Improving the Delta-hedging risk-adjusted performance: the standard VG volatility space model

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Abstract

This paper provides a comparison of the Delta-hedging strategy under the Black-Scholes model and under a particular VG space volatility model, the so-called standard VG space model. This model is obtained by replacing the standard Normal distribution by the symmetric VG distribution with a parameter ν equal to 1. In particular, this paper focuses on the performance of the P&L of liquid vanilla options written on two major indices quoted on the US market: the Dow Jones and the S&P500. In a first time we look at the optimal historical VG space model by considering one of the most straightforward simple risk measure: the P&L variance. We then compare the P&L variance evolution through time under the Black-Scholes model and the standard VG space model for options traded on a monthly basis from the 4th of January 1999 on. Finally, we compare different performance measures and acceptability indices for the P&L of liquid in-the-money vanilla options, i.e. for writing the option, hedging the position on a daily basis and paying out the option payoff at maturity, focusing therefore on the typical hedging strategy adopted by financial institutions.

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1 Introduction

Although several advanced asset return models have been developed these last two decades, including jumps and stochastic volatility characteristics, the Black-Scholes model has remained the standard quoting tool for many banks and financial institutions. This is partly due to the simple and widespread used concept of the Black-Scholes implied volatility. However, this model parameter needs to be adjusted separately for each individual contract given the inadequacy of the underlying Black-Scholes model. Moreover, historical log-returns exhibit some skewness and excess of kurtosis and have fatter tails than those the Normal distribution can provide. Hence, Corcuera et al. have developed a similar concept but under a Lévy framework and therefore based on distributions that match more closely historical returns (see [7]).

The implied Lévy volatility models are obtained by replacing the Gaussian distribution modeling the diffusion part of the log-return process of the Black-Scholes model by a more flexible distribution (characterised by skewness, excess kurtosis, fatter tails, ...). By considering the class of Lévy distributions, the stochastic part of the stock price process is ensured to have the same properties as the Brownian motion (starting at zero, independent and stationary increments) but with increments of a log-return over a unit interval of time distributed according to a more suited mother distribution. Corcuera et al. have proposed two models, the Lévy implied space and time volatility models; the first arising when the Lévy distribution is multiplied by the volatility and the second one when the time argument of the Lévy distribution is multiplied by the square of the volatility. By switching from the Black-Scholes world to the Lévy world, additional degrees of freedom (i.e. parameters that can be set freely) are introduced which can be used in order to minimize either the skew adjustment that we need to consider in the model to replicate the option market prices or the absolute value of the mean and the square root of the variance of the daily hedging error (see [7] for more details).

In this paper, we will focus on a particular sub-class of Lévy implied volatility models, the Variance Gamma (VG) space volatility models, and more particularly on the standard VG space model obtained by setting the parameters θ and ν equal to zero and one, respectively. In particular, we investigate the Delta-hedging performance of a portfolio of liquid vanilla options with different strike prices and times to maturity written on the Dow Jones and the S&P500 index and traded from the 4th of January 1999 on.

In the financial literature, two major classes of risk measures $\rho(X)$ have been proposed to assess the risk of a financial position X: the coherent risk measures introduced by Artzner et al. (see [1]) and the convex risk measures proposed by Föllmer and Schied (see [9] and [10]), the second class being an extension of the first one. Cherny and Madan (see [6]) have proposed a new class of performance measures, called accessibility indices and based on coherent risk measures. Under the accessibility index framework, a risk measure associated to a zero cost cash-flow X, denoted by $\rho_\gamma(X)$, is defined as minus the expectation of X under a risk-adjusted distribution function obtained by applying a concave distortion function Ψ_γ to the original cdf. More particularly, a sequence of concave distortions increasing pointwise in γ is applied to the distribution and the accessibility index is then defined as the highest stress level γ such that the risk measure ρ_γ remains negative, or equivalently, such that the stressed cash-flow remains acceptable. Hence, the risk of a financial position can be assessed by a single parameter for each family of distortion functions, which can turn out to be a significant advantage in practice.

In this paper, we focus on the accessibility indices to assess the risk-adjusted performance of the Delta-hedging strategy and therefore to determine the best performing model and we compare the results with more traditional performance measures (Sharpe ratio and GLR ratio). It is shown that making use of the standard VG model leads to a significant improvement of the variance of the Profit and Loss (P&L) and to a more profitable Delta-hedging strategy for a wide range of in-the-money vanilla options.

The outline of this paper is as follows. Section 2 recalls the implied Lévy volatility space model and details the particular case of the standard VG space model. Section 3 details the computation of the Profit and Loss of the Delta-hedging strategy and features the evolution of the variance of the P&L through time. In section 4, we recall several traditional performance measures as well as the class of accessibility indices proposed by Cherny and Madan in [6] and in Section 5, we investigate in details the performance results of the P&L of a Delta-hedging strategy under the Black-Scholes and standard VG space models for different option sets. Section 6 formulates our conclusions.

2 The Lévy space volatility model

This section briefly recalls the concept of Lévy processes as well as the Lévy space implied volatility model proposed by Corcuera et al. and details the particular case of the VG process.

2.1 Lévy processes

Suppose $\phi(x)$ is the characteristic function of the mother distribution (the log-return distribution over an interval of unit length). If for every positive integer n, $\phi(x)$ is also the nth power of a characteristic function, we say that the distribution is infinitely divisible. One can define for every such an infinitely divisible distribution a stochastic process, $X = \{X_t, t \geq 0\}$, called Lévy process, which starts at zero, has independent and stationary increments and such that the distribution of an increment over $[s, s+t]$, $s, t \geq 0$, i.e. $X_{t+s} - X_s$, has $(\phi(x))^t$ as characteristic function (see [12] for a complete overview of Lévy processes).

2.2 The Lévy space implied volatility

Under the Lévy space implied volatility model, the stochastic part of the log-return process is modeled by a Lévy process $X = \{X_t, t \geq 0\}$:

$$S_t = S_0 \exp((r - q + \omega)t + \sigma X_t), t \geq 0,$$

where the parameter ω is the mean correcting term and can be expressed in function of the characteristic function of the mother distribution X_1, $\phi(u) = \mathbb{E}[\exp(\mathrm{i}uX_1)]$, by

$$\omega = -\log(\phi(-\sigma \mathrm{i})).$$

In order to match the first two moments of the diffusion component of the Black-Scholes model, the mean and the variance of X_1 is set equal to zero and one, respectively.

In analogy to the implied Black-Scholes volatility concept, the volatility parameter σ needed to match the model price with a given market price is called the *implied Lévy space volatility* of the option.

2.3 The Variance Gamma process

The Variance Gamma process is a Lévy process built on the Variance Gamma distribution (see for instance [12], [11] or [4]). The characteristic function of the Variance Gamma distribution $\mathbf{VG}(\delta,\nu,\theta,\mu)$ with parameters $\delta > 0$, $\nu > 0$, $\theta \in \mathbb{R}$ and $\mu \in \mathbb{R}$ is given by:

$$\phi_{\mathtt{VG}}(u;\delta,\nu,\theta,\mu) = \exp\Big(\mathrm{i}u\mu\Big)\left(1 - \mathrm{i}u\theta\nu + \frac{u^2\delta^2\nu}{2}\right)^{\frac{-1}{\nu}}, \quad u \in \mathbb{R}.$$

The Variance Gamma process $X = \{X_t, t \geq 0\}$ is a Lévy process such that X_t follows a $\mathbf{VG}(\sqrt{t}\delta, \frac{\nu}{t}, \theta t, \mu t)$ distribution. The VG distribution satisfies the following scaling property: if $X \sim \mathbf{VG}(\delta,\nu,\theta,\mu)$ then $cX \sim \mathbf{VG}(c\delta,\nu,c\theta,c\mu)$.

The first four moments of the VG distribution are given in Table 1 for the general and symmetric case.

	$\mathbf{VG}(\delta,\nu,\theta,\mu)$	$\mathbf{VG}(\delta,\nu,0,\mu)$
mean	$\theta+\mu$	μ
variance	$\delta^2+\nu\theta^2$	δ^2
skewness	$\frac{\theta\nu\left(3\delta^2+2\nu\theta^2\right)}{\left(\delta^2+\nu\theta^2\right)^{\frac{3}{2}}}$	0
kurtosis	$3\left(1+2\nu-\frac{\nu\delta^4}{(\delta^2+\nu\theta^2)^2}\right)$	$3(1+\nu)$

Table 1: Characteristics of the Variance Gamma distribution: general case (left) and symmetric case (right)

A parameter θ equal to zero indicates a symmetric distribution around μ whereas negative and positive values of θ lead to negative and positive skewness, respectively. The parameter ν primarily controls the kurtosis (see Table 1).

The standard VG model is obtained by setting the parameters θ and ν equal to zero and one, respectively. The corresponding VG distribution, $\mathbf{VG}(1,1,0,0)$, is a double exponential distribution and has a zero skewness and an excess of kurtosis equal to three.

3 Improving the Delta-hedge: P&L of Vanilla options

In order to evaluate the model performance, we first look at the Profit and Loss of the Delta-hedging strategy. At time t_0 we sell the option and buy stocks for an amount equal to $\Delta_{t_0}S_{t_0}$. At time t_i we spend $\left(\Delta_{t_i} - \Delta_{t_{i-1}}\right)S_{t_i}$ in stocks in order to rebalance our position. At the option maturity, we close both the stock and the option position.

