Budget Allocation for Permanent and Contingent Capacity under Stochastic Demand

Nico Dellaert [1], Jully Jeunet [2], Gergely Mincsovics [1]

[1] Technische Universiteit Eindhoven, Industrial Engineering School, Postbus 513,
5600MB Eindhoven, The Netherlands
n.p.dellaert@tue.nl, g.z.mincsovics@tue.nl

[2] CNRS, Lamsade, Université Paris Dauphine, Place de Lattre de Tassigny,
75775 Paris Cedex 16, France
jully.jeunet@dauphine.fr

Abstract

We develop a model of budget allocation for permanent and contingent workforce under stochastic demand. The level of permanent capacity is determined at the beginning of the horizon and is kept constant throughout whereas the number of temporary workers to be hired must be decided in each period. Compared to existing budgeting models, this paper explicitly considers a budget constraint. Under the assumption of a restricted budget, the objective is to minimize capacity shortages. When over-expenditures are allowed, both budget deviations and shortage costs are to be minimized. The capacity shortage cost function is assumed to be either linear or quadratic with the amount of shortage, which corresponds to different market structures or different types of services. We thus examine four variants of the problem that we model and solve either approximately or to optimality when possible. A comprehensive simulation study is designed to analyze the behavior of our models when several levels of demand variability and parameter values are considered. The parameters consist of the initial budget level, the unit cost of temporary workers and the budget deviation penalty/reward rates. Varying these parameters produce several trade-off between permanent and temporary workforce levels, and between capacity shortages and budget deviations. Simulation results also show that the quadratic cost function leads to smooth and moderate capacity shortages over the time periods, whereas all shortages are either avoided or accepted when the cost function is linear.

Keywords: capacity planning, contingent workers, budget allocation, non-linear stochastic dynamic programming, optimization, stochastic processes.

1. Introduction

Intensifying demand for better education and skills leads organizations to increasingly rely on contingent workers' knowledge, with the side benefit of avoiding long-term hiring costs such as retirement costs. Besides, globalization implies a need for flexibility and agility to remain competitive. In this respect, the recourse to contingent workers is cost-effective and allows for adjustments in employment levels to quickly respond to demand changes.

Recent research tackles the problem of hiring and firing permanent and contingent workers so as to face unexpected spikes in demand. Techawiboonwong et al. (2006) consider the problem of assigning temporary workers to skilled and unskilled workstations so as to minimize the wage costs, the hiring/firing costs and the permanent overtime costs as well as the inventory holding costs and the backorder costs. The demand here is highly uncertain but treated as deterministic. In Bhatnagar et al. (2007), the objective is to minimize the cost of permanent workers, overtime and contingent workers (wage and induction costs), and the idle time cost for unutilized permanent workers; each worker being dedicated to a certain stage, line, shift and day. Permanent workers are endowed with several skill levels and contingent workers are used at the bottom of the skills. These multi-skilled models address the issue of selecting an appropriate contingent workforce and consider the demand as deterministic.

Apart from heterogeneous workforce management models, existing research has shown that staffing methods that account for the stochastic nature of demand (and supply) of labor can lead to significant reductions in labor costs. Wild and Schneeweiss (1993) develop a

hierarchical decision model to optimize the use of temporary workers, overtime and floaters (workers able to work in different departments) under uncertain demand for labor and absenteeism. In the same vein, Bard et al. (2007) design a 2-stage stochastic program for staff planning and scheduling decisions. In the first stage, the size of the permanent workforce is determined before demand is known. In a second stage, demand is revealed so overtime and casual workers can be used to satisfy the demand.

Uncertainty in labor supply is also included in the following references. Berman and Larson (1993) simultaneously determine the number of full time, part time and temporary employees to respond to day-to-day fluctuations in workload, while accounting for random absenteeism. Later on, Berman and Larson (1994) considered a similar problem derived from the postal sector, with a workload varying on a day-to-day basis and the restriction that all work received during a day must be processed on that day. Availability of both full time employees and temporary workers is uncertain so unmet work requirements for the day are provided by overtime shifts. In the last four references above cited, backlogs are not allowed. In Pinker and Larson (2003), the objective is to determine the number of regular and contingent workers over the whole planning horizon so as to minimize the expected labor and backlog costs. The model allows for backlogging or unfinished work, uncertainty in demand for labor, absenteeism and heterogeneous productivity amongst workers.

In this paper, we develop a model for contingent and permanent workforce management that allows for stochastic demand while considering a budget constraint. Budgeting models related to human resources planning can be found in the more specific OR-literature on health care. For instance, Trivedi (1981) develops a model for nursing service budgeting that ensures a balanced staff to meet a deterministic demand while satisfying cost control and regulation constraints. Kao and Queyranne (1985) design a stochastic model to provide the weekly pattern of permanent nurses and emphasize the detrimental impact of demand uncertainty on budget estimates. By considering the usage of temporary workers, our model provides a better dynamic usage of capacities under stochastic demand conditions. Furthermore, this paper is one of the first that explicitly takes into account a budget constraint. The budget allocation problem we consider consists in determining *(i)* a suitable permanent workforce level that will be available throughout the horizon and *(ii)* the number of temporary workers to be hired in each period. Several variants of the problem are examined, depending on the shape of the shortage cost function (linear or quadratic) and on the assumptions related to budget deficits as they can either be allowed or forbidden.

The next section provides a general description of the problem and introduces the assumptions and notations. In Section 3, we derive the budget allocation model when the capacity shortage cost function is assumed to be linear with the amount of shortages. Section 4 extends the analysis to the case of quadratic shortage costs to account for higher loss when shortages affect important clients. For each type of cost function, we distinguish the case of a fixed budget from the situation in which budget deficits are allowed. In Section 5, we design a comprehensive simulation study to assess the behavior of our models when several levels of demand variability and cost parameter values are considered. Section 6 draws the main conclusions of this paper.

2. Model framework

We start with a general description of the problem under examination and we discuss the underlying assumptions. We then describe the variants of the problem we have developed to reflect practical aspects of budget allocation.

The objective of our models is to allocate a given budget to both a permanent and a contingent capacity under stochastic demands so as to minimize the capacity shortage cost over a fixed horizon, usually a year. We assume the demands in each period to be independent of each other and to be identically distributed with some known distribution. This standard assumption covers a wide range of situations; for instance, it may be appropriate to model the demand for raw materials emanating from several customers to a supplier or to model the demand for healthcare. Our generic capacity planning model consists of permanent and contingent capacity decisions. At the tactical level, we decide on the permanent capacity level P, which will be held fixed and valid throughout the whole year. At the operational level, resorting to momentary capacity (M_t) may take place in each period $t = 1,...,T$ of the year. In every period we assume the following order of events:

- Demand of the coming period, d_t, is revealed and represents a realization of D_t which designates the demand as a random variable. The remaining budget b_t is observed;
- Contingent capacity may be called, which amounts to determine M_t;
- The capacity shortage cost is recorded.

Thus, demand of the current period is known before the contingent capacity decision is taken, whereas only the distribution of demand is known for later periods. In line with the capacity shortage penalty cost concept in Warner and Prawda (1972), surplus capacities in any periods are assumed to be lost.

We consider two assumptions regarding the budget use. First, we assume the budget can not be overspent, thus the objective is to minimize capacity shortages under a fixed, given budget. Second, budget deviations are allowed for over-expenditure so the objective consists in a joint minimization of capacity shortages and budget deviations. We borrow the budget deviation penalty assumption from Trivedi (1981): the end-of-year budget surplus is rewarded linearly with rate c_B^+, whereas the budget deficit is penalized linearly with a penalty cost c_B^- such that $c_B^- \geq c_B^+$.

We examine two types of *capacity shortage cost functions:* linear and quadratic, with the restriction that these functions return a value of zero when the capacity exceeds the demand, since surplus capacities are assumed to be lost. In competitive markets where a lot of producers offer the same product at same price, it is reasonable to assume linear shortage costs. In more oligopolistic market structures, companies have incentives to deal carefully with their most important and loyal customers. Thus, in case of shortages these customers will be served first. As shortages become higher, orders of such customers may be not honored with a risk that these customers go to competitors. Thus, the shortage cost should not only involve the cost of lost sales but also the cost of loosing the biggest clients. This is properly reflected by an increasing convex cost function of shortages, where marginal costs are increasing: as shortages increase, the cost of loosing an additional unit of demand increases.
Combining the two types of cost function for capacity shortages and the two assumptions for the budget leads to the four models developed and analyzed in the next sections of this paper. Table 1 summarizes the notations that will be used throughout the paper.

