First passage percolation on random graphs with finite mean degrees

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Abstract

We study first passage percolation on the configuration model. Assuming that each edge has an independent exponentially distributed edge weight, we derive explicit distributional asymptotics for the minimum weight between two randomly chosen connected vertices in the network, as well as for the number of edges on the least weight path, the so-called hopcount.

We analyze the configuration model with degree power-law exponent \( \tau > 2 \), in which the degrees are assumed to be i.i.d. with a tail distribution which is either of power-law form with exponent \( \tau - 1 > 1 \), or has even thinner tails (\( \tau = \infty \)). In this model, the degrees have a finite first moment, while the variance is finite for \( \tau > 3 \), but infinite for \( \tau \in (2, 3) \).

We prove a central limit theorem for the hopcount, with asymptotically equal means and variances equal to \( \alpha \log n \), where \( \alpha \in (0, 1) \) for \( \tau \in (2, 3) \), while \( \alpha > 1 \) for \( \tau > 3 \). Here \( n \) denotes the size of the graph. For \( \tau \in (2, 3) \), it is known that the graph distance between two randomly chosen connected vertices is proportional to \( \log \log n \) [3], i.e., distances are ultra small. Thus, the addition of edge weights causes a marked change in the geometry of the network. We further study the weight of the least weight path, and prove convergence in distribution of an appropriately centered version.

This study continues the program initiated in [1] of showing that \( \log n \) is the correct scaling for the hopcount under i.i.d. edge disorder, even if the graph distance between two randomly chosen vertices is of much smaller order. The case of infinite mean degrees (\( \tau \in [1, 2] \)) is studied in [2], where it is proved that the hopcount remains uniformly bounded and converges in distribution.

Key words: Flows, random graph, first passage percolation, hopcount, central limit theorem, coupling to continuous-time branching processes, universality.

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References


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