We define the hedge error at time t_i as the value of the hedging portfolio just before the rebalancing occurring at time t_i:

$$\mathrm{HE}(t_i) = -C_{t_i}(K, T - t_i) + \Delta_{t_{i-1}}S_{t_i} - \left(\Delta_{t_{i-1}}S_{t_{i-1}} - C_{t_{i-1}}(K, T - t_{i-1})\right)(1 + r_{t_i}\Delta t_{t_i}).$$

The balance at time t_i is defined as the amount spent until time t_i to build the hedging portfolio:

$$\mathrm{Balance}(t_0) = \Delta_{t_0}S_{t_0} - C_{t_0}(K,T)$$

and

$$\text{Balance}(t_i) = \text{Balance}(t_{i-1})\,(1 + \Delta t_{t_i} r_{t_i}) + \text{Rebalance CF}(t_i), \quad 0 < t_i \leq T$$

where

$$\text{Rebalance CF}(t_i) = \left(\Delta_{t_i} - \Delta_{t_{i-1}}\right) S_{t_i}.$$

The Mark to market is defined as the amount spent until time t_i to build the hedging portfolio after the closing of the option and stock position:

$$\text{MtM}(t_i) = \text{Balance}(t_i) + C_{t_i}(K, T - t_i) - \Delta_{t_i} S_{t_i}.$$

The profit and loss of the Delta-hedging strategy is by definition equal to the Mark to market at the option maturity T:

$$\text{P\&L} = -\text{MtM}(T).$$

where the minus sign is introduced such that a positive value corresponds to a profit and a negative one to a loss for a short option position, which is the typical position taken by financial institutions.

3.1 The historical optimal models

For the numerical study, we first consider a portfolio composed of 491 liquid Vanilla options written on the Dow Jones with different strikes and times to maturity traded on the third of January 2000 on. We delta hedge daily each of the options composing the portfolio on their entire life. We consider two cases: we either buy each option once (unweighted portfolio) or we buy each option for a fixed amount equal to one dollar (weighted portfolio).

Figure 1 shows the variance of the weighted and unweighted portfolios for different symmetric VG space volatility models. Considering VG space models allows to significantly reduce the variance of the profit and loss. In particular, the optimal historical parameter ν, defined as the parameter which leads to the smallest P&L variance, amounts to 1.35 and 1.6 for the unweighted and weighted portfolios, respectively and allows to reduce the variance by 10.71 and 27.47 percents, respectively.

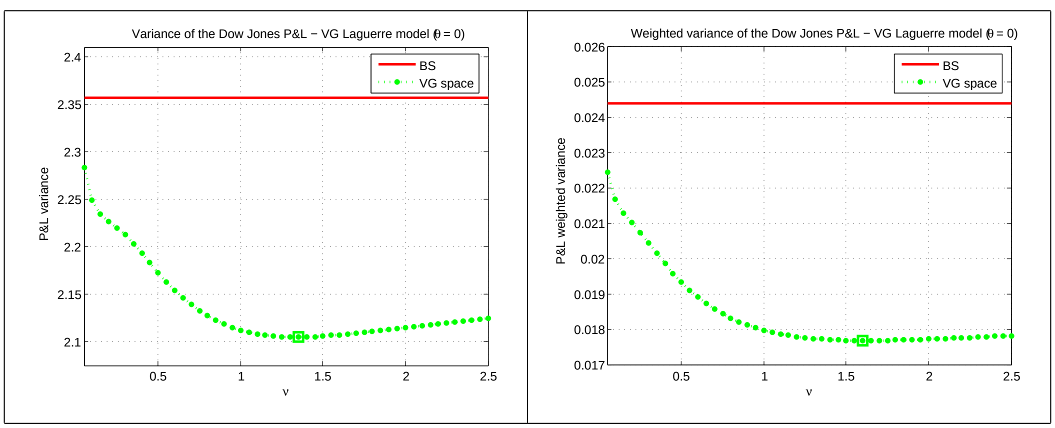

Figure 1: Variance of the unweighted (left) and weighted (right) P&L for the symmetric VG space volatility model (Dow Jones)

We perform a similar study for a portfolio composed of 1177 liquid Put and Call options written on the S&P500 index and characterised by different maturities and strikes. Figure 2 shows the variance of the weighted and unweighted portfolios for different symmetric VG space volatility models. The historical optimal symmetric VG space model is obtained by considering a parameter ν equal to 1.95 and higher than 5 for the weighted and unweighted portfolios, respectively and leads to a variance reduction amounting to 11.51 and to approximately 6.43 percents, respectively.

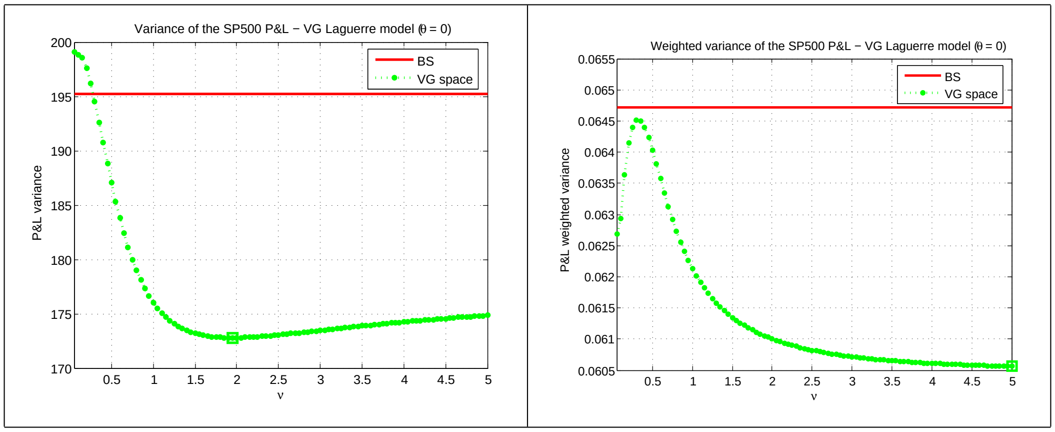

Figure 2: Variance of the unweighted (left) and weighted (right) P&L for the symmetric VG space volatility model (S&P500)

As it can be seen from Table 2, for both the Dow Jones and the S&P500 indices and for both the weighted and the unweighted portfolios, opting for the symmetric VG space model with a parameter ν equal to one already allows to significantly reduce the P&L variance. In particular the variance reduction of the standard model is pretty close to the variance reduction of the historical optimal model for the Dow Jones options sets and the unweighted S&P500 options set. Hence, in the following, we will focus on this particular model, although it does not turn out to be the optimal historical model for the particular options sets we considered. The choice of the standard VG space model as benchmark can be justified by the simplicity of the mother distribution ($\mathbf{VG}(1, 1, 0, 0)$) which is nothing else than a double exponential distribution.

index	Dow Jones		S&P500	
portfolio	unweighted	weighted	unweighted	weighted
historical optimal model	10.71 % ($\nu = 1.35$)	27.47 % ($\nu = 1.6$)	11.51 % ($\nu = 1.95$)	$\approx$ 6.43 % ($\nu > 5$)
standard model	10.40 % ($\nu = 1$)	26.28 % ($\nu = 1$)	9.85 % ($\nu = 1$)	4.00 % ($\nu = 1$)

Table 2: P&L relative variance reduction for the historical optimal models and the standard model

3.2 The standard VG model

Figure 3 and Figure 4 show the evolution of the variance of the weighted and unweighted portfolios through time for options written on the Dow Jones and S&P500 index, respectively. At each of the quoting date, we take a short position in all the liquid vanilla options traded on the market and we delta-hedge them daily until maturity.

For each option set (i.e. for both the Dow Jones and the S&P500 indices and for both the weighted and unweighted portfolios), the variance of the P&L under the standard VG model moves in line with the variance of the P&L under the Black-Scholes model. Moreover, the P&L variance of the unweighted and the weighted portfolios is almost always smaller under the standard VG model than under the Black-Scholes model (see Table 3).

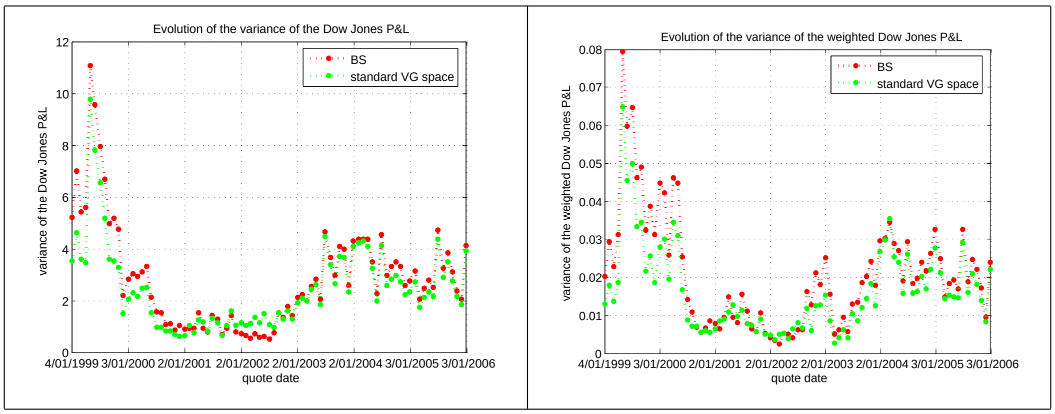

Figure 3: Evolution of the variance of the unweighted (left) and weighted (right) portfolios through time (Dow Jones)