3. Linear cost for capacity shortages

This section is devoted to the analytical results we derive under the assumption that the capacity shortage cost function is linear. We first examine the situation in which the budget may not be overspent and we provide an approximation to the optimal permanent capacity

level that requires only a few computations. We then turn to the development and analysis of the model when budget deviations are allowed.

P	Permanent capacity expressed in number of supplied units per period
M_t	Contingent capacity expressed in number of supplied units in period t
T	Horizon length, with $t = 1,...,T$
D_t	Random variable used to designate the demand in period t
d_t	A realization of D_t
B	Fixed budget available over the whole horizon (yearly budget)
b_t	Remaining budget observed at the beginning of period t
c_S	Unit cost of shortage (equivalent to the cost of one unit of lost sale)
c_M	Unit cost of contingent capacity
c_P	Unit cost of permanent capacity
c_B^-	Penalty rate for one unit of budget deficit (recorded at the end of the year)
c_B^+	Reward rate of one unit of budget excess

Table 1. Notations

3.1 Restricted budget and linear cost for capacity shortages

Let $(a)^+$ be defined as $(a)^+ = \max(0,a)$. The total annual cost amounts to the shortage cost, $C_S(P)$, given by

$$C_S(P) = \sum_{t=1}^{T} c_S\left(D_t - P - M_t\right)^+, \tag{1}$$

with an expected value $E\left[C_S(P)\right]$ equal to

$$E\left[C_S(P)\right] = \sum_{t=1}^{T} c_S \int_{P+M_t}^{\infty} \left(x - P - M_t\right)^+ dF(x), \tag{2}$$

where $F(x)$ denotes the demand probability distribution function per period.

Since the cost is linear with the amount of shortage, there is no incentive to accept small shortages in some periods to save budget that could be used to face higher shortages, as the cost is strictly proportional to the shortage. This feature makes simple the decision about contingent capacity: as long as the remaining budget is large enough we choose the contingent capacity to exactly meet the demand, otherwise we spend all remaining budget on contingent capacity to cover as much demand as possible. Formally, we have

$$M_t = \begin{cases} \left(d_t - P\right)^+ & \text{if } b_t \geq c_M (d_t - P)^+, \\ \dfrac{b_t}{c_M} & \text{if } b_t < c_M (d_t - P)^+. \end{cases} \tag{3}$$

This means that the only decision left is the determination of the permanent capacity level *P*. When the budget is restricted to B, the total annual contingent capacity is a function of *P*. This function $M(P)$ has the form

$$M(P) = \left(B - c_P TP\right)/c_M \geq \sum_{t=1}^{T} M_t \tag{4}$$

The annual shortage cost, $C_S(P)$, can be expressed as the difference between the total excess demand that the permanent capacity can not satisfy and the total contingent capacity used. This difference represents the demand that can not be covered neither by permanent capacity nor by contingent capacity. Capacity shortages will only occur when the sum of the excess demand over the periods exceeds the available number of temporary workers $M(P)$. As demands are random, the annual shortage cost also is random. We have

$$C_S(P) = c_S\left(\sum_{t=1}^{T}\left(D_t - P\right)^+ - M(P)\right)^+ \tag{5}$$

Our objective is to find the optimal permanent capacity level P^* that minimizes the expected annual shortage cost. Since it is difficult to obtain analytical expression of P^* we provide an approximation to the expected annual shortage cost $E\left[C_S(P)\right]$:

$$E\left[C_S(P)\right] \approx c_S\left(T \cdot E\left[\left(D_t - P\right)^+\right] - M(P)\right)^+, \tag{6}$$

where D_t is the demand in an arbitrary period t. Letting $R(P)$ designate $E\left[\left(D_t - P\right)^+\right]$ to simplify the writing of the next equations, we can express the approximation to the expected total shortage cost as

$$E\left[C_S(P)\right] \approx c_S\left(T \cdot R(P) - M(P)\right)^+ \tag{7}$$

Minimizing the approximation in the right-hand side of (7) is considerably easier than minimizing (5). Minimizing (7) is equivalent to the minimization of the term inside the positive part operator. Replacing $M(P)$ with its expression given in (4), we obtain a newsvendor problem, having the following straightforward solution for the optimal permanent capacity level

$$P^* \approx P^*_{nv} = \arg\min_{P \geq 0}\left\{T \cdot R(P) - \frac{B - c_P TP}{c_M}\right\} = F^{-1}\left(\frac{c_M - c_P}{c_M}\right). \tag{8}$$

This permanent capacity level P^*_{nv} is an approximate value to the optimal capacity P^* that minimizes the exact total shortage cost in (5). Another approximation P^*_{sim} may be obtained through simulations by performing numerous draws of random demands, and by computing the observed shortage cost for some P values and then choosing the value of P that minimizes the shortage cost. The approximation P^*_{sim} is then obtained by averaging, over the replications of random demand draws, these values of P minimizing the shortage cost. The higher the number of replications is, the higher the quality of the approximation is.

To illustrate, let us consider a budget $B = 3250$ available over $T = 50$ periods. The demand is assumed to follow either a normal distribution or a gamma distribution, both with mean 50 and standard deviation 20 (which amounts to parameters $\alpha = 6.25$ and $\lambda = 8$ for the gamma

distribution). We set the following values for the unit costs: $c_S = c_P = 1$ and $c_M = 2.5$. For each demand distribution, we perform 1000 replications of demand draws $\{d_t\}_{t=1,\ldots,50}$ and we consider values of P in the range $\{30,\ldots,65\}$. For each value of P we compute the exact shortage cost $C_S(P)$, with $C_S(P) = c_S\left(\sum_{t=1}^{T}(d_t - P)^+ - (B - c_P TP)/c_M\right)^+$. We then record the value of P that minimizes the shortage cost within one replication. We get P^*_{sim} by averaging the 1000 values of P obtained this way. Table 2 summarizes the results we obtain for both demand distributions.

Parameters	Demand	Newsboy P^*_{nv}	Simulated P^*_{sim}
$B = 3250;\ T = 50$	$D_t \sim N(50,20)$	55.07	55
$c_S = c_P = 1;\ c_M = 2.5$	$D_t \sim \Gamma(6.25,8)$	52.44	53

Table 2. Approximated values to the optimal permanent capacity

With both demand distributions, the approximated value of P^* provided by the solution to the newsvendor problem P^*_{nv} is very close to the value P^*_{sim}. We thus have an approximation of good quality that requires very few computations compared to P^*_{sim}.

In the next paragraph, we examine the situation in which we include costs for budget deviations. The objective is to allocate the budget dynamically so as to find a good balance between capacity shortages and budget deficit for a given annual budget.

3.2 Unrestricted budget and linear cost for capacity shortages

We consider linear penalties for budget deficits, and linear rewards for budget surpluses. The budget deficit is assumed to be at least as much penalized as the budget surplus is rewarded, that is $c_B^- \geq c_B^+$. In this situation, hiring temporary workers is still possible when the budget is totally spent. We will see that the various cost coefficients will largely influence the use of temporary workers.

The total annual expense equals $c_P TP + c_M \sum_{t=1}^{T} M_t$ and can either be superior to the budget B or inferior to it. The budget deficit cost $C_B^-(P)$ and the budget surplus reward $-C_B^+(P)$ may therefore be expressed as follows

$$C_B^-(P) = c_B^-\left(c_P TP + c_M \sum_{t=1}^{T} M_t - B\right)^+ = c_B^- c_M \left(\sum_{t=1}^{T} M_t - \frac{B - c_P TP}{c_M}\right)^+, \tag{9}$$

$$C_B^+(P) = -c_B^+\left(B - c_P TP - c_M \sum_{t=1}^{T} M_t\right)^+ = -c_B^+ c_M \left(\frac{B - c_P TP}{c_M} - \sum_{t=1}^{T} M_t\right)^+,$$

where $\{M_t\}_{t=1,\ldots,T}$ are the decision variables. As over-expenditure is only dedicated to hiring temporary workers to cover demand excess that can not be covered by the permanent capacity, the cost of a budget deficit not only includes the penalty cost of overspending the budget but also the cost of hiring temporary workers. Under-expenditure represents a saving in terms of costs of contingent workers we do not need to hire.

