A Characterization Related to the Equilibrium Distribution Associated with a Polynomial Structure

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Abstract

Let \( f \) be a probability density function on \((a, b) \subset (0, \infty)\) and consider the class \( \mathcal{C}_f \) of all probability density functions of the form \( Pf \) where \( P \) is a polynomial. Assume that if \( X \) has its density in \( \mathcal{C}_f \) then the equilibrium probability density \( x \mapsto P(X > x)/\mathbb{E}(X) \) also belongs to \( \mathcal{C}_f \): this happens for instance when \( f(x) = Ce^{-\lambda x} \) or \( f(x) = C(b-x)^{\lambda-1} \). The present paper shows that actually they are the only possible two cases. This surprising result is achieved with an unusual tool in renewal theory, by using ideals of polynomials.

Keywords: Renewal theory; excess lifetime; polynomial densities; ideals of polynomials.

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