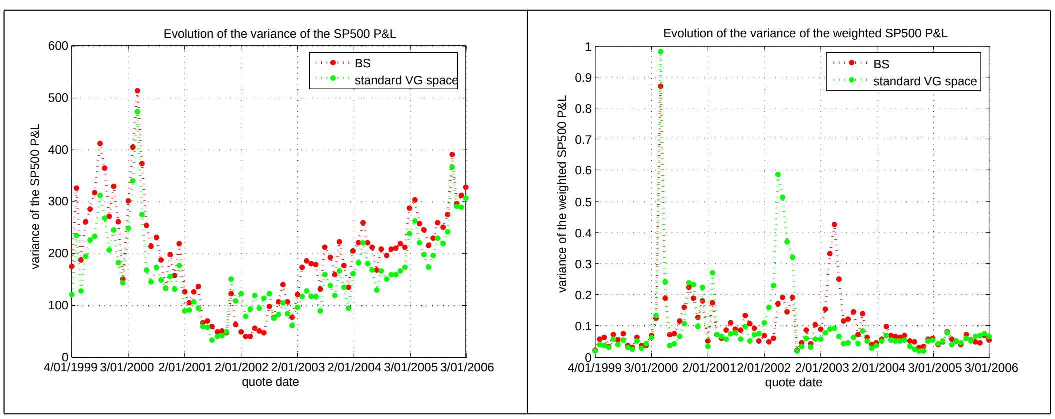

Figure 4: Evolution of the variance of the unweighted (left) and weighted (right) portfolios through time (S&P500)

index	Dow Jones		S&P500	
portfolio	unweighted	weighted	unweighted	weighted
variance	82.353 %	85.882 %	88.235 %	70.588 %

Table 3: Percentage of the quoting dates for which the variance of the P&L is lower under the standard VG model than under the Black-Scholes model.

In order to estimate adequately the model performance, we should take into account both the mean and the variance of the P&L, or even better the whole historical distribution. The next section defines several performance measures which can be used as benchmarks to assess the attractiveness of the Delta-hedging strategy.

4 Performance measures

This section recalls several performance measures and acceptability indices previously introduced in the literature (see [2], [6] and [13]) and allowing to measure the attractiveness of an investment strategy.

4.1 The Sharpe ratio

The Sharpe ratio, introduced by W.F. Sharpe in 1966 is defined as the ratio of the mean to the standard deviation of the differential return of an investment portfolio and hence takes into account both the return of the investment strategy and its risk (see [13]):

$$\mathrm{SR}(X) = \frac{\mathbb{E}(X)}{\mathrm{std}(X)} \tag{1}$$

where X stands for any return or cash-flow at zero cost. Note that Cherny and Madan exclude the negative value of the Sharpe ratio in order to satisfy the expectation consistency axiom of their acceptability indices (see [6]):

$$\begin{cases} \mathrm{SR}(X) = \frac{\mathbb{E}(X)}{\mathrm{std}(X)} & \text{if } \mathbb{E}(X) \geq 0 \\ \mathrm{SR}(X) = 0 & \texttt{if } \mathbb{E}(X) < 0 \end{cases}$$

By considering the random variable X equal to the P&L of a vanilla option, the Sharpe ratio as defined in equation (1) can be interpreted as the risk-adjusted profit of the Delta-hedging strategy.

4.2 The Gain Loss ratio

The Gain Loss ratio was proposed by Bernardo and Ledoit in [2] to circumvent some weaknesses of the commonly used Sharpe ratio, including the non-replication of arbitrage opportunities. It was originally defined as the ratio of the expectation of the investment profits to the expectation of the absolute value of the investment losses:

$$\mathrm{GLR(X)} = \frac{\mathbb{E}(\max(0, X))}{\mathbb{E}(\max(0, -X))};$$

a GLR higher than one indicating an attractive investment strategy. Cherny and Madan considered a modified version of the Gain Loss ratio (see [6]):

$$\begin{cases} \mathrm{GLR}(X) = \frac{\mathbb{E}(X)}{\mathbb{E}(\max(0,-X))} & \text{if } \mathbb{E}(X) \geq 0 \\ \mathrm{GLR}(X) = \quad 0 & \texttt{if } \mathbb{E}(X) < 0 \end{cases}$$

which is the ratio of the expected cash-flow to the absolute value of the expected loss. Note that the modified GLR, contrary to the original one, satisfies the expectation consistency axiom of the acceptability indices defined in [6] and recalled below (see Definition 3).

4.3 Risk measures

Under the risk measure framework, the risk of any financial position X is quantified by a risk measure $\rho(X)$, which is the minimal amount of money that should be added to the position to obtain an acceptable position (see for instance Artzner et al. [1], Föllmer and Shied [10] or Cherny and Madan [6]):

$$\rho(X) = \inf \{m \in \mathbb{R} : X + m \in \mathcal{A}\}$$

where $\mathcal{A}$ denotes the set of acceptable positions, referred as *acceptability* or *acceptance set* and defined as the set of trades which do not require additional capital:

$$\mathcal{A} := \{X \in \mathcal{L}^\infty : \rho(X) \leq 0\},$$

where $\mathcal{L}^\infty$ stands for the set of bounded random variables.

4.3.1 Convex and coherent risk measures

The convex risk measures were defined by Föllmer and Schied in [9] as:

Definition 1 *A mapping $\rho : \mathcal{X} \to \mathbb{R}$ where $\mathcal{X}$ is a set of real-valued random variables, is a convex risk measure if it satisfies the following axioms $\forall X, Y \in \mathcal{X}$:*

1. ***monotonicity**: if $X \geq Y$ a.s. then $\rho(X) \leq \rho(Y)$*
2. ***translation invariance**: $\rho(X + m) = \rho(X) - m$, $\forall m \in \mathbb{R}$*
3. ***convexity**: $\rho(\lambda X + (1 - \lambda)Y) \leq \lambda\rho(X) + (1 - \lambda)\rho(Y)\, \forall\lambda \in [0, 1]$*

Moreover, any convex risk measure which is subadditive and positive homogeneous is called coherent risk measure (see [1]):

Definition 2 *A mapping $\rho : \mathcal{X} \to \mathbb{R}$ where $\mathcal{X}$ is a set of real-valued random variables, is a coherent risk measure if it satisfies the following axioms $\forall X, Y \in \mathcal{X}$:*

1. ***monotonicity**: if $X \geq Y$ a.s. then $\rho(X) \leq \rho(Y)$*
2. ***translation invariance**: $\rho(X + m) = \rho(X) - m$, $\forall m \in \mathbb{R}$*
3. ***sub-additivity**: $\rho(X + Y) \leq \rho(X) + \rho(Y)$*
4. ***positive homogeneity**: $\rho(\lambda X) = \lambda\rho(X)$, $\forall\lambda > 0$*

According to the representation theorem, a risk measure relative to a random variable X defined on the probability space $(\Omega, \mathcal{F}, P)$ is coherent if and only if

$$\rho(X) = -\inf_{\mathcal{Q}\in\mathcal{M}} \mathbb{E}^{\mathcal{Q}}(X) \tag{2}$$

where $\mathcal{M}$ is a set of probability measures on $(\Omega, \mathcal{F})$. Hence, any coherent risk measure is computed as the worst case expectation over a set of test measures $\mathcal{Q} \in \mathcal{M}$.

One of the most widespread used coherent risk measure is the Tail Value at Risk, also called Conditional Tail Expectation, Expected Shortfall or Expected Tail Loss and defined as the average of the λ 100 % worst losses:

$$\text{TV@R}_\lambda(X) = -\mathbb{E}(X|X \leq q_\lambda(X))$$

where q_λ denotes the λ-quantile. Since TV@R_λ is a family of risk measures decreasing in λ, a TV@R acceptability index can be constructed as follow:

$$\text{AIT}(X) = \frac{1}{\inf\{\lambda \in (0,1] : \mathbb{E}(X|X \leq q_\lambda(X)) \geq 0\}} - 1.$$

4.4 Acceptability indices

Cherny and Madan have defined a particular sub-class of performance measures, called accessibility indices, denoted by α and satisfying the eight following axioms (see [6] or [8]):

Definition 3 *A mapping $\alpha : \mathcal{L}^\infty \to [0,\infty]$ where $\mathcal{L}^\infty$ is a set of bounded real-valued random variables, is an accessibility index (also called gamma) if it satisfies the following axioms $\forall X, Y \in \mathcal{L}^\infty$:*

1. ***monotonicity**: if $X \leq Y$ a.s. then $\alpha(X) \leq \alpha(Y)$;*
2. ***scale invariance**: $\alpha(\lambda X) = \alpha(X)\ \forall\ \lambda > 0$;*
3. ***Fatou property**: If (X_n) is a sequence of random variables such that $|X_n| \leq 1$, $\alpha(X_n) \geq x$ and X_n converges to X in probability (i.e. $\lim_{n\to\infty} \mathbb{P}(|X_n - X| > \epsilon) = 0, \quad \forall \epsilon > 0$) then $\alpha(X) \geq x$;*
4. ***Quasi-concavity**: if $\alpha(X) \geq x$ and $\alpha(Y) \geq x$ then $\alpha(\lambda X + (1-\lambda)Y) \geq x\ \forall\, \lambda \in [0,1]$;*
5. ***law invariance**: if $X =^d Y$ then $\alpha(X) = \alpha(Y)$ where $=^d$ stands for equal in distribution ;*
6. ***consistence with second order stochastic dominance**: if $\mathbb{E}(U(X)) \leq \mathbb{E}(U(Y))$ then $\alpha(X) \leq \alpha(Y)$ where U is any increasing concave function ;*
7. ***arbitrage consistence**: $X \geq 0$ a.s. if and only if $\alpha(X) = \infty$;*
8. ***expectation consistency**: $\begin{cases} \text{if } \ \mathbb{E}(X) < 0 \text{ then } \alpha(X) = 0 \\ \text{if } \ \mathbb{E}(X) > 0 \text{ then } \alpha(X) > 0. \end{cases}$*