The sum $C_B^-(P)+C_B^+(P)$ represents the cost of the overall deviation from budget B. Depending on the unit cost values, it may be advantageous not to exceed the budget even if lost sales are incurred. Thus, the total cost $C(P)$ to be minimized not only includes $C_B^-(P)+C_B^+(P)$ but also the shortage cost $C_S(P)$ as defined by Eq. (1). We minimize the expected value of the total cost $C(P)=C_S(P)+C_B^-(P)+C_B^+(P)$, that is

$$\min_{P\geq 0} E\left[C_S(P)+C_B^-(P)+C_B^+(P)\right]. \tag{10}$$

Depending on the order of magnitude of the cost parameter values, several cases can be distinguished that lead to specific uses of temporary workers, and therefore to simplified expressions of the total cost for which we derive solutions to find the best permanent capacity level.

Case 1. If the unit shortage cost exceeds the cost of overspending the budget to hire temporary workers: $c_S > c_B^- c_M$, then it is more advantageous to avoid capacity shortages. All demands will be satisfied by using permanent capacity and by hiring temporary workers when necessary. We thus have $C_S(P)=0$. Consequently the total cost only have two components and is expressed as

$$C(P)=C_B^-(P)+C_B^+(P) \tag{11}$$

Since $M_t=(D_t-P)^+$, we can use the same recipe as the one we developed to derive the approximation of the expected shortage cost under restricted budget (Eq. (7)): using $R(P)\equiv E\left[(D_t-P)^+\right]$ and replacing $\sum_{t=1}^{T} E\left[(D_t-P)^+\right]$ with $T\cdot R(P)$, we get the following approximation

$$E\left[C(P)\right]\approx c_B^- c_M\left(T\cdot R(P)-\frac{B-c_P TP}{c_M}\right)^+ - c_B^+ c_M\left(\frac{B-c_P TP}{c_M}-T\cdot R(P)\right)^+, \tag{12}$$

The approximation suggests that the optimal capacity level, P^*, is close to the maximizer of the approximated budget excess: $(B-c_P TP)/c_M - T\cdot R(P)$, and this maximizer also is a minimizer of the approximated budget deficit $T\cdot R(P)-(B-c_P TP)/c_M$. Thus, the maximization reduces to the same newsvendor problem as in (8) and we have exactly the same expression for P_{nv}^*

$$P^*\approx P_{nv}^*=\arg\min_{P\geq 0}\left\{T\cdot R(P)-\frac{B-c_P TP}{c_M}\right\}=F^{-1}\left(\frac{c_M-c_P}{c_M}\right). \tag{13}$$

Case 2. If, contrary to case 1, the unit shortage cost is inferior to the cost of overspending the budget to hire temporary workers: $c_S < c_B^- c_M$ then it is less costly to have capacity shortages than having a budget deficit. To avoid budget deficits, demand is satisfied only if there is budget left. We thus have $C_B^-(P)=0$. If, furthermore, we have $c_S > c_B^+ c_M$ then saving money by not hiring temporary workers is less rewarded than a shortage is penalized. Thus, it is still advantageous to hire temporary workers. Minimizing the total cost results in finding a good balance between the shortage cost and the budget excess reward, as saving budget on

temporary workforce can lead to higher shortages. Thus if $c_B^+ c_M < c_S < c_B^- c_M$, the total cost is written as

$$C(P) = C_S(P) + C_B^+(P) \tag{14}$$

We can proceed similarly to Case 1 to obtain the approximation

$$E\left[C(P)\right] \approx c_S\left(T \cdot R(P) - \frac{B - c_P TP}{c_M}\right)^+ - c_B^+ c_M\left(\frac{B - c_P TP}{c_M} - T \cdot R(P)\right)^+, \tag{15}$$

which leads to the same observation and approximation as in Case 1, equation (13).

Case 3. We still consider the no budget deficit situation implied by $c_S < c_B^- c_M$ but we now examine the case where $c_S < c_B^+ c_M$. Saving budget on temporary workers is more rewarded than a lost sale is penalized. Compared to Case 2, now there is no longer incentive to spend budget on temporary capacity to diminish lost sales. As the expense on temporary workers is zero, to get the total cost we take the expression of the shortage cost in Eq. (1) and the expression of the budget excess cost in Eq. (9) in which we replace M_t with zero for all $t = 1,...,T$. From Eq. (1), we obtain $C_S(P) = c_S \sum_{t=1}^{T} (D_t - P)^+$ and from Eq. (9) we get $C_B^+(P) = -c_B^+ (B - c_P TP)^+$. The total cost is the sum of these two costs

$$\begin{aligned} C(P) &= C_S(P) + C_B^+(P) \\ &= c_S \sum_{t=1}^{T} (D_t - P)^+ - c_B^+ (B - c_P TP)^+. \end{aligned} \tag{16}$$

From Eq. (16), let us notice that one unit of permanent capacity that is saved is rewarded $c_B^+ c_P$. Thus we still invest in permanent capacity as long as the unit shortage cost exceeds $c_B^+ c_P$. Consequently, the expression of the total cost in (16) is valid if the three above mentioned conditions on the unit shortage cost hold, that is $c_S < c_B^- c_M$ and $c_S < c_B^+ c_M$ and $c_S > c_B^+ c_P$. As we assumed $c_B^+ \le c_B^-$, this finally amounts to have $c_B^+ c_P < c_S < c_B^+ c_M$. Using again $R(P)$, we derive the following approximation to the total cost given in Eq. (16)

$$E\left[C(P)\right] \approx c_S T \cdot R(P) - c_B^+ (B - c_P TP)^+, \tag{17}$$

which is another newsvendor equation, yielding

$$P^* \approx P_{nv}^* = F^{-1}\left(\frac{c_S - c_B^+ c_P}{c_S}\right). \tag{18}$$

As it is more advantageous to avoid capacity shortages than saving budget on permanent capacity ($c_S > c_B^+ c_P$), if the budget B is not too large there is a big chance it is totally spent, meaning that $P^* = B / Tc_P$. It should be noted that if $c_S < c_B^+ c_P$ there is no longer incentive to avoid shortages by investing in permanent capacity, as capacity shortages are less penalized than the savings on permanent capacity are rewarded. In such a situation, no budget is spent at all and condition $c_S < c_B^+ c_P$ is actually a necessary and sufficient condition, since we assumed

$c_P < c_M$ (so it is needless to combine $c_S < c_B^+ c_P$ and $c_S < c_B^+ c_M$). This extreme situation where no budget is spent constitutes Case 4 that is fully characterized by $c_S < c_B^+ c_P$.
It should be noted that the situation where $c_B^+ = c_B^-$ is compatible with all cases except case 2.

To illustrate the three cases, let us consider a gamma-distributed demand with mean 50 and standard deviation of 20. We set $c_S = c_P = 1$, $c_B^+ = 0.3$ (that is 30%) and $c_B^- = 0.6$, with a budget level $B = 3250$. Figure 1 displays the remaining budget in each period over 50 periods in each case.
Case 1. With a unit cost of temporary work $c_M = 1.1$, we have $c_S > c_B^- c_M$. The optimal permanent capacity available per period equals 53. The budget use exceeds the budget level, as it is more advantageous to avoid capacity shortages by accepting budget deficits. Thus, the expected remaining budget becomes negative from period 37.
Case 2. Inequalities $c_B^+ c_M < c_S < c_B^- c_M$ are satisfied with a unit cost of temporary work $c_M = 2.5$. The optimal permanent capacity equals 53. Capacity shortages are borne to avoid budget deficits, thus the expected remaining budget curve always stays above the x-axis.
Case 3. We have $c_B^+ c_P < c_S < c_B^+ c_M$ for $c_M = 6$. Temporary workers are so expensive that they are not hired at all. The optimal permanent capacity equals 43, thus the remaining budget is constant throughout and equal to $B - c_P TP = 1100$.

