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# Conic Financial Markets and Corporate Finance

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**Abstract**

Markets passively accept a convex cone of cash flows that contains the the nonnegative cash flows. Different markets are defined by different cones and conditions are established to exclude the possibility of arbitrage between markets. Operationally these cones are defined by positive expectation under a concave distortion of the distribution function of the cash flow delivered to market. Firms access risky assets and risky liabilities and regulatory bodies ensure that sufficient capital is put at stake to make the risk of excess loss acceptable to taxpayers. Firms approach equity markets for funding and can come into existence only if they can raise sufficient equity capital. Firms that are allowed to exist approach debt markets for favorable funding opportunities. The costs of debt limit the amount of debt. Firms with lognormally distributed and correlated assets and liabilities are analysed for their required capital, their optimal debt levels, the value of the option to put losses back to the taxpayer, the costs of debt and equity, and the level of finally reported equity in the balance sheet. The relationship between these entities and the risk characteristics of a firm are analysed and reported in detail.

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# 1 Introduction

Financial markets are generally modeled as counterparties for market participants permitting such entities to trade in both directions, buy from or sell to market at the going market price. Though market participants are seen as optimizing agents, the financial market itself passively accepts a range of so-called traded cash flows. When we analyze the behavior of economic agents in market economies, the counterparty to each transaction by an economic agent is not another economic agent, but the market. These are the basic tenets of the modeling of markets in general equilibrium as set out for example in Arrow and Hahn (1971), Debreu (1959). By virtue of accepting trades in both directions the set of cash flows acceptable to the market is closed under negation. It is also closed under addition and scaling and is therefore a linear subspace of the space of all possible cash flows. This model of marketed cash flows is central to the no arbitrage theory of financial markets as described for example in Duffie (2003).

The above view of markets is central to much of modern financial analysis and it is central to the law of one price and the no arbitrage theory of markets with its many implications. We shall abandon this view of markets by recognizing up front that terms of trade with the market depend on the direction of trade. When buying from market one pays the ask price but when we sell to market we receive the lower bid price. The market buys at bid and sells at ask. As a consequence the set of cash flows acceptable to market is not closed under negation. We shall however, continue to treat it as closed under addition and scaling.

With our revised view of markets we address the classical questions of corporate finance. Traditionally a corporation accesses random cash flows as assets and finances the purchase in debt and equity markets (Merton, 1973, 1974, 1977). The firm consists of a nonnegative but random asset value that is unchanged by financing choices (Modigliani and Miller (1958)). Debt is issued to capture tax advantages that are limited by exposure to default. In addition, there may be additional costs associated with bankruptcy (Leland (1994), Leland and Toft (1997)).

We shall expand this view by modeling a corporation as accessing both random cash flows as assets and another set of potentially unbounded random cash flows as liabilities. For examples one may take long short hedge funds, insurance liabilities and a variety of swap contracts. Such a model has recently been proposed in Madan (2009) and studied further in Eberlein and Madan (2009). It is now possible that the liabilities may dominate the assets and the firm cannot be permitted to exist as a limited liability entity if it is insufficiently capitalized. We follow Madan (2009) and determine the required capital levels as a function of the risk structure of the firm. It is possible that capital markets are unwilling to supply the capital required at a sufficient enough level. In this case the firm is not allowed to exist. Hence, contrary to the structure of firms in the Modigliani and Miller (1958) model, financial markets determine the size of the real economy by a precise computation that we illustrate. The size of the real economy and the number of risk structures that are allowed to exist depends critically on risk exposures, the associated capital requirements and the willingness of capital markets to fund the same.

Once a firm is allowed to exist as a limited liability entity, it accesses for free the option to put excessive losses back to the economy. This is an asset of the firm that should be reported on its balance sheet and is held by the equity holders. It was valued recently for the major US banks in Eberlein and Madan (2009) as at the end of the calendar year 2008. Furthermore, the firm may also decide to issue debt securities. In our model, unlike Leland (1994) and Leland and Toft (1997) there are no tax advantages to debt nor are there any bankruptcy costs. Debt is issued to access favorable financing terms by designing securities that appeal to investors in debt markets. However, there are costs to issuing debt, described here in some detail, that limit the extent of debt that may be issued. Our firms issue the maximum allowable debt and the optimal debt level is reached on meeting a funding constraint.