The accessibility indices are constructed by replacing the cumulative distribution function of X, $F_X(x) = \mathbb{P}(X \leq x)$, by a risk adjusted distribution, $G_X(x) = \Psi_\gamma(F_X(x))$. The corresponding risk measure[1] is then given by minus the expectation of the zero cost cash-flow under the distorted distribution function:

$$\rho_\gamma(X) = -\int_{\mathbb{R}} yd(\Psi_\gamma(F_X(y))), \quad \gamma \in \mathbb{R}_+$$

where Ψ_γ is a family of concave distortion functions on $[0,1]$ increasing pointwise in its stress level parameter γ. Hence, considering a higher value of γ leads to a more severe distortion of the distribution function of X. The acceptability index is defined as the largest stress level γ such that the expectation of X under the distortion Ψ_γ remains positive:

$$\alpha(X) = \sup\{\gamma \in \mathbb{R}_+ : \rho_\gamma(X) \leq 0\}$$

or equivalently such that the distorted cash-flow remains acceptable. Note that $\alpha(X)$ satisfies the arbitrage consistency axiom of Definition 3 if and only if $\Psi_\gamma(y)$ tends toward unity as γ tends toward infinity and the expectation consistency axiom of Definition 3 if and only if $\Psi_\gamma(y)$ tends pointwise to y as γ tends to zero.

The acceptability indices are more relevant to valuate the performance of a trading strategy for non Gaussian returns (or zero cost cash-flows) since they take into account the whole distribution function of X and not only its first two moments as the Sharpe ratio.

Cherny and Madan have introduced four acceptability indices: AIMIN, AIMAX, AIMAXMIN and AIMINMAX based on four different families of distortion functions Ψ_γ.

- The **MINVAR acceptability index** rests on the following family of distortion functions:

$$\Psi_\gamma(y) = 1 - (1-y)^{\gamma+1}, \quad \gamma \in \mathbb{R}_+,\ y \in [0,1]$$

  and the corresponding risk measure is equal to minus the expectation of the minimum of $\gamma+1$ realisations of the cash-flow.

- The **MAXVAR acceptability index** is obtained by considering the following family of distortion functions:

$$\Psi_\gamma(y) = y^{\frac{1}{\gamma+1}}, \quad \gamma \in \mathbb{R}_+,\ y \in [0,1]$$

  and was first introduced by Wang in [14] under the name proportional hazards transform in the insurance field. The non-distorted cdf $F_X(x)$ is obtained from the distorted cdf $G_X(x) = \Psi_\gamma(F_X(x))$ by generating $1+\gamma$ independent draws and by taking the maximum of the distorted cash-flow outcomes.

  The MAXMINVAR and MINMAXVAR acceptability indices are obtained by combining the distortion functions MINVAR and MAXVAR.

[1] Any accessibility index is closely linked to coherent risk measures since

$$\rho_\gamma(X) = -\inf_{\mathcal{Q}\in\mathcal{M}_\gamma} \mathbb{E}^{\mathcal{Q}}(X), \quad \gamma \in \mathbb{R}_+$$

is a coherent risk measure since it satisfies the representation theorem (2).

- The **MAXMINVAR acceptability index** is built from the following family of distortion functions:
$$\Psi_\gamma(y) = \left(1-(1-y)^{\gamma+1}\right)^{\frac{1}{\gamma+1}}, \quad \gamma \in \mathbb{R}_+,\ y \in [0,1]$$
and is obtained by first using a MINVAR procedure and then a MAXVAR procedure.

- The **MINMAXVAR acceptability index** rests on the following family of distortion functions:
$$\Psi_\gamma(y) = 1-\left(1-y^{\frac{1}{\gamma+1}}\right)^{\gamma+1}, \quad \gamma \in \mathbb{R}_+,\ y \in [0,1]$$
and results of a MAXVAR procedure followed by a MINVAR procedure.

Figure 5 shows the original and distorted (or risk-adjusted) distribution functions for several stress levels γ, starting from a standard Gaussian random variable. As it can be seen from Figure 5, the acceptability indices are built on a distortion function which associates more weight on the down side (i.e. to losses) than on the up side (i.e. to profits) than the original distribution function. In particular, the MINVAR distortion procedure results into a distribution characterised by higher negative peakness and lighter upper tail whereas the MAXVAR distortion function leads to a distribution characterised by heavier lower tail and lighter upper tail. Moreover, the higher the stress level γ, the more severe the distortion of the distribution function. Note that the MAXMINVAR and MINMAXVAR distribution exhibit a similar trend which is quite intuitive since they rest on the combination of the same two distortion functions.

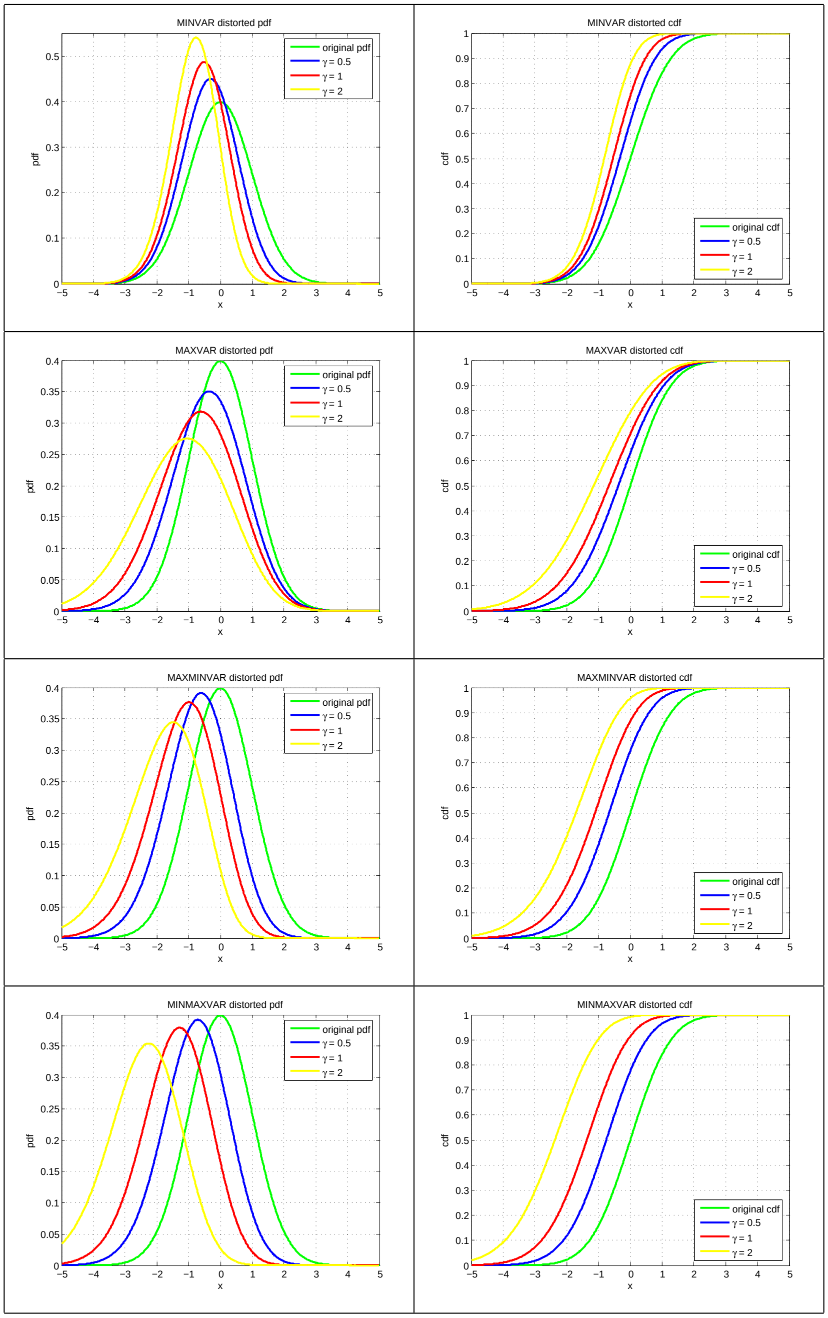

Figure 5: Distorted probability distribution function (left) and cumulative distribution function (right) for the MINVAR, MAXVAR, MAXMINVAR and MINMAXVAR distortion functions for a standard normal random variable X for several values of the stress level parameter γ.

5 Performance measures for the Delta-hedging strategy

This section features a study of the Black-Scholes and standard VG model performance for the zero cost P&L of vanilla options, i.e. for writing the option, hedging the position on a daily basis and paying out the option payoff at maturity. In particular, we compute the acceptability indices detailed in Section 4 for vanilla options written on two major indices: the Dow Jones and the S&P500. The more traditional performance measures (Sharpe ratio and GLR ratio) are included in the appendix A. We consider a period extending from January 1999 until January 2006 using option prices quoted monthly and sort out the options with respect to six maturity buckets and to eight strike buckets.