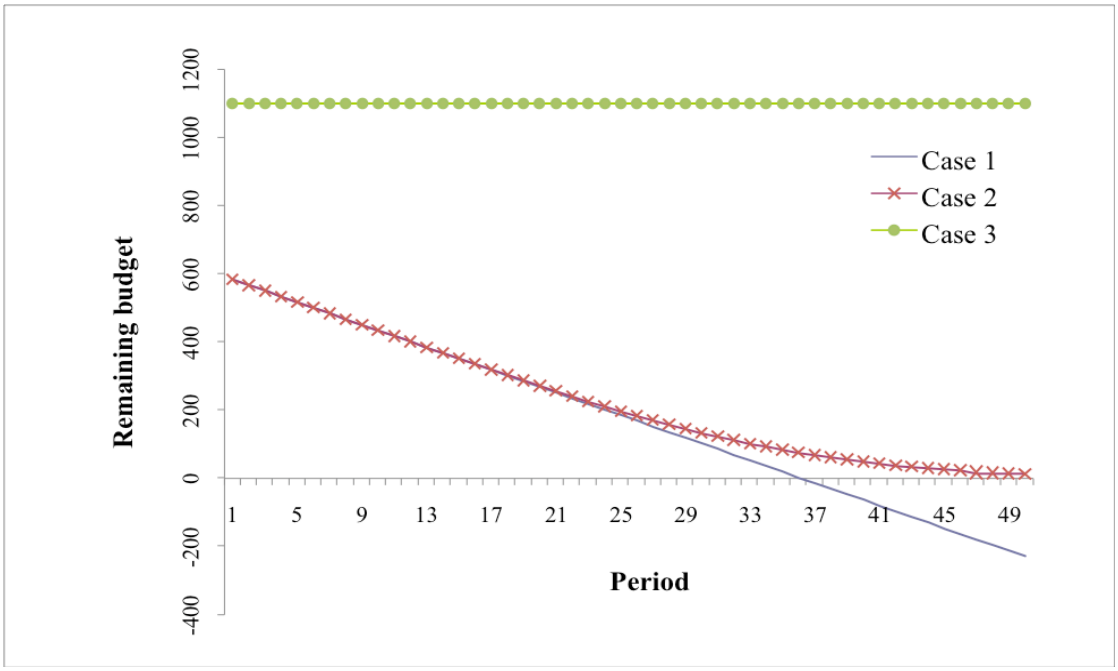

Fig. 1. Remaining budget per period for each case - unrestricted budget and linear cost

4. The quadratic shortage cost situation

Next to studying the linear capacity shortage cost function, we introduce the quadratic cost function, which we consider as being more realistic in less competitive market structures. As already mentioned in Section 2, an increasing convex cost function of capacity shortages is appropriate to reflect the fact that higher shortages imply a higher cost as the cost not only involves the cost of lost sales but also the cost of loosing the biggest customers. We choose the following shortage cost

$$C_S(P) = c_S \sum_{t=1}^{T} \frac{\left[\left(D_t - P - M_t\right)^+\right]^2}{D_t}. \tag{19}$$

The shortage cost function in (19) is increasing and convex with the amount of shortage. Such a function is also appropriate to model the cost of nursing care shortages in hospitals. If the demand for care slightly exceeds the supply, this means that nurses can not socialize with patients and this has little consequence so the associated shortage cost should be low. Conversely, if the demand largely exceeds the supply, nurses can no longer be able to provide all essential cares with possible serious consequences that should be reflected by a high shortage cost. With a quadratic cost function, there is a strong incentive to accept small capacity shortages in some periods in order to save some budget which will be used to avoid large capacity shortages that would be more likely to happen in later periods if we had fulfilled all demands in the first periods. Consequently, contrary to the linear shortage cost situation, here it is no longer optimal to avoid any amount of shortage. A better strategy to impede large shortages (at any time) would consist in accepting small shortages in some periods (when the excess demand over the permanent capacity is not very high) so as to save money that can be used later when there are demand spikes and/or less budget.

We model this situation as a dynamic program solved via backward induction. A limited state space is necessary to compute the decisions. Hence we assume that we always use an integer amount of contingent capacity per period and that the demand per period has some discrete distribution. In the linear case, we considered a continuous demand as we wanted to use well-known distributions like the normal distribution and the gamma distribution. When we use dynamic programming however, it becomes practically impossible to solve the model with continuous demand, so we discretize the demand. With a high number of possible demand values, the discretization will hardly influence the results. In the next paragraphs, we develop the dynamic programming models for the restricted and the unrestricted budget situations.

4.1 Restricted budget and quadratic cost for capacity shortages

In each period we minimize the sum of the current capacity shortage costs and the expected future costs in later periods, so $f_t(b_t)$ denotes the expected costs from period t to the last period T when the remaining budget at the beginning of period t equals b_t. The remaining budget is the state variable. We have

$$f_t(b_t) = \min_{M_t \geq 0} c_S \frac{\left[\left(D_t - P - M_t\right)^+\right]^2}{D_t} + E\left[f_{t+1}(b_t - c_M M_t)\right] \text{ for all } t = 1,2,...,T. \tag{20}$$

Before the start of the year, in period 0 we pay in advance for our permanent capacity:

$$f_0(B) = f_1\left(B - c_P TP\right). \tag{21}$$

The budget expense for permanent workers being reserved at the beginning of the year, we have $b_1 = B - c_P TP$ and for later periods, we have $b_{t+1} = b_t - c_M M_t$. For the restricted budget case, the closing cost-to-go is defined as

$$f_{T+1}(b_{T+1}) \equiv 0. \tag{22}$$

Since the demand of the last period, d_T, is known before the last decision is taken, all remaining budget will be spent, as far as this demand makes it necessary.

Obviously, when $D_t \le P$, the optimal contingent capacity decision is $M_t = 0$. Consequently, we can write $f_t(b_t)$ as

$$\begin{aligned} f_t(b_t) &= \sum_{i=1}^{P} \Pr\{D_t = i\} f_{t+1}(b_t) \\ &+ \sum_{i \ge P+1}^{\infty} \Pr\{D_t = i\} \left[\min_{M_t \ge 0} c_S \frac{(i - P - M_t)^2}{i} + f_{t+1}(b_t - c_M M_t) \right], \end{aligned} \tag{23}$$

for all $t = 1,2,...,T$.

In order to find the optimal solution, we calculate the expected costs over the whole horizon for a set of relevant *P*-values. For every *P*-value the model is solved recursively as follows. Starting with period $T+1$, we use Eq. (22) that provides the values of $f_{T+1}(b_{T+1})$. We then turn to period T and we use Eq. (23) to calculate $f_T(b_T)$ for all possible values for b_T and continue this way until we calculate the expected costs for this *P*-choice using formula (21).

4.2 Budget deviations and quadratic cost for capacity shortages

Compared to the model given in the previous paragraph, only the final stage is changed. To account for budget deviations that are rewarded or penalized, we rewrite equation (22) as

$$f_{T+1}(b_{T+1}) = c_B^-(-b_{T+1})^+ - c_B^+(b_{T+1})^+. \tag{24}$$

The implementation of the model solution remains the same as in the restricted budget situation.

5. Simulation experiment

We first describe our experimental framework as well as the indicators we chose to analyze the simulation results. We then present the results of the restricted budget case and that of the unrestricted case.

5.1 Parameter setting and indicators

Demand distribution. In the linear case with restricted budget (Section 3.1), we considered both a normal and a gamma distribution as they are very popular. In our experiment, we decided to keep only the gamma distribution for several reasons. The gamma distribution has the advantage of not producing any negative demand value. Contrary to the normal distribution which is not suitable to model fast moving items, the gamma distribution is ideal for modeling slow moving items and can easily be adapted for fast moving items as well. In our dynamic programming approach, we have to discretize the distribution, which is not a problem as the mean and variability are high enough. We thus consider a discretized gamma distribution with mean $\mu = 50$ and three levels of demand variability $\sigma = \{10,20,30\}$ that

represent respectively 20%, 40% and 60% of the mean. The mean of the distribution was kept constant throughout as other parameters were varied to cover all types of relevant situations.

Unit cost of temporary workforce c_M *and permanent workforce* c_P. As setting the ratio c_M / c_P is more relevant than setting each unit cost value separately, it is reasonable to normalize $c_P = 1$. We chose $c_M = \{1.1, 1.5, 1.9, 2.5, 6\}$. A value of $c_M = 1.1$ allows for an analysis of the situation when the unit cost of temporary capacity is very close to the cost of permanent workers. A value of $c_M = 1.9$ represents the French case for unqualified work. The highest value of c_M was chosen to account for high productivity workers.

