Abstract: Let $X_1, \ldots, X_n$ be i.i.d. observations, where $X_i = Y_i + \sigma_n Z_i$ and the $Y$'s and $Z$'s are independent. Assume that the $Y$'s are unobservable and that they have the density $f$ and also that the $Z$'s have a known density $k$. Furthermore, let $\sigma_n$ depend on $n$ and let $\sigma_n \to 0$ as $n \to \infty$. We consider the deconvolution problem, i.e. the problem of estimation of the density $f$ based on the sample $X_1, \ldots, X_n$. A popular estimator of $f$ in this setting is the deconvolution kernel density estimator. We derive its asymptotic normality under two different assumptions on the relation between the sequence $\sigma_n$ and the sequence of bandwidths $h_n$. We also consider several simulation examples which illustrate different types of asymptotics corresponding to the derived theoretical results and which show that there exist situations where models with $\sigma_n \to 0$ have to be preferred to the models with fixed $\sigma$.

Keywords: Asymptotic normality, deconvolution, Fourier inversion, kernel type density estimator.