The option set we consider consists of in-the-money (ITM) vanilla options, i.e. of call options with a moneyness $\frac{K}{S_0}$ lower than one and of put options with a moneyness greater than one. Moreover, we restrict the set to liquid options, i.e. to options with a bid price strictly positive. Each option is delta-hedged daily until maturity. The number of options for each strike-maturity bucket is given in Table 4 and in Table 5 for the Dow Jones and S&P500 index, respectively.

	maturity range					
moneyness range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-5	> 1.5
< 0.85	296	268	322	307	292	404
0.85-0.9	406	148	131	126	104	126
0.9-0.95	749	173	137	131	98	129
0.95-1	1034	189	142	132	84	120
1-1.05	968	179	134	114	87	115
1.05-1.1	440	111	104	103	94	109
1.1-1.15	160	56	49	49	37	60
> 1.15	179	61	60	72	81	119

Table 4: Number of ITM options for each strike-maturity bucket (Dow Jones)

	maturity range					
moneyness range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-5	> 1.5
< 0.85	760	485	485	473	518	361
0.85-0.9	669	217	191	183	150	116
0.9-0.95	1049	228	211	197	164	137
0.95-1	1492	229	210	192	168	152
1-1.05	1500	229	208	182	136	120
1.05-1.1	499	168	165	135	125	97
1.1-1.15	181	73	86	86	70	59
> 1.15	143	105	119	126	156	98

Table 5: Number of ITM options for each strike-maturity bucket (S&P500)

For the Sharpe ratio and the Gain Loss ratio, we adopt the original convention, i.e. we do not set any negative Sharpe ratio and Gain Loss ratio equal to zero as in [6]. The accessibility indices are obtained by replacing the empirical P&L distribution by a risk-adjusted distribution. More precisely, for each strike-maturity bucket, we sort out the historic P&L by increasing order: $X_1^* = \min\left(\{X_i, i = 1, \ldots, n\}\right)$, $X_2^* = \min\left(\{X_i, i = 1, \ldots, n\}\backslash\{X_1^*\}\right)$, etc., where n is the number of options composing the bucket. The empirical cdf is then given by $F_X\left(X_i^*\right) = \frac{i}{n}, i = 1, \ldots, n$ and the risk-adjusted cdf is directly obtained by $G_X\left(X_i^*\right) = \Psi_\gamma\left(F_X\left(X_i^*\right)\right), i = 1, \ldots, n$.

The performance measures allow us to determine the best performing model, i.e. the model which leads to the most profitable Delta-hedging strategy. Indeed, since the aim is to compare the risk-adjusted profit of the same financial position under different models, and not to determine the most profitable strategy among several, an higher value of the gamma level is consistent with a better performing model. The acceptability indices relative to the different strike-maturity buckets are given in Table 6 to 9 and in Table 10 to 13 for the Dow Jones and the S&P500 options, respectively. For each strike-maturity combination, the best model (standard VG versus Black-Scholes) is mentioned in green type for each performance measure.

The highest levels of gamma are those obtained with the MINVAR distortion function, followed by the MAXVAR procedure and then by the dual cases MINMAXVAR and MAXMINVAR; MINMAXVAR leading to the lowest gamma's. Nevertheless, we observe several strike-maturity buckets for which the MAXVAR distortion leads to an higher gamma than the MINVAR distortion function.

An infinite level of gamma means that the Delta-hedging strategy represents, for the period and the option set we considered, an historic arbitrage opportunity, or equivalently, there is no empirical loss for the particular strike-maturity bucket. Empirical arbitrage opportunities only occur under the standard VG model for deep in-the-money put options[2]. Moreover an accessibility index equal to zero means that the financial position is not profitable since the expected P&L is negative.

5.1 Dow Jones ITM options

The MINVAR, MAXMINVAR and MINMAXVAR distortion functions lead to the same model selection for all the strike-maturity buckets whereas the MAXVAR procedure opts for the Black-Scholes model for one additional bucket ($\frac{K}{S_0} \in [1, 1.05]$ and T lower than three months). For the unweighted portfolio, the percentage of buckets for which the Delta-hedging strategy is not attractive amounts to 25 percents for all the distortion functions. Furthermore, the standard VG model performs better than the Black-Scholes model in 50 and 47.92 percents of the cases for AIMIN, AIMAXMIN or AIMINMAX and for AIMAX, respectively. For the weighted portfolio, the Black-Scholes model outperforms the standard VG model for only 20.83 and 18.75 percents of the buckets if we consider the MAXVAR procedure and the MINVAR, MAXMINVAR or MINMAXVAR procedure, respectively whereas no model leads to an attractive Delta-hedging strategy in 31.25 percents of the scenarios. More particularly, for both the unweighted and weighted portfolios, the standard VG model outperforms the Black-Scholes model for all the ITM puts whereas the Black-Scholes model leads to a more profitable Delta-hedging strategy for ITM calls with a moneyness $\frac{K}{S_0}$ greater than 0.9. Moreover, call options characterised by a low moneyness (i.e. lower than 0.9) correspond to a loss for the Delta-hedging strategy under both the standard VG and Black-Scholes models.

For both the weighted and unweighted portfolios, the level of gamma is at least of one order of magnitude higher for the ITM puts than for the ITM calls for each distortion function. Hence, it is more profitable to Delta-hedge ITM puts than ITM calls. The same conclusion can be drawn from the Sharpe ratio and the GLR ratio (see appendix A). Moreover, the relative distance of the accessibility index between the standard VG and the Black-Scholes models, $\frac{\alpha_{\mathtt{VG}}(X)-\alpha_{\mathtt{BS}}(X)}{\alpha_{\mathtt{BS}}(X)}$, is typically significantly higher in absolute value when the standard VG model outperforms the Black-Scholes model than when the Black-Scholes model outperforms the standard VG model.

Note that the Sharpe ratio and the GLR ratio select the standard VG model for all the in-the-money puts and the Black-Scholes model for close to the money call options for both the weighted and unweighted portfolios whereas both models lead to a non profitable strategy for deep ITM calls. In particular, the Black-Scholes model leads to a more attractive Delta-hedging strategy than the standard VG model in 25 and 18.75 percents of the scenarios for both the Sharpe ratio and the GLR ratio, for the weighted and unweighted portfolios, respectively. On the other hand, the standard VG model outperforms the Black-Scholes model in 50 percents of the scenarios for both the SR and GLR ratio and for both the weighted and unweighted portfolios. Hence, the accessibility indices and the more traditional performance measures select the same model for all the moneyness-maturity buckets.

[2] The infinite level of gamma should be considered with caution since an infinite accessibility index may only be due to a too refined moneyness-maturity bucket. Hence, it is more relevant to speak of *empirical* arbitrage opportunity than arbitrage opportunity.

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0	0	0	0	0	0.044955	0	0	0	0	0	0
0.9-0.95	0	0.056369	0.19207	0.1448	0.25588	0.61703	0	0	0.095212	0	0.052657	0.31438
0.95-1	0.25924	0.5634	0.88257	0.64423	0.6666	0.85575	0.21058	0.60744	0.79321	0.46467	0.45309	0.4904
1-1.05	1.3052	4.7517	5.2693	5.1466	7.7945	4.2319	1.2207	4.8398	5.27	5.1388	7.9249	4.0701
1.05-1.1	1.3825	3.9059	6.0169	7.1358	18.259	7.3037	1.3322	3.9407	6.1315	7.2299	19.181	7.4196
1.1-1.15	2.0164	3.9557	5.3355	6.1412	7.7732	5.8338	2.0779	3.9497	5.2468	6.0901	7.4469	5.5748
> 1.15	1.7371	9.3837	8.6818	6.579	10.839	12.313	1.6201	8.5479	7.9343	5.9592	8.7407	9.018
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0	0	0	0	0	0	0	0	0	0	0	0
0.9-0.95	0	0	0.041377	0	0	0.3883	0	0	0	0	0	0.12921
0.95-1	0.21039	0.34832	0.53538	0.28213	0.25884	0.60678	0.17362	0.4252	0.49895	0.18547	0.054817	0.29344
1-1.05	1.5327	6.2627	11.776	9.5561	11.193	6.4719	1.3947	6.23	11.156	9.134	11.304	6.1529
1.05-1.1	1.9857	8.1994	33.073	24.138	39.695	13.698	1.865	7.9095	35.063	25.035	39.765	13.865
1.1-1.15	5.5106	55.871	111.53	24.718	20.113	9.4243	5.5345	56.269	111.53	25.138	20.383	8.9906
> 1.15	7.4552	45.218	∞	27.864	28.507	22.476	6.6273	43.615	∞	27.886	26.552	18.231

Table 6: MINVAR acceptability index for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, Dow Jones)

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0	0	0	0	0	0.046514	0	0	0	0	0	0
0.9-0.95	0	0.063337	0.19964	0.15549	0.25567	0.5502	0	0	0.094068	0	0.048899	0.25004
0.95-1	0.21602	0.52319	0.79376	0.62306	0.57706	0.68247	0.16569	0.54435	0.67709	0.3961	0.35332	0.3393
1-1.05	0.75899	2.3615	3.7933	4.0963	3.2843	2.5256	0.61973	2.271	3.5899	3.9129	3.2244	2.1603
1.05-1.1	0.90965	2.6156	4.5615	5.8561	4.5866	3.5639	0.84797	2.5038	4.6323	6.0044	4.8985	3.4168
1.1-1.15	1.4483	2.8555	4.6138	5.8653	5.1368	3.564	1.4585	2.9684	4.6021	5.8232	4.6436	3.1217
> 1.15	2.1097	4.8968	5.5229	4.2396	4.2548	5.5522	1.6321	4.1936	4.7212	3.9938	3.5202	4.1168
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0	0	0	0	0	0	0	0	0	0	0	0
0.9-0.95	0	0	0.042302	0	0	0.36592	0	0	0	0	0	0.10943
0.95-1	0.16571	0.31702	0.538	0.27021	0.23446	0.55459	0.1309	0.38202	0.49578	0.17188	0.047019	0.23065
1-1.05	0.75719	2.8348	8.5502	6.589	5.4716	3.3185	0.6125	3.0517	7.319	6.2477	5.3439	2.8421
1.05-1.1	0.92615	3.5919	9.142	8.2423	9.0162	4.6901	0.85492	3.3572	9.9654	9.0554	9.5894	4.5236
1.1-1.15	3.5571	100.08	∞	13.534	23.258	4.8231	3.6912	102.94	∞	13.769	23.31	4.2336
> 1.15	5.1722	24.873	∞	13.963	9.699	8.8448	3.8988	19.166	∞	13.169	8.052	6.3505