Unit shortage cost c_S *and budget deviation rates* $\left(c_B^-, c_B^+\right)$. As budget deviation rates may be chosen freely, it would make no sense to also vary the unit shortage cost. Both types of unit cost are indeed involved in the trade-off which consists in possibly accepting budget deficits to avoid shortages. Consequently, we set $c_S = 1$ throughout. To represent properly all situations of the linear case, we chose the following rates expressed in percent: $\left(c_B^-, c_B^+\right) = \{(30,30),(60,30),(60,60),(120,60)\}$. In the quadratic case, we set the following rates in percent: $\left(c_B^-, c_B^+\right) = \{(1,1),(4,2),(8,4),(16,8),(16,16)\}$. In both linear and quadratic cases, the restricted budget situation was modeled by setting $\left(c_B^-, c_B^+\right) = (100000,0)$.

Budget level. Setting artificially $P = 50$ to cover the demand mean per period implies a yearly budget expense of $c_P PT = 1 \times 50 \times 50 = 2500$. We thus start with a budget level of 2500 which was incremented by 250 until the highest budget level of 3500 was reached. This maximum budget level of 3500 corresponds to a number of periodic permanent workers equal to 70.

Indicators. For each level of demand variability σ, each value of the unit costs c_M and $\left(c_B^-, c_B^+\right)$ and for each budget level B, we computed several indicators over the $T = 50$ periods: the optimal permanent workforce level P (per period); the number of hired temporary workers $M = \sum_{t=1}^{50} M_t$; the budget use which is equal to $c_P TP + c_M M$; the capacity shortage in each period which is the expected value of $(D_t - P - M_t)^+$; the related expected capacity shortage cost (per year) in the linear case equals $\sum_{t=1}^{T} E\left[(D_t - P - M_t)^+\right]$ (recall that $c_S = 1$) and in the quadratic case, it equals $\sum_{t=1}^{T} (1/D_t)\left[\left(D_t - P - M_t\right)^+\right]^2$.

Under the assumption of an unrestricted budget, we also consider the budget deviation cost equal to $c_B^-\left(\text{Budget Use} - B\right)^+ - c_B^+\left(B - \text{Budget Use}\right)^+$ and the expected budget deficit where the budget deficit is defined as $\left(c_P TP + c_M \sum_{t=1}^{T} M_t - B\right)^+$.

5.2 Result analysis for restricted budget cases

Table 3 displays the mean of indicators for each level of demand variability and over all parameter values, for both types of shortage cost function, when the budget is restricted. As demand variability increases, permanent capacity decreases in favor of additional temporary workers more capable to respond to demand spikes. In the quadratic case, more temporary workers are hired than in the linear case for there is a bigger incentive to avoid capacity shortages. The budget use slightly increases because temporary workers are increasingly hired

and cost more than permanent ones. Shortages per period also increase with demand variability and this entails an increasing shortage cost.

Shortage cost function	*Demand Variability*	**Opt. Perm.**	**Temps**	**Budget Use**	**Shortage per period**	**Shortage Cost (=Total)**
Linear	Low	50.88	167.92	2791.68	0.98	48.99
	Average	46.20	395.41	2873.78	2.97	148.34
	High	41.56	585.03	2903.03	5.45	272.56
Quadratic	Low	50.20	177.29	2782.63	1.07	7.28
	Average	45.12	403.82	2854.18	3.24	36.92
	High	39.88	599.34	2870.84	5.92	89.57

Table 3. Overall average of indicators under restricted budget

For each level of demand variability, and for each parameter value (factor), Table A1 in the Appendix provides the average of the indicators for both shortage cost functions under the assumption of a restricted budget. As the budget level (B) increases, the permanent capacity also increases but remains lower for higher demand variability so as to favor the employment of temporary workers. For low and intermediate levels of demand variability, and over a budget threshold of approximately 3000, the number of hired temporary workers start to decrease with the budget level: permanent workers are cheaper than temporary workers and are increasingly hired because they also are able to respond to reasonable demand fluctuations but at a lower cost. As the budget level is augmented, we can afford more capacity so the shortages per period decrease. However, capacity shortages remain higher for higher demand variability. The budget level clearly impacts the total cost (equal to the shortage cost) as more budget implies less shortages by allowing the employment of more workers, both temporary and permanent.

The unit cost of temporary workforce (c_M) has a strong influence on the hiring of temporary workers. There is a trade-off between permanent work and temporary work as c_M increases. When demand fluctuates a lot and when temporary workers are very expensive, the number of hired temporary workers in the linear case is even zero, leading to large shortages.

5.3 Result analysis for unrestricted budget situations

We shall now examine the simulation results when the budget is unrestricted. Table 4 displays the average of each indicator over all parameter values and for each level of demand variability, for both types of shortage cost function.

Shortage cost function	*Demand Variability*	**Opt. Perm.**	**Temps**	**Budget Use**	**Budget Dev. Cost**	**Shortage per Period**	**Budget Deficit**	**Shortage Cost**	**Total Cost**
Linear	Low	46.08	229.19	2616.42	-193.21	2.07	27.04	103.72	-89.51
	Average	41.63	433.34	2671.18	-185.90	4.57	73.10	228.44	42.54
	High	36.32	627.90	2683.65	-201.35	7.29	139.00	364.39	163.04
Quadratic	Low	46.42	236.77	2679.17	-27.60	1.92	42.65	10.91	-16.69
	Average	43.94	496.08	2978.22	-9.14	2.46	191.43	18.09	8.95
	High	41.30	743.01	3275.76	12.10	2.97	423.23	25.04	37.13

Table 4. Overall average of indicators under unrestricted budget

When demand variability increases, the permanent workforce level decreases and the number of temporary workers increase as they are more capable to respond to demand changes. More temporary workers also imply a higher budget use as they are more costly than permanent workers. This tendency is stronger in the quadratic case for the incentive to avoid capacity shortages is bigger. Still, shortages per period increase but this increase is definitely flatter in the quadratic case than in the linear case. Compared to the restricted budget situation, here, even more temporary workers can be afforded in the detriment of permanent workforce.

A high demand variability implies more periodic shortages as budget deficits are limited by a natural trade-off between the capacity shortage cost and the budget deficit cost. This is particularly true in the linear case as budget deficits are lower than those observed in the quadratic case. All demands are not covered to avoid too much budget deficits, even if these budget deficits increase with the demand variability. In the linear case, the most advantageous situation is the lowest demand variability one, as budget can be saved while limiting periodic shortages. The resultant total cost is indeed negative, whereas it becomes increasingly positive with demand variability. In the quadratic case, budget deviation costs are increasingly positive and larger than they are in the linear case, because higher budget deficits are necessary to avoid capacity shortages so far as we can.

For each level of demand variability, and for each parameter value (factor), Table A2 in the Appendix displays the average of the indicators for both shortage cost functions when budget deviations are allowed. We shall now analyze the impact of the three factors (budget level, budget deviation rates and unit cost of temporary workforce) on the indicators.

Budget level. The budget level has hardly any influence on the permanent capacity, as witnessed by the steady averages in Table A2. As already revealed by the global averages in Table 4, the permanent capacity decreases with the demand variability, in favor of temporary capacity. For a budget of 3000 and above, the number of hired temporary workers remains pretty stable, as well as the periodic shortages together with the shortage costs. Since the budget use exhibits only a slight increase, the gap with the budget level increases which entails increasingly negative budget deviation costs (budget deficits experience a strong decrease). This is the result of a trade-off between capacity shortage costs and budget excess rewards.

Budget deviation rates. Permanent capacity decreases as budget deviation rates (c_B^-, c_B^+) are increased. Temporary capacity follows the same decreasing path. We have less budget deficits and increased capacity shortages per period entailing increased shortage costs, meaning that it is more advantageous to accept capacity shortages so as to limit budget deficits that become increasingly expensive and more detrimental to the total cost than capacity shortages. Capacity shortages are also borne in favor of higher budget excesses that are more rewarded than shortages are penalized. Thus, with higher budget deviation rates, there is a tendency to spend less, both on permanent and temporary workers. The trade-off between capacity shortages and budget deviations is clearly in favor of a strong limitation of budget deficits that even reach a zero value for the highest budget deviation rates in the linear case, whatever the level of demand variability.

Unit cost of temporary capacity. This factor is obviously the most influent on the workforce capacity: as temporary workers become increasingly expensive, more permanent workers and less temporary workers are hired. It should be noted that when permanent workers are almost equally expensive as temporary workers, a few permanents are hired when the demand variability is strong, as more flexibility is achieved at low cost with temporary workers. On

the contrary, when temporary workers are extremely expensive they are no longer hired in the linear case but a few are still utilized in the quadratic case for the incentive to avoid capacity shortages is stronger. Capacity shortages increase since less and less temporary workers are employed.