The static model of Leland (1994) has been generalized to a dynamic version by a number of authors, including Hilberink and Rogers (2002), Goldstein, Ju and Leland (2001) and Chen and Kou (2009). A dynamic extension of our approach requires the use of dynamic versions of risk acceptability as described in Peng (2005), Delbaen, Peng, Gianin (2008), Jobert and Rogers (2008) and Cohen and Elliott (2008). For the moment this is a subject for future research.

Our plan is to first formally define financial markets. We then take a stochastic balance sheet seen as the joint random process for assets and liabilities and show how one may compute the capital required for existence. The second step is to determine whether the firm can exist as an all equity firm. If it can exist, we value the limited liability option to put excessive losses back to the taxpayer or the general economy. Finally we determine the amount of debt the firm will issue. From the inputs of balance sheet risk characteristics that describe the joint probability law of the random assets and liabilities, we determine as outputs,

- the required capital,
- the indicator variable for firm existence, if this is unity,
    - we determine the value of the option to put losses to the taxpayer, and
    - finally the debt level.

We close the theoretical discussion by exhibiting a typical balance sheet that takes into consideration all these items. We then go on to construct a variety of selectively generated bivariate Gaussian stochastic balance sheets and we report on the relationships between a firm's risk characteristics and its final balance sheet.

The outline of the rest of the paper is as follows. Section 2 describes our model for financial markets. In Section 3 we present the methods for computing capital required for existence. Section 4 explains how to value the option to put losses to the taxpayer if the firm is allowed to exist. The determination of the debt level is detailed in Section 5. Section 6 presents a typical balance sheet recognizing all the items analysed here. The analysis of our selectively generated balancesheets is presented in Section 7. Section 8 concludes.

# 2 Modeling Financial Markets

We model markets as satisfying the law of two prices as opposed to one price. In keeping with the classical model for markets, economic agents may trade prospective cash flows with the market in any amount at the going bid or ask price. The market sells at the ask price and buys at the bid price so agents buy at ask and sell at bid. The bid price is below the ask price. The set of zero cost cash flows that one may deposit in the market therefore include any positive multiple of such a cash flow and hence this set of cash flows is closed under scaling by a positive constant. Furthermore, if two different zero cost cash flows may be deposited in the market then one may also deposit the sum. The set of zero cost cash flow that one may deposit in the market is then closed under addition. It follows that the set of zero cost cash flows economic agents may deposit in the market is a convex cone. We further recognize that any agent may deposit in any market at zero cost any nonnegative cash flow. The convex cone of cash flows acceptable to a market therefore contains the set of nonnegative cash flows. Markets therefore accept at zero cost a set of risks that satisfy the axioms for acceptable risks as set out in Artzner, Delbaen, Eber and Heath (1999), and studied further in Carr, Geman and Madan (2001), Jaschke and Kuchler (2001) and Follmer and Schied (2004).

The classical market of liquid markets with its law of one price allows one to trade in both directions at the same price and so it is also closed under negation. This property makes the set of cash flows acceptable to the market a half space defined as the set of all cash flows with positive expectation under a single risk neutral and unique probability measure $Q$. Once we pass to the law of two prices we lose closure under negation and we have just a convex cone containing the nonnegative cash flows but no longer a half space associated with one risk neutral measure. However, we recognize that every convex set is also defined by the intersection of all the half spaces that contain it, and this is also true for convex cones. Since our cone of acceptable cash flows contains the nonnegative cash flows, we have a whole collection $\mathcal{M}$ of probability measures $Q \in \mathcal{M}$, with the property that a cash flow is acceptable to the market at zero cost just if its expectation is positive under each measure $Q \in \mathcal{M}$. Without loss of generality we may take the set $\mathcal{M}$ of measures to be a convex set as we may always include all the convex combinations. Every market is then defined by a convex cone of zero cost cash flows acceptable to the market, and this cone has associated with it a convex set of probability measures $Q \in \mathcal{M}$ with acceptability equivalently defined as positive expectation under each $Q \in \mathcal{M}$. We therefore refer to financial markets for the law of two prices as conic, given that they are defined by convex cones of acceptable cash flows.