Table 7: MAXVAR acceptability index for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, Dow Jones)

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0	0	0	0	0	0.022671	0	0	0	0	0	0
0.9-0.95	0	0.029511	0.094703	0.073086	0.12271	0.26556	0	0	0.046554	0	0.02513	0.13293
0.95-1	0.1132	0.2493	0.36671	0.28615	0.28169	0.33615	0.089981	0.26271	0.32577	0.19992	0.18675	0.18764
1-1.05	0.41627	1.1437	1.4985	1.5523	1.5529	1.1598	0.36732	1.14	1.4828	1.537	1.567	1.0775
1.05-1.1	0.46335	1.1318	1.695	2.0283	2.2799	1.6104	0.44381	1.1194	1.7317	2.0641	2.4067	1.6143
1.1-1.15	0.65606	1.1736	1.6437	1.9223	1.9979	1.5353	0.67272	1.1912	1.624	1.91	1.8676	1.4243
> 1.15	0.72665	1.9936	2.0599	1.6974	1.9433	2.3238	0.64087	1.7341	1.8249	1.5451	1.6101	1.781
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0	0	0	0	0	0	0	0	0	0	0	0
0.9-0.95	0	0	0.020767	0	0	0.17638	0	0	0	0	0	0.057972
0.95-1	0.089766	0.15714	0.24682	0.13163	0.11814	0.26285	0.072795	0.18889	0.22989	0.086478	0.025081	0.12345
1-1.05	0.43579	1.3621	2.6774	2.2959	2.2246	1.5105	0.37779	1.4229	2.4833	2.2235	2.2327	1.3998
1.05-1.1	0.52189	1.6416	3.4325	3.1214	3.9127	2.1286	0.49263	1.5733	3.6551	3.3027	4.0619	2.14
1.1-1.15	1.3566	10.971	15.959	4.0543	5.0553	2.0178	1.3906	10.977	15.809	4.1079	5.1047	1.8794
> 1.15	1.4468	6.512	∞	4.3817	3.7763	3.4378	1.3035	5.5	∞	4.181	3.2491	2.6174

Table 8: MAXMINVAR acceptability index for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, Dow Jones)

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0	0	0	0	0	0.022526	0	0	0	0	0	0
0.9-0.95	0	0.029277	0.092256	0.071599	0.11858	0.2472	0	0	0.04595	0	0.024951	0.12812
0.95-1	0.10992	0.23376	0.33323	0.26494	0.26119	0.30754	0.087929	0.24532	0.29881	0.18943	0.17737	0.17831
1-1.05	0.37597	0.87545	1.0611	1.0864	1.1006	0.88243	0.33651	0.87525	1.0547	1.08	1.1077	0.83435
1.05-1.1	0.41329	0.86057	1.1582	1.3064	1.4489	1.1307	0.39761	0.85482	1.1757	1.3218	1.4987	1.1317
1.1-1.15	0.55937	0.8875	1.1307	1.259	1.2943	1.0827	0.57156	0.89702	1.1213	1.2536	1.2388	1.0266
> 1.15	0.61649	1.3028	1.325	1.1636	1.2888	1.4477	0.5534	1.188	1.223	1.0911	1.1336	1.2185
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0	0	0	0	0	0	0	0	0	0	0	0
0.9-0.95	0	0	0.020648	0	0	0.16818	0	0	0	0	0	0.057047
0.95-1	0.087704	0.15075	0.2312	0.12704	0.11431	0.24505	0.071452	0.17965	0.21603	0.084467	0.024903	0.11932
1-1.05	0.39314	1.0038	1.5727	1.427	1.4014	1.0784	0.3463	1.0319	1.5035	1.3962	1.4045	1.0214
1.05-1.1	0.46263	1.1512	1.8695	1.7584	2.0058	1.3803	0.43946	1.1186	1.9356	1.815	2.0438	1.3845
1.1-1.15	0.99854	3.2158	3.722	2.0119	2.2344	1.3162	1.0163	3.2183	3.7105	2.0265	2.2427	1.2558
> 1.15	1.08	2.5546	∞	2.0946	1.9449	1.8442	0.99717	2.3678	∞	2.0455	1.7946	1.5791

Table 9: MINMAXVAR acceptability index for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, Dow Jones)

5.2 S&P500 ITM options

The MAXVAR, MAXMINVAR and MINMAXVAR distortions lead to the same best model selection for all the buckets, whereas under the MINVAR procedure there exist three additional buckets for which the standard VG model performs better than the Black-Scholes model. The standard VG model leads to a more attractive Delta-hedging strategy of S&P500 in-the-money options with respect to the acceptability indices than the Black-Scholes model except for a strike price close to the spot price and for a short maturity. The same conclusions can be drawn from the Sharpe ratio and the GLR ratio. Furthermore, any strike-maturity bucket for which the Delta-hedging of a weighted portfolio is more attractive under the Black-Scholes model corresponds to a more attractive investment under the Black-Scholes model when we buy a fixed number of options (i.e. for the unweighted portfolio). Moreover, Delta-hedging of deep in-the-money call options (i.e. with a low moneyness, $\frac{K}{S_0} < 0.85$ for a weighted portfolio under the Black-Scholes model and $\frac{K}{S_0} < 0.9$ under the standard VG model and for an unweighted portfolio under the Black-Scholes model) is not attractive since the corresponding accessibility indices are equal to zero. In particular, no model is selected for 14.58 and 20.83 percents of the scenarios for the unweighted and weighted portfolios, respectively. Moreover, the standard VG model outperforms the Black-Scholes model for 54.17 and 47.92 percents of the buckets and for 58.33 and 47.92 percents of the buckets for the MINVAR procedure and MAXVAR, MAXMINVAR or MINMAXVAR procedure, for the weighted and unweighted portfolios, respectively.

As it was the case for the Dow Jones options, all the acceptability indices are significantly lower (of one order of magnitude or more) for the ITM call options than for the ITM put options for both the unweighted and weighted portfolios and under both the Black-Scholes and standard VG models. Hence, any option seller and hedger should opt for in-the-money put options, or even better deep in-the-money put options in order to maximise his risk-adjusted profit. A similar conclusion can be drawn from the more traditional performance measures. Moreover, the relative distance of the accessibility index between the standard VG and the Black-Scholes model is typically higher in absolute value when the VG model is the best performing model than when the Black-Scholes model is selected.

Note that the Black-Scholes model leads to a more attractive Delta-hedging strategy than the standard VG model in 31.25 and 25 percents of the scenarios for the SR ratio and in 31.25 and 20.83 percents of the cases for the GLR ratio, for the weighted and unweighted portfolios, respectively. On the other hand, the standard VG model outperforms the Black-Scholes model in 50 and 54.17 percents of the scenarios for the Sharpe ratio and in 54.17 and 58.33 percents of the buckets for the GLR ratio, for the weighted and unweighted portfolios, respectively. Hence, the traditional performance measures select the same model as the accessibility indices for almost all the option buckets.

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0.021347	0	0.011281	0.10432	0.20832	0.12054	0	0	0	0	0.0013768	0
0.9-0.95	0.13139	0.48095	0.6865	0.5286	0.25814	0.22006	0.09496	0.38691	0.52244	0.31825	0.045173	0
0.95-1	0.27008	1.1863	1.477	1.2914	0.71901	0.26322	0.2862	1.2379	1.3089	1.0648	0.50461	0.01042
1-1.05	0.7594	3.4444	6.3167	7.4003	9.4646	6.9225	0.76165	3.5247	6.041	7.13	8.9745	6.7376
1.05-1.1	1.0525	4.407	4.9712	10.457	8.9448	12.323	1.0275	4.3744	4.494	9.3337	8.4035	12.203
1.1-1.15	1.5987	6.3308	15.385	18.536	25.021	8.8363	1.5072	6.0346	14.683	17.923	24.311	8.5691
> 1.15	4.1227	30.827	64.094	75.254	78.613	42.997	3.0989	22.84	57.292	67.004	75.524	36.984
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0.027896	0	0	0	0.1616	0.12521	0.020802	0	0	0	0	0
0.9-0.95	0.10068	0.38757	0.64675	0.66403	0.38194	0.24546	0.094369	0.35161	0.56463	0.47649	0.19999	0.041305
0.95-1	0.17988	0.91842	1.3272	1.3643	0.86033	0.30391	0.22976	1.0892	1.309	1.1555	0.64444	0.067994
1-1.05	0.81006	3.2106	6.9034	20.681	82.892	14.482	0.78782	3.4339	7.1252	20.177	81.038	13.809
1.05-1.1	1.5207	3.676	3.5787	10.347	6.2667	27.381	1.4717	3.8355	3.4375	9.4568	5.1299	27.363
1.1-1.15	5.4185	7.053	11.455	∞	∞	16.905	4.8996	7.535	12.442	∞	∞	16.36
> 1.15	14.926	∞	∞	∞	∞	∞	11.931	∞	∞	∞	∞	∞