The total cost is the sum of the budget deviation cost and the shortage cost. With more budget, we have decreasing costs. This is less true for high demand variability. For a low demand variability and as the budget deviation rates increase, we have more budget excesses as they are increasingly rewarded. For higher demand variability however, budget deficits are larger and even more penalized. The total cost clearly degrades with high values of the unit cost of temporary capacity, especially when demand variability is high. Shortage costs are minimum when it is possible to hire temporary workers at the lowest cost, when a high budget level is available and when budget deviations are the least penalized. Finally, for all levels of demand variability and whatever the shape of the shortage cost function, the total cost is minimum for the lowest unit cost of temporary capacity, the highest budget level and the highest budget deviation rates: a high budget deficit rate tends to limit costly budget deficits, whereas budget excesses are rewarded at most.

5.4 Further illustrations and comments

To complete the analysis of the quadratic case, we plot in Figure 2 the probability distribution of the remaining budget expressed in terms of number of temporary workers that can be hired, for each combination of budget deviation rates (in the legend, (1,1) corresponds to $\left(c_B^-, c_B^+\right) = (1,1)$). We selected a "typical" quadratic case with a budget level of 3250, a unit cost of temporary work equal to 2.5 and an intermediate level of demand variability ($\sigma = 20$). For the restricted budget case, the probability of a zero remaining budget was 0.31 but was reduced to a maximum value of 0.02 to get a proper figure. As the budget deviation rates increase the probability distributions move to the right: it becomes less and less advantageous to have budget deficits, so the remaining budget takes more and more positive values. For the restricted budget case, we have of course the highest probability for a zero remaining budget.

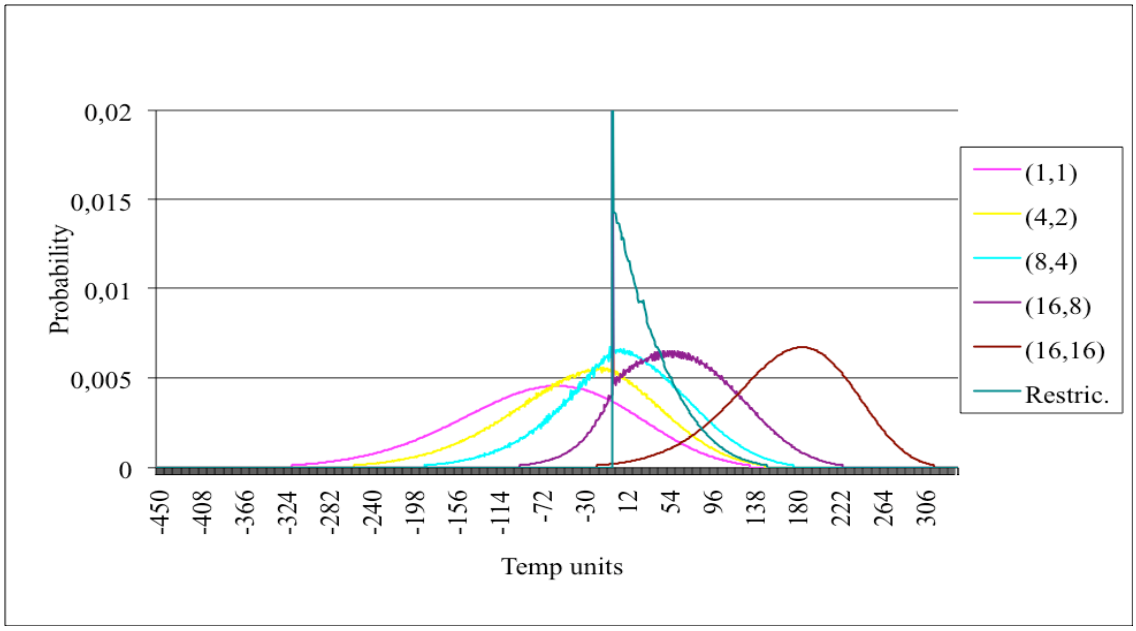

Fig. 2. Probability distribution of remaining budget expressed in number of temporary workers - quadratic instance

Figure 3 illustrates the periodic behavior of capacity shortages and their associated costs under a restricted budget with both types of cost function. We took a typical instance characterized by intermediate values for the parameters, as in the previous illustration (same values for σ, B and c_M) and we chose $\left(c_B^-, c_B^+\right) = (100000,0)$ (restricted budget). In the linear case, we have $P = 53$ and in the quadratic case, $P = 52$.

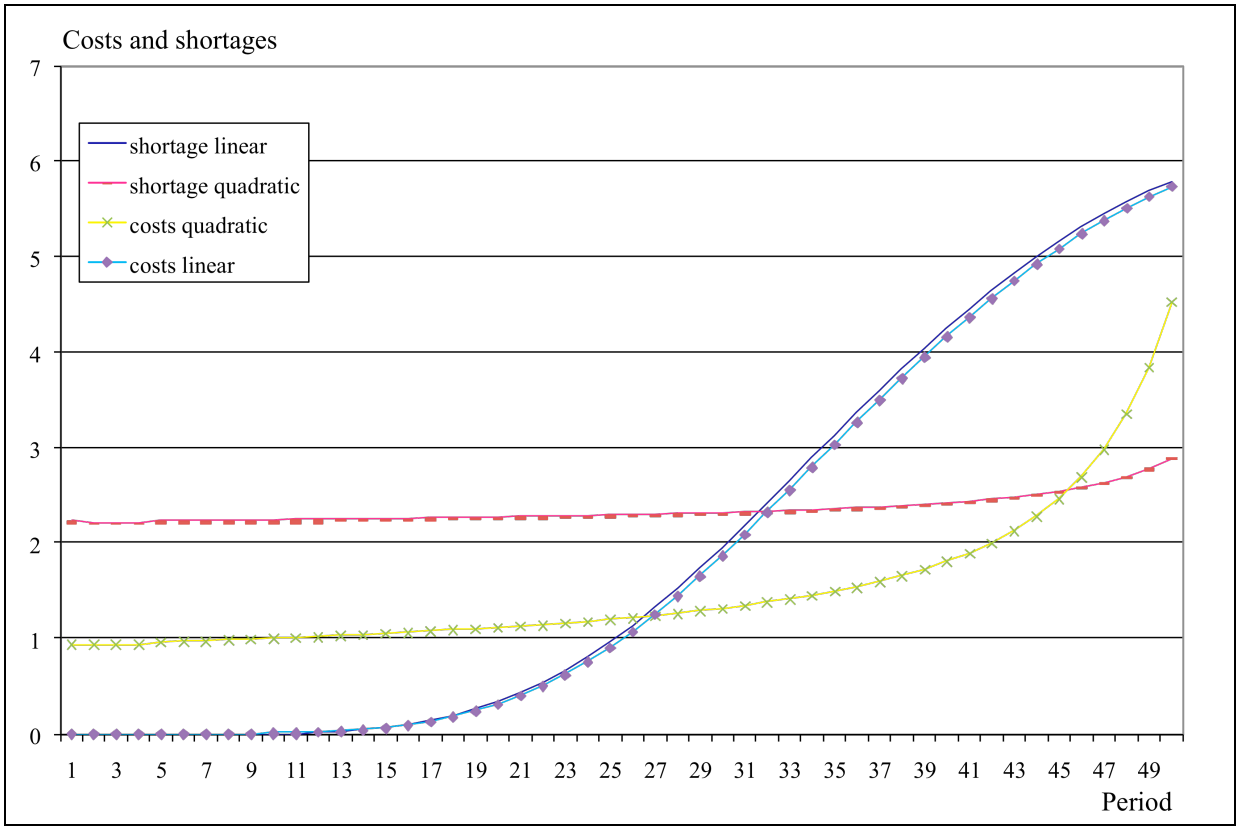

Fig. 3. Costs and shortages as a function of time periods - restricted budget

Compared to the linear case, shortages are pretty steady in the quadratic case since big shortages are avoided by accepting small shortages throughout. There is a slight shortage increase in the end accompanied with a faster increase of the cost due to the convex shape of the cost function. In the linear case, shortages are zero in the first 15 periods and then continuously increase since there is no incentive to avoid large shortages in any period.

