notes, for different combinations of default and prepayment models and mean cumulative default rate $\mu_d = 0.10, 0.20, 0.40$ and mean cumulative prepayment rate $\mu_p = 0.20$.

Model pair	Note A								
	Rating			DIRR (bp)			WAL (year)		
	$\mu_p = 10\%$	$\mu_p = 20\%$	$\mu_p = 40\%$	$\mu_p = 10\%$	$\mu_p = 20\%$	$\mu_p = 40\%$	$\mu_p = 10\%$	$\mu_p = 20\%$	$\mu_p = 40\%$
Logistic – CPR	Aa1	Aa1	Aa1	0.31365	0.3466	0.27714	5.4309	5.2742	4.9611
Logistic – Lévy portfolio	Aa1	Aa1	Aa1	0.34552	0.48683	0.90665	5.365	5.1311	4.618
Logistic – Normal one-factor	Aa1	Aa1	Aa1	0.30488	0.3278	0.25706	5.431	5.2745	4.9633
Lévy portfolio – CPR	Aaa	Aaa	Aa1	0.10416	0.16105	0.42327	5.4093	5.2529	4.9404
Lévy portfolio – Lévy portfolio	Aaa	Aa1	Aa1	0.14828	0.34616	1.4266	5.3439	5.1101	4.5982
Lévy portfolio – Normal one-factor	Aaa	Aa1	Aa1	0.11976	0.20977	0.51304	5.4096	5.2532	4.9424
Normal one-factor – CPR	Aaa	Aaa	Aaa	0.023787	0.034631	0.046599	5.3995	5.2427	4.9292
Normal one-factor – Lévy portfolio	Aaa	Aaa	Aaa	0.03208	0.055516	0.1094	5.3335	5.0986	4.5842
Normal one-factor – Normal one-factor	Aaa	Aaa	Aaa	0.016874	0.017135	0.018373	5.3995	5.2427	4.9291
Gamma one-factor – CPR	Aa3	Aa3	Aa2	5.811	4.6682	2.8855	5.4149	5.2599	4.9497
Gamma one-factor – Lévy portfolio	Aa3	Aa3	Aa2	6.7492	4.9614	3.2188	5.3487	5.1167	4.6043

Model pair	Note B								
	Rating			DIRR (bp)			WAL (year)		
	$\mu_p = 10\%$	$\mu_p = 20\%$	$\mu_p = 40\%$	$\mu_p = 10\%$	$\mu_p = 20\%$	$\mu_p = 40\%$	$\mu_p = 10\%$	$\mu_p = 20\%$	$\mu_p = 40\%$
Logistic – CPR	A1	A1	A2	8.9089	10.581	14.756	5.4642	5.3124	5.0111
Logistic – Lévy portfolio	A1	A1	A3	9.9097	13.624	26.681	5.4011	5.1771	4.695
Logistic – Normal one-factor	A1	A1	A2	9.0211	10.906	14.436	5.4646	5.3135	5.0123
Lévy portfolio – CPR	A1	A2	A3	14.216	17.801	27.506	5.4994	5.3525	5.0628
Lévy portfolio – Lévy portfolio	A1	A2	Baa1	15.687	21.891	42.04	5.4384	5.2204	4.7511
Lévy portfolio – Normal one-factor	A1	A2	A3	14.318	18.35	28.531	5.4992	5.353	5.0644
Normal one-factor – CPR	Aa1	Aa1	Aa2	1.3334	1.5642	2.0323	5.4064	5.2502	4.9375
Normal one-factor – Lévy portfolio	Aa1	Aa2	Aa3	1.4404	1.9829	3.4481	5.3406	5.1071	4.5943
Normal one-factor – Normal one-factor	Aa1	Aa1	Aa1	1.1397	1.156	1.2153	5.4059	5.2491	4.9356
Gamma one-factor – CPR	Baa1	A2	A1	54.297	20.736	11.785	5.4614	5.3022	4.9848
Gamma one-factor – Lévy portfolio	A3	A3	A2	42.16	28.406	17.871	5.3945	5.1588	4.6418

Table 5: Ratings, DIRR and WAL of the ABS notes, for different combinations of default and prepayment models and mean cumulative default rate $\mu_d = 0.20$ and mean cumulative prepayment rate $\mu_p = 0.10, 0.20, 0.40$.