Table 10: MINVAR acceptability index for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, S&P500)

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0.020519	0	0.011661	0.11799	0.23035	0.12928	0	0	0	0	0.0014419	0
0.9-0.95	0.12612	0.47665	0.62534	0.54426	0.28132	0.2278	0.090066	0.3532	0.43292	0.29239	0.0451	0
0.95-1	0.20872	0.87588	1.1093	1.056	0.71121	0.26322	0.21373	0.85636	0.88556	0.77307	0.43733	0.0091539
1-1.05	0.45848	1.7677	3.5084	3.3599	5.204	3.9441	0.41216	1.6505	3.2174	3.8604	5.0369	3.8096
1.05-1.1	0.59873	2.4686	2.7735	6.3041	4.354	6.9246	0.57678	2.3012	2.2425	5.2783	4.129	7.4665
1.1-1.15	1.1262	3.0193	5.3215	13.165	10.317	6.7307	1.0368	2.8212	4.8119	11.576	9.3506	6.2102
> 1.15	1.9719	5.1634	14.14	30.18	27.975	19.686	1.49	3.9912	11.17	21.212	20.862	13.358
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0	0	0	0	0.17377	0.1398	0.016298	0	0	0	0	0
0.85-0.9	0.021873	0	0	0	0	0	0	0	0	0	0	0
0.9-0.95	0.080667	0.33693	0.60417	0.64519	0.40527	0.26893	0.073945	0.28698	0.47143	0.41474	0.19169	0.04024
0.95-1	0.1277	0.57464	0.99416	1.2523	0.88598	0.32707	0.16095	0.65932	0.97793	0.92488	0.57278	0.063904
1-1.05	0.42829	1.2927	2.2843	8.4605	50.863	7.4511	0.3836	1.2506	2.3568	8.001	47.947	7.0842
1.05-1.1	0.61598	1.4438	1.818	3.5021	3.4906	20.152	0.59582	1.4093	1.5238	2.8066	2.8062	21.907
1.1-1.15	3.2953	1.9274	3.4649	∞	∞	16.065	2.9597	2.065	3.7402	∞	∞	14.221
> 1.15	5.973	∞	∞	∞	∞	∞	4.577	∞	∞	∞	∞	∞

Table 11: MAXVAR acceptability index for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, S&P500)

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0.010426	0	0.0057228	0.054277	0.10544	0.061059	0	0	0	0	0.00070412	0
0.9-0.95	0.062958	0.22243	0.29728	0.2472	0.12872	0.10775	0.045507	0.17471	0.22083	0.14516	0.022382	0
0.95-1	0.11312	0.43438	0.53083	0.49565	0.32029	0.12599	0.11759	0.43646	0.45403	0.39601	0.21741	0.0048644
1-1.05	0.25996	0.8914	1.5394	1.587	2.1361	1.6641	0.24676	0.87956	1.4804	1.667	2.0588	1.6527
1.05-1.1	0.33761	1.1364	1.2419	2.3751	1.9266	2.6265	0.32792	1.1049	1.0969	2.0886	1.8266	2.7307
1.1-1.15	0.54698	1.3589	2.3195	3.8296	3.688	2.3084	0.51414	1.3026	2.1913	3.6191	3.5233	2.2235
> 1.15	0.96437	2.5941	5.4728	7.9956	8.1923	5.8027	0.75367	2.0206	4.4061	6.3206	6.6552	4.349
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0.012207	0	0	0	0.081313	0.064575	0.0091095	0	0	0	0	0
0.9-0.95	0.044078	0.17032	0.28483	0.29757	0.18401	0.12278	0.040862	0.15036	0.23839	0.20696	0.094497	0.020234
0.95-1	0.072726	0.31674	0.48838	0.54963	0.38129	0.14942	0.09165	0.36309	0.48183	0.44642	0.27543	0.032557
1-1.05	0.25469	0.72912	1.2328	3.2255	9.6748	2.8191	0.23792	0.73595	1.2795	3.0567	9.1667	2.7041
1.05-1.1	0.3796	0.79985	0.90738	1.7447	1.5188	5.3826	0.36967	0.80488	0.82392	1.4902	1.2724	5.542
1.1-1.15	1.3582	1.0995	1.7226	∞	∞	4.0728	1.2523	1.1639	1.8535	∞	∞	3.8752
> 1.15	2.4201	∞	∞	∞	∞	∞	1.9513	∞	∞	∞	∞	∞

Table 12: MAXMINVAR acceptability index for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, S&P500)

strike range	maturity range						maturity range					
	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0.010397	0	0.0057136	0.053456	0.10242	0.060025	0	0	0	0	0.00070398	0
0.9-0.95	0.061953	0.20955	0.27466	0.23093	0.12423	0.10455	0.044976	0.16672	0.2081	0.13939	0.022241	0
0.95-1	0.10992	0.389	0.46393	0.43596	0.29402	0.12166	0.11413	0.39048	0.40447	0.35696	0.2047	0.0048576
1-1.05	0.24372	0.71958	1.0886	1.116	1.3623	1.1488	0.2324	0.71334	1.0603	1.1498	1.3313	1.1435
1.05-1.1	0.31161	0.86799	0.92945	1.4548	1.2841	1.5506	0.30317	0.85028	0.84691	1.342	1.24	1.5853
1.1-1.15	0.47795	0.99772	1.4546	1.9388	1.9171	1.4222	0.45294	0.96793	1.4037	1.8797	1.8695	1.3876
> 1.15	0.76747	1.5983	2.3962	2.8277	2.8571	2.428	0.62991	1.3622	2.1596	2.5585	2.6273	2.1153
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0	0	0	0	0	0	0	0	0	0	0	0
0.85-0.9	0.012169	0	0	0	0.079517	0.063431	0.0090884	0	0	0	0	0
0.9-0.95	0.043591	0.16277	0.26418	0.27456	0.17505	0.11872	0.040441	0.14442	0.22371	0.19554	0.092041	0.020118
0.95-1	0.071403	0.29224	0.43179	0.47715	0.34492	0.14341	0.089545	0.331	0.42574	0.39715	0.25554	0.032258
1-1.05	0.23953	0.61311	0.93805	1.7802	3.0681	1.6284	0.22489	0.61978	0.9643	1.732	3.0045	1.5905
1.05-1.1	0.34902	0.66255	0.7289	1.2072	1.0728	2.3149	0.34052	0.66754	0.67576	1.0865	0.94418	2.3491
1.1-1.15	1.0012	0.86294	1.1962	∞	∞	1.9947	0.94273	0.90141	1.2575	∞	∞	1.9413
> 1.15	1.4913	∞	∞	∞	∞	∞	1.298	∞	∞	∞	∞	∞

Table 13: MINMAXVAR acceptability index for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, S&P500)

6 Conclusion

This paper provides a detailed comparison of the Delta-hedging performance under the standard VG space model and the Black-Scholes model.

It has been shown that the standard VG model is close to the optimal historical symmetric VG space model for both the unweighted and weighted portfolios and for both options written on the Dow Jones and S&P500 indices. Moreover, the variance of the P&L is usually lower under the standard VG model than under the Black-Scholes model.

Finally, we have defined a model selection procedure based on the value of the accessibility indices proposed by Cherny and Madan. In particular, we have shown that, for options written on the Dow Jones, the standard VG model leads to a significantly more attractive Delta-hedging strategy for all the in-the-money put options whereas the Black-Scholes model is typically selected for in-the-money Call options with a moneyness greater than 0.9, although the accessibility index are of the same magnitude order under the two models. Furthermore the standard VG model significantly outperforms the Black-Scholes model for ITM S&P500 options, except for short maturities and for strikes close to the spot. Moreover for deep in-the-money call options, none of the two models lead to a profitable hedging strategy for both the Dow Jones and the S&P500 indices. We have also noticed that the accessibility indices are in line with more traditional performance measures since similar conclusions can be drawn from the Sharpe ratio and GLR ratio.

A Performance measures: The Sharpe ratio and the GLR ratio

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	-0.23303	-0.57158	-0.58446	-0.68321	-0.51918	-0.3066	-0.23162	-0.53396	-0.57479	-0.67673	-0.59507	-0.37876
0.85-0.9	-0.14383	-0.3431	-0.23362	-0.30286	-0.24192	0.039191	-0.15933	-0.3933	-0.323	-0.41143	-0.38458	-0.15248
0.9-0.95	-0.017159	0.051214	0.15688	0.1214	0.19826	0.40319	-0.036785	-0.00054186	0.07991	-0.00080801	0.043016	0.21727
0.95-1	0.19262	0.37242	0.5119	0.41826	0.42628	0.49828	0.15437	0.39722	0.48057	0.32038	0.29835	0.29869
1-1.05	0.63044	1.2785	1.3345	1.3631	1.5875	1.2651	0.59599	1.3633	1.4184	1.4086	1.7488	1.313
1.05-1.1	0.64051	1.1992	1.3512	1.5357	2.1532	1.5383	0.64625	1.2295	1.4171	1.5677	2.4241	1.6968
1.1-1.15	0.69092	1.1526	1.3304	1.4052	1.6951	1.4925	0.75587	1.1472	1.2964	1.4203	1.6416	1.508
> 1.15	0.50831	1.5555	1.508	1.4487	1.838	1.9185	0.52468	1.2223	1.2611	1.1977	1.5035	1.5017
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	-0.21189	-0.50439	-0.54586	-0.67564	-0.51285	-0.38785	-0.20806	-0.46527	-0.53433	-0.67928	-0.6081	-0.46262
0.85-0.9	-0.13166	-0.3281	-0.30835	-0.44354	-0.4058	-0.096395	-0.13984	-0.36467	-0.37959	-0.52263	-0.52374	-0.27791
0.9-0.95	-0.033969	-0.029897	0.036483	-0.11958	-0.021113	0.28024	-0.044421	-0.068014	-0.017782	-0.18965	-0.17388	0.098713
0.95-1	0.15425	0.25487	0.36491	0.21644	0.1943	0.39456	0.12488	0.30253	0.3485	0.14707	0.042702	0.2035
1-1.05	0.66799	1.4007	1.5215	1.5821	1.7205	1.4823	0.6173	1.4959	1.6458	1.6349	1.942	1.5839
1.05-1.1	0.72412	1.5152	1.62	1.8361	2.4405	1.7266	0.73618	1.5222	1.7493	1.8957	2.805	2.0276
1.1-1.15	0.79799	1.8065	2.065	1.8866	2.0622	1.6681	0.92191	1.7875	1.9883	1.9573	2.01	1.7572
> 1.15	0.55193	2.3395	2.29	2.2554	2.4602	2.2802	0.59651	1.7732	1.9279	1.8263	1.9986	1.7741