6. Conclusion

In many companies, fixed yearly budgets are allocated to the heads of departments to cover their fixed and variable expenses during the year. In our paper, we addressed the problem of periodical budget allocation to fixed and variable expenditures. We developed four different models to determine the permanent and contingent capacity levels so as to minimize the capacity shortage, and budget deviation penalty costs when over-expenditures are allowed.

When the capacity shortage cost function is linear, in both the restricted and unrestricted budget cases we developed analytic formulas and found that near-optimal solutions can be obtained by using a newsvendor equation. For quadratic cost functions, we proposed a solution with stochastic dynamic programming.

Numerical experiments show that the service level over the year is more stable when the cost function is quadratic since large shortages are avoided by accepting small shortages throughout. With a linear cost function, there is no incentive to obviate big shortages so we have extreme behaviors: we either avoid or accept all capacity shortages. Thus, contrary to the linear cost function, the quadratic shortage cost function leads to smooth and moderate shortages over the time periods.

We studied the impact of several factors on the behaviour of our models. As demand variability increases, less permanent workers are hired to favor the employment of temporary ones as they allow for more flexibility necesary to respond at best to demand spikes. However, capacity shortages inevitably augment even in the unrestricted budget case as over-expenditures are limited by a natural trade-off between the budget deficit cost and the capacity shortage cost. In the restricted budget situation, the initial budget level impacts both types of workforce: with more budget, more temporary workers and more permanent workers are hired, which implies less shortages. In the unrestricted buget case however, the budget level has hardly any influence on the level of permanent capacity, whereas temporary workers are increasingly hired. In both restricted and unrestricted budget situations, the unit cost of the temporary capacity is the most influent factor on the temporary workforce level. As the unit cost of the temporary capacity increases, there is a trade-off between permanent and temporary workers: more permanent and less temporary workers are hired. In the linear case, the temporary capacity even reaches a zero value when it becomes extremely expensive; a few temporary workers are still hired however in the quadratic case due to a stronger incentive to avoid shortages. With higher budget deviation rates, there is a tendency to spend less on both permanent and temporary workers to favor higher budget excesses that are more rewarded than capacity shortages are penalized. It is also more advantageous to accept capacity shortages so as to limit budget deficits that become increasingly expensive and more detrimental to the total cost than capacity shortage costs.

In the present paper, no demand backlogs were allowed and the uncertainty only affects the demand side. A possible extension of this work would then consist in allowing for backlogs and considering the labour supply as uncertain, due to absenteeism or due to the difficulty to hire the desired level of temporary workforce. Our models could also be refined by endowing workers with different skill levels and use demand forecasts with a forecast error distribution.

Appendix

Shortage Cost Function	*Factor*	**Opt. Permanent Capa.**			**Temps**			**Budget Use**			**Shortage/period**			**Shortage Cost (=Total)**		
		$\sigma = 10$	$\sigma = 20$	$\sigma = 30$	$\sigma = 10$	$\sigma = 20$	$\sigma = 30$	$\sigma = 10$	$\sigma = 20$	$\sigma = 30$	σ=10	σ=20	σ=30	σ=10	σ=20	σ=30
Linear	*B*=2500	46.60	42.60	39.00	140.00	294.42	429.94	2498.80	2497.30	2495.01	3.41	6.62	9.57	170.29	330.76	478.58
	B=2750	48.00	44.00	40.00	220.28	386.73	542.88	2710.95	2724.04	2724.49	1.18	4.13	7.05	58.89	206.43	352.30
	B=3000	50.60	45.80	41.20	204.50	435.25	613.61	2841.02	2914.67	2929.73	0.26	2.33	5.01	12.92	116.51	250.46
	B=3250	53.80	47.60	42.80	146.00	455.06	661.41	2929.11	3061.38	3108.15	0.05	1.18	3.40	2.70	58.77	170.23
	B=3500	55.40	51.00	44.80	128.83	405.61	677.29	2978.49	3171.50	3257.78	0.00	0.58	2.22	0.16	29.23	111.23
	$c_M = 1.1$	41.80	29.80	19.20	450.71	1025.43	1523.91	2585.78	2617.97	2636.30	0.32	0.66	1.07	15.88	33.05	53.50
	$c_M = 1.5$	49.20	41.60	34.60	188.34	508.35	783.87	2742.51	2842.53	2905.81	0.81	2.24	4.09	40.66	111.98	204.72
	$c_M = 1.9$	51.40	47.20	42.40	123.10	299.72	450.54	2803.88	2929.46	2976.02	1.07	3.24	6.20	53.49	162.03	309.78
	$c_M = 2.5$	53.80	52.40	51.60	68.18	143.57	166.81	2860.44	2978.92	2997.03	1.23	4.01	7.63	61.60	200.57	381.30
	$c_M = 6.0$	58.20	60.00	60.00	9.30	0.00	0.00	2965.77	3000.00	3000.00	1.47	4.68	8.27	73.33	234.07	413.50
Quadratic	*B*=2500	45.20	40.40	35.80	171.26	337.28	489.77	2491.22	2482.32	2472.63	3.60	6.98	10.08	27.21	91.08	173.25
	B=2750	47.60	43.00	37.80	221.67	394.54	567.82	2699.04	2703.91	2694.92	1.32	4.44	7.55	7.43	49.74	116.65
	B=3000	50.00	45.40	40.20	206.80	428.95	613.42	2832.96	2891.19	2896.62	0.33	2.61	5.44	1.50	25.62	76.80
	B=3250	52.80	46.80	41.60	158.03	453.19	657.78	2912.21	3037.73	3069.74	0.08	1.43	3.89	0.25	12.33	49.63
	B=3500	55.40	50.00	44.00	128.70	405.15	667.90	2977.73	3155.75	3220.27	0.01	0.73	2.63	0.02	5.82	31.53
	$c_M = 1.1$	41.80	29.80	19.60	446.57	1012.47	1481.39	2581.23	2603.72	2609.53	0.40	0.92	1.56	1.33	5.04	11.27
	$c_M = 1.5$	48.40	41.40	33.60	206.31	500.71	784.12	2729.46	2821.07	2856.17	0.90	2.54	4.76	4.68	21.30	51.17
	$c_M = 1.9$	50.80	46.20	40.60	135.33	311.89	471.86	2797.13	2902.58	2926.54	1.17	3.58	6.78	7.33	35.83	88.48
	$c_M = 2.5$	53.00	50.00	46.80	81.19	180.20	249.70	2852.99	2950.50	2964.25	1.35	4.38	8.10	9.69	50.33	125.53
	$c_M = 6.0$	57.00	58.20	58.80	17.06	13.84	9.62	2952.35	2993.03	2997.69	1.52	4.76	8.38	13.37	72.09	171.41

Table A1. Average of indicators for each factor and for each demand variability under restricted budget