Model pair	Note A								
	Rating			DIRR (bp)			WAL (year)		
	Reserve (Sq)	Reserve (PR)	No Reserve (PR)	Reserve (Sq)	Reserve (PR)	No Reserve (PR)	Reserve (Sq)	Reserve (PR)	No Reserve (PR)
Logistic – CPR	Aaa	Aa1	Aa1	0.02813	0.3466	0.71815	4.3424	5.2742	5.2752
Logistic – Lévy portfolio	Aaa	Aa1	Aa1	0.036137	0.48683	1.0068	4.1747	5.1311	5.1327
Logistic – Normal one-factor	Aaa	Aa1	Aa1	0.032607	0.327	0.7184	4.3475	5.2745	5.2755
Lévy portfolio – CPR	Aaa	Aaa	Aa1	0.064217	0.16105	0.71116	4.3189	5.2529	5.28
Lévy portfolio – Lévy portfolio	Aaa	Aa1	Aa1	0.094711	0.34616	1.1772	4.1503	5.1101	5.1379
Lévy portfolio – Normal one-factor	Aaa	Aa1	Aa1	0.056669	0.0977	0.80489	4.323	5.2532	5.2802
Normal one-factor – CPR	Aaa	Aaa	Aaa	0.020445	0.034631	0.16448	4.2894	5.2427	5.2437
Normal one-factor – Lévy portfolio	Aaa	Aaa	Aa1	0.018883	0.055516	0.26287	4.1185	5.0986	5.0997
Normal one-factor – Normal one-factor	Aaa	Aaa	Aaa	0.013992	0.017135	0.051144	4.2858	5.2427	5.2437
Gamma one-factor – CPR	Aa3	Aa3	Aa3	4.1521	4.6682	5.9435	4.3125	5.2599	5.264
Gamma one-factor – Lévy portfolio	Aa3	Aa3	Aa3	4.2954	4.9614	6.5872	4.1417	5.1167	5.1207

Model pair	Note B								
	Rating			DIRR (bp)			WAL (year)		
	Reserve (Sq)	Reserve (PR)	No Reserve (PR)	Reserve (Sq)	Reserve (PR)	No Reserve (PR)	Reserve (Sq)	Reserve (PR)	No Reserve (PR)
Logistic – CPR	Aa3	A1	A3	10.792	10.581	38.957	9.0526	5.3124	5.4739
Logistic – Lévy portfolio	Aa3	A1	Baa1	13.567	13.624	46.316	9.0201	5.1771	5.3522
Logistic – Normal one-factor	Aa3	A1	A3	10.952	10.906	39.955	9.0348	5.3135	5.4763
Lévy portfolio – CPR	Aa3	A2	Baa1	17.089	17.801	67.004	9.1082	5.3525	5.6466
Lévy portfolio – Lévy portfolio	A1	A2	Baa2	20.412	21.891	75.017	9.0832	5.2204	5.5242
Lévy portfolio – Normal one-factor	Aa3	A2	Baa1	17.566	18.35	67.608	9.0939	5.353	5.6467
Normal one-factor – CPR	Aa1	Aa1	A1	1.5244	1.5642	8.5988	9.0657	5.2502	5.3062
Normal one-factor – Lévy portfolio	Aa1	Aa2	A1	1.9526	1.9829	10.739	9.0305	5.1071	5.171
Normal one-factor – Normal one-factor	Aa1	Aa1	Aa3	1.1428	1.156	5.4548	9.078	5.2491	5.2995
Gamma one-factor – CPR	A1	A2	A3	20.322	20.736	30.589	9.0956	5.3022	5.341
Gamma one-factor – Lévy portfolio	A1	A3	A3	28.065	28.406	37.646	9.063	5.1588	5.1986

Table 6: Ratings, DIRR and WAL of the ABS notes, for different combinations of default and prepayment models with and without reserve account for sequential (Sq) and pro rata (PR) payment. Mean cumulative default rate $\mu_d = 0.20$ and mean cumulative prepayment rate $\mu_p = 0.20$.

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