Table 14: Sharpe Ratio for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, Dow Jones)

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0.51955	0.23415	0.24686	0.19068	0.29643	0.45989	0.51525	0.24255	0.23832	0.17924	0.23785	0.36296
0.85-0.9	0.65104	0.42475	0.56867	0.48632	0.57615	1.0977	0.62324	0.37831	0.46339	0.38018	0.41613	0.69784
0.9-0.95	0.95177	1.1468	1.4811	1.3426	1.6158	2.7069	0.90039	0.99861	1.2129	0.99812	1.105	1.6654
0.95-1	1.6995	2.9337	4.1405	2.8993	2.8717	3.4444	1.5441	2.9589	3.4944	2.1895	1.9919	2.0166
1-1.05	5.2916	19.399	24.177	26.484	34.403	18.813	4.6075	20.143	23.806	26.441	33.672	15.781
1.05-1.1	6.1237	14.765	30.961	44.548	80.567	45.268	5.7142	15.313	30.957	46.271	88.66	41.231
1.1-1.15	10.762	17.871	25.024	38.683	39.337	28.81	11.249	18.504	25.642	38.166	36.158	24.983
> 1.15	23.384	48.359	47.844	34.731	59.792	74.296	17.57	40.026	40.101	32.567	46.164	60.17
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0.4977	0.24317	0.24368	0.17764	0.29437	0.37023	0.49509	0.25177	0.23208	0.16066	0.22549	0.27918
0.85-0.9	0.64148	0.42862	0.46054	0.33376	0.38378	0.78891	0.62754	0.38906	0.38373	0.27439	0.28296	0.5043
0.9-0.95	0.90086	0.92435	1.0937	0.74886	0.95108	2.036	0.87422	0.84098	0.95853	0.63598	0.66227	1.271
0.95-1	1.5417	2.0301	2.6824	1.7501	1.5908	2.8077	1.4308	2.217	2.4473	1.4552	1.1059	1.6426
1-1.05	6.1795	29.818	110.82	83.758	56.487	35.534	5.1833	29.875	98.489	76.723	55.784	29.534
1.05-1.1	8.6803	55.133	348.09	271.42	221.38	98.8	7.8137	53.473	381.41	295.95	239.82	91.884
1.1-1.15	38.387	2727.1	3444.7	278.94	226.69	69.388	40.471	2836.2	3496.1	282.9	215.49	58.617
> 1.15	203.66	597.83	∞	224.49	178.15	232.52	141.82	474.03	∞	202.17	139.19	173.94

Table 15: GLR ratio for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, Dow Jones)

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	-0.044802	-0.40987	-0.48513	-0.45668	-0.26286	-0.19979	-0.057761	-0.39352	-0.49622	-0.47789	-0.33574	-0.34465
0.85-0.9	0.018501	-0.040616	0.010068	0.091087	0.17101	0.10252	-0.0017521	-0.11273	-0.087624	-0.047065	0.0012447	-0.091127
0.9-0.95	0.10771	0.33858	0.43978	0.36491	0.20515	0.17537	0.079006	0.2799	0.34941	0.23331	0.038831	-0.015652
0.95-1	0.19095	0.61455	0.71036	0.66885	0.45504	0.20382	0.19993	0.63603	0.65991	0.58865	0.33821	0.0083516
1-1.05	0.42246	1.1063	1.4743	1.6705	1.8539	1.4805	0.40805	1.1787	1.5527	1.6369	1.7941	1.6215
1.05-1.1	0.52194	1.2343	1.2593	1.8314	1.7311	1.9404	0.51447	1.2888	1.3078	1.7447	1.636	2.0723
1.1-1.15	0.70767	1.253	1.6041	1.7867	2.1127	1.6856	0.68657	1.2898	1.7759	1.9454	2.2443	1.7406
> 1.15	1.0624	2.0397	2.299	2.149	2.2263	2.2739	0.88346	1.5434	1.6679	1.6623	1.5222	1.8348
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	-0.042724	-0.3124	-0.46173	-0.57566	-0.4154	-0.2845	-0.042727	-0.30003	-0.46382	-0.5851	-0.4761	-0.42578
0.85-0.9	0.02066	-0.043297	-0.053284	-0.0038598	0.13589	0.10727	0.015372	-0.086868	-0.10495	-0.10782	-0.0055416	-0.068644
0.9-0.95	0.075065	0.27823	0.42132	0.4324	0.28312	0.19513	0.069446	0.25043	0.37509	0.32608	0.15777	0.03508
0.95-1	0.12247	0.50099	0.67111	0.68352	0.50357	0.23462	0.15554	0.56828	0.67396	0.62683	0.41552	0.055726
1-1.05	0.41081	1.0458	1.5388	2.1052	2.4843	1.9589	0.38684	1.1129	1.6507	2.0078	2.2981	2.0288
1.05-1.1	0.57534	1.102	1.1185	1.8278	1.5274	2.3196	0.56927	1.1546	1.1401	1.7006	1.3713	2.4019
1.1-1.15	1.0957	1.3671	1.5721	1.9453	2.0226	1.854	1.0629	1.459	1.7843	2.1342	2.1418	2.0008
> 1.15	1.6031	2.6298	3.5006	3.2166	2.8893	2.3604	1.3554	1.9112	2.3014	2.2479	1.8658	1.9049

Table 16: Sharpe ratio for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, S&P500)

	maturity range						maturity range					
strike range	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5	0-0.25	0.25-0.5	0.5-0.75	0.75-1	1-1.5	> 1.5
	Black-Scholes model - unweighted portfolio						**Black-Scholes model - weighted portfolio**					
< 0.85	0.88385	0.3557	0.31109	0.34227	0.52256	0.62078	0.84922	0.34298	0.28791	0.31195	0.42668	0.44338
0.85-0.9	1.0541	0.90552	1.0243	1.2474	1.5431	1.2945	0.99498	0.75829	0.81348	0.89647	1.003	0.80376
0.9-0.95	1.3727	2.3826	2.9608	2.4313	1.6824	1.5642	1.2572	1.9983	2.2748	1.7057	1.096	0.96275
0.95-1	1.7498	5.3144	6.5611	5.4214	3.2242	1.6706	1.7572	5.1777	5.264	3.9496	2.1667	1.0199
1-1.05	3.1487	12.264	27.377	33.237	50.228	33.811	3.0098	12.193	26.398	34.278	49.763	33.43
1.05-1.1	4.6965	16.385	21.799	61.793	64.639	66.438	4.4612	15.934	18.319	55.072	61.198	68.34
1.1-1.15	7.3854	25.888	77.024	190.86	271.8	48.525	6.9196	24.958	70.025	168.89	237.58	44.346
> 1.15	15.077	146.77	515.43	1440.1	1035.1	669.25	11.658	112.73	426.11	1043.1	815.01	472.69
	Standard VG model - unweighted portfolio						**Standard VG model - weighted portfolio**					
< 0.85	0.88219	0.40678	0.29737	0.25599	0.36529	0.50443	0.87616	0.38834	0.27696	0.23575	0.30028	0.3594
0.85-0.9	1.0674	0.88982	0.87372	0.99057	1.419	1.3282	1.0499	0.78815	0.76794	0.77109	0.98658	0.84302
0.9-0.95	1.2682	2.0619	3.0315	2.8968	2.1068	1.6965	1.2425	1.8716	2.5324	2.129	1.4639	1.0918
0.95-1	1.4524	3.6376	6.0054	6.0671	4.1026	1.8402	1.5675	3.9327	5.4253	4.4512	2.7203	1.1414
1-1.05	3.2617	10.522	30.321	203.51	2801.2	109.63	3.0634	11.066	32.805	217.1	2729.2	107.44
1.05-1.1	6.3957	12.11	13.179	61.309	23.907	375.95	6.0277	12.48	11.806	56.517	19.669	398.24
1.1-1.15	35.714	25.773	49.786	∞	∞	177.44	31.587	28.216	55.607	∞	∞	158.13
> 1.15	90.264	∞	∞	∞	∞	∞	65.507	∞	∞	∞	∞	∞

Table 17: GLR ratio for the Black-Scholes and standard VG models and for the unweighted and weighted portfolios (ITM options, S&P500)

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