Shortage Cost Function	*Factor*	**Opt. Permanent Capa.**			**Temps**			**Budget Use**			**Budget Dev. Cost**			**Shortage/period**			**Budget Deficit**			**Shortage Cost**		
		$\sigma=10$	$\sigma=20$	$\sigma=30$	$\sigma=10$	$\sigma=20$	$\sigma=30$	$\sigma=10$	$\sigma=20$	$\sigma=30$	$\sigma=10$	$\sigma=20$	$\sigma=30$	$\sigma=10$	$\sigma=20$	$\sigma=30$	$\sigma=10$	$\sigma=20$	$\sigma=30$	$\sigma=10$	$\sigma=20$	$\sigma=30$
Linear	B=2500	46.05	41.25	35.80	197.41	394.66	590.08	2562.44	2584.89	2587.53	15.61	8.62	-5.38	2.63	5.49	8.30	108.69	180.18	287.53	131.67	274.53	414.99
	B=2750	46.05	41.50	36.30	230.96	426.70	609.53	2615.16	2652.37	2646.85	-81.11	-82.64	-99.87	2.07	4.74	7.64	24.53	101.12	192.36	103.73	237.18	382.22
	B=3000	46.10	41.60	36.40	238.66	448.32	634.17	2633.53	2696.18	2698.41	-188.07	-181.93	-196.87	1.90	4.31	7.13	1.94	46.88	114.06	94.94	215.46	356.53
	B=3250	46.10	41.75	36.50	239.46	454.73	648.72	2635.46	2719.88	2730.48	-300.00	-287.32	-299.74	1.88	4.08	6.80	0.02	16.80	66.12	94.14	203.98	340.03
	B=3500	46.10	42.05	36.60	239.47	442.30	657.02	2635.48	2702.61	2755.01	-412.49	-386.25	-404.89	1.88	4.22	6.56	0.00	20.53	34.93	94.13	211.08	328.17
	$(c_B^-,c_B^+)=(30,30)$	47.64	44.68	40.84	275.45	529.13	803.35	2792.11	3020.84	3281.92	-62.37	6.25	84.58	0.42	1.26	1.62	76.67	213.47	410.06	20.84	62.83	81.03
	$(c_B^-,c_B^+)=(60,30)$	47.40	43.96	40.20	252.74	472.99	667.36	2731.11	2866.82	2944.98	-75.95	-28.11	5.39	0.95	2.61	4.58	15.73	39.49	72.98	47.72	130.52	229.10
	$(c_B^-,c_B^+)=(60,60)$	44.68	38.92	32.04	198.47	380.61	549.70	2479.20	2418.34	2290.50	-312.48	-349.00	-425.70	3.35	6.92	10.96	15.74	39.44	72.98	167.46	346.17	547.91
	$(c_B^-,c_B^+)=(120,60)$	44.60	38.96	32.20	190.11	350.64	491.21	2463.24	2378.74	2217.22	-322.06	-372.76	-469.67	3.58	7.49	11.99	0.00	0.00	0.00	178.87	374.26	599.50
	$c_M=1.1$	37.15	25.95	16.70	654.56	1217.34	1675.00	2577.51	2636.58	2677.50	-189.48	-162.21	-142.94	0.08	0.17	0.27	13.15	27.06	44.14	4.06	8.66	13.61
	$c_M=1.5$	45.20	39.85	33.20	325.79	642.25	981.83	2748.69	2955.87	3132.75	-110.84	-13.84	71.13	0.21	0.63	1.04	45.92	121.05	229.61	10.28	31.65	52.19
	$c_M=1.9$	47.90	44.50	39.55	101.10	203.07	306.29	2587.09	2610.83	2559.46	-221.37	-244.25	-304.61	3.06	6.61	10.69	25.47	73.28	146.56	152.97	330.26	534.26
	$c_M=2.5$	49.40	47.90	44.40	64.51	104.06	176.35	2631.29	2655.15	2660.89	-208.11	-215.96	-274.18	3.10	7.04	11.04	38.13	136.62	237.21	154.95	351.85	552.04
	$c_M=6.0$	50.75	49.95	47.75	0.00	0.00	0.03	2537.50	2497.50	2387.68	-236.25	-293.25	-356.15	3.93	8.40	13.40	12.50	7.50	37.50	196.36	419.80	669.84
Quadratic	B=2500	45.88	43.40	41.20	225.86	486.38	730.05	2632.02	2932.60	3244.13	2.63	24.48	49.45	2.37	2.89	3.30	152.86	442.61	755.86	14.47	22.85	28.53
	B=2750	46.44	43.84	41.24	235.91	486.85	736.04	2673.82	2952.19	3260.55	-12.21	5.90	29.79	1.93	2.67	3.12	48.67	259.68	554.70	10.82	20.48	26.57
	B=3000	46.56	43.96	41.28	240.09	497.91	744.41	2692.84	2979.51	3274.22	-27.38	-10.16	10.91	1.79	2.42	2.97	10.41	143.82	382.24	9.88	17.48	25.03
	B=3250	46.60	44.28	41.48	240.93	501.22	745.96	2698.29	3004.86	3289.70	-42.77	-25.40	-6.86	1.77	2.22	2.82	1.29	74.20	254.44	9.69	15.39	23.55
	B=3500	46.60	44.24	41.32	241.05	508.06	758.59	2698.88	3021.95	3310.19	-58.26	-40.53	-22.82	1.77	2.12	2.66	0.05	36.82	168.91	9.68	14.24	21.49
	$(c_B^-,c_B^+)=(1,1)$	48.40	46.24	44.08	275.90	561.88	825.85	2864.18	3259.12	3641.95	-1.36	2.59	6.42	0.23	0.29	0.38	113.80	395.31	743.08	0.23	0.26	0.48
	$(c_B^-,c_B^+)=(4,2)$	47.60	45.52	42.96	261.70	530.85	793.95	2791.87	3140.82	3492.85	-2.83	8.62	21.91	0.81	1.09	1.34	66.70	290.37	602.78	1.73	3.03	4.10
	$(c_B^-,c_B^+)=(8,4)$	46.92	44.36	41.68	244.00	504.80	750.10	2713.66	3006.13	3305.98	-10.32	7.33	29.43	1.45	2.05	2.59	28.31	177.05	429.74	5.21	10.31	14.88
	$(c_B^-,c_B^+)=(16,8)$	45.56	42.64	39.40	223.10	462.97	688.63	2599.09	2824.14	3019.23	-31.80	-10.00	15.72	2.58	3.63	4.72	3.38	50.84	177.22	13.33	28.22	46.29
	$(c_B^-,c_B^+)=(16,16)$	43.60	40.96	38.40	179.15	419.92	656.52	2427.04	2660.90	2918.78	-91.67	-54.26	-12.99	4.56	5.26	5.85	1.08	43.55	163.34	34.03	48.61	59.43
	$c_M=1.1$	36.64	25.60	16.52	601.74	1157.08	1608.12	2493.91	2552.79	2594.93	-35.55	-31.76	-28.80	1.61	1.70	1.77	6.51	18.33	34.37	5.01	5.41	5.70
	$c_M=1.5$	44.12	38.68	32.68	283.10	606.91	929.02	2630.66	2844.36	3027.54	-28.51	-14.22	-0.82	1.82	2.13	2.46	25.23	92.52	190.53	7.60	9.31	11.07
	$c_M=1.9$	47.24	44.68	41.08	176.75	406.20	649.47	2697.83	3005.77	3288.00	-25.41	-4.69	16.76	1.94	2.44	2.93	39.13	164.90	362.08	9.70	13.34	16.92
	$c_M=2.5$	49.84	49.92	49.04	104.01	260.35	432.63	2752.02	3146.88	3533.58	-23.71	2.48	32.16	2.04	2.74	3.40	54.90	251.47	568.10	12.45	19.40	25.93
	$c_M=6.0$	54.24	60.84	67.20	18.24	49.89	95.79	2821.42	3341.32	3934.74	-24.80	2.47	41.19	2.21	3.31	4.31	87.50	429.91	961.07	19.78	42.99	65.56

Table A2. Average of indicators for each factor and for each demand variability under unrestricted budget

References

Bard, J.F., Morton, D.P., Min Wang, Y., 2007. Workforce planning at USPS mail processing and distribution centers using stochastic optimization. *Annals of Operations Research*, 155, 51-78.

Berman, O., Larson, R.C., 1993. Optimal workforce configuration incorporating absenteeism and daily workload variability. *Socio-Economic Planning Science*, 27, 91-96.

Berman, O., Larson, R.C., 1994. Determining optimal pool size of a temporary call-in work force. *European Journal of Operational Research*, 73, 55-64.

Bhatnagar, R., Saddikutti, V., Rajgopalan, A., 2007. Contingent manpower planning in a high clock speed industry. *International Journal of Production Research*, 45(9), 2051-2072.

Kao, E.P.C., Queyranne, M., 1985. Budgeting Costs of Nursing in a Hospital. *Management Science*. 31(5), 608-621.

Pinker, E.J., Larson, R.C., 2003. Optimizing the use of contingent labor when demand is uncertain. *European Journal of Operational Research*, 144, 39-55.

Techawiboonwong, A., Yenradee, P., Das, S.K., 2006. A master scheduling model with skilled and unskilled temporary workers. *International Journal of Production Economics*, 103, 798-809.

Trivedi, V.M., 1981. A Mixed-Integer Goal Programming Model for Nursing Service Budgeting. *Operations Research*, 29(5), 1019-1034.

Warner, D.M., and Prawda, J. 1972. A mathematical programming model for scheduling nursing personnel in a hospital. Management Science, 19(4), 411-422.

Wild, B., Schneeweiss, C., 1993. Manpower capacity planning – A hierarchical approach. *International Journal of Production Economics*, 30-31, 95-106.

Zimmerman, J.L. 1976. Budget Uncertainty and the Allocation Decision in a Nonprofit Organization. Journal of Accounting Research, 14(2), 301-319.