We may model different markets using different convex cones of acceptable cash flows. For two markets we may have cones of acceptable zero cost cash flows $\mathcal{A}_1$, $\mathcal{A}_2$ with associated sets of measures $\mathcal{M}_1, \mathcal{M}_2$. One may then wonder if the two markets may be arbitraged by buying some cash flow at the ask price from one market and selling it to the other market at its higher bid price. Now for any cash flow $X$ we determine the markets ask price $a(X)$ by noting that

$$a(X) - X \in \mathcal{A}$$

or equivalently that

$$a(X) - E^Q[X] \geq 0, \text{ all } Q \in \mathcal{M},$$

and so

$$a(X) = \sup_{Q \in \mathcal{M}} E^Q[X].$$

Similarly one shows that

$$b(X) = \inf_{Q \in \mathcal{M}} E^Q[X].$$

Now provided the set of supporting measures $\mathcal{M}_1, \mathcal{M}_2$ have a common element $\overline{Q}$ then

$$a_1(X) \geq E^{\overline{Q}}[X] \geq b_2(X)$$

and the bid price of market two is never above the ask price of market one. Hence one may employ different cones to conceptualize different markets provided the set of supporting measures have a nonempty intersection. One may show by classic separation arguments, separating $\mathcal{A}_1$ from $-\mathcal{A}_2$ that this condition for the absence of arbitrage is also necessary.

Models for markets may be constructed by specifying intersecting sets of supporting measures. However, this is not that simple a task. Operational cones were defined by Cherny and Madan (2009) for cones that depend solely on the probability law of the cash flow being assessed. For such cones one may proceed with acceptability being linked to positive expectation under concave distortion. One may take any concave distribution function on the unit interval $\Psi(u)$, $0 \leq u \leq 1$ and define a random variable $X$ with distribution function $F(x)$ to be acceptable provided

$$\int_{-\infty}^{\infty} x d\Psi(F(x)) \geq 0.$$

In this case the set of supporting measures consist of all equivalent probabilities with densities $Z$ such that

$$E\left[(Z-a)^+\right] \leq \Phi(a), \quad \text{for all } a \geq 0$$

where $\Phi(a)$ is the conjugate of $\Psi$,

$$\Phi(a) = \sup_{u \in [0,1]} \left( \Psi(u) - ua \right).$$

Cherny and Madan (2009) introduced parametric families of distortions $\Psi^{\gamma}(u)$, that define a decreasing sequence of cones as one raises $\gamma$ with the limit as $\gamma$ goes to infinity being just the nonnegative cash flows. Observing that expectations under concave distortions are also expectations under the measure change $\Psi^{\gamma\prime}(F(x))$ they suggested that for $x$ tending to negative infinity and $F(x)$ tending to zero, one should induce loss aversion by ensuring that $\Psi^{\gamma\prime}(u)$ tends to infinity as $u$ tends to zero. Similarly as $x$ tends to positive infinity and $u$ tends to unity one should ensure against being enticed by large gains by requiring that $\Psi^{\gamma\prime}(u)$ tends to zero. A suggested distortion that has these properties is the distortion termed *minmaxvar* for which

$$\Psi^{\gamma}(u) = 1 - \left(1 - u^{\frac{1}{1+\gamma}}\right)^{1+\gamma}.$$

We generalize this distortion to a two parameter one that allows different rates of convergence to infinity and zero at the left and right end points of the unit interval defined by

$$\Psi^{\lambda,\gamma}(u) = 1 - (1 - u^{\frac{1}{1+\lambda}})^{1+\gamma}.$$

For this distortion the rate of loss aversion is determined by $\lambda$ while the absence of gain enticement is controlled by $\gamma$. We call this distortion *minmaxvar*2 for its two parameters. We could model different markets using different levels of loss aversion and absence of gain enticement. However, we have to first enquire whether the sets of supporting measures associated with different distortions have a nonempty intersection, thereby ensuring the absence of arbitrage opportunities between markets.

For this we first show how one evaluates bid and ask prices using distortions. For the bid price one must have that

$$Y = X - b$$

is acceptable. This requires that

$$\int_{-\infty}^{\infty} y d\Psi(F_Y(y)) \geq 0.$$

Note now that

$$F_Y(y) = F_X(y + b)$$

and so we must have that

$$\int_{-\infty}^{\infty} y d\Psi(F_X(y + b)) = \int_{-\infty}^{\infty} (x - b) d\Psi(F_X(x)) \geq 0,$$

or that

$$b(X) = \int_{-\infty}^{\infty} x d\Psi(F_X(x)).$$

From the relationship between suprema and infima we observe that

$$\begin{aligned} a(X) &= -b(-X) \\ &= -\int_{-\infty}^{\infty} x d\Psi \left(F_{(-X)}(x)\right). \end{aligned}$$

Let $\Psi(u)$ be a distortion. Suppose we have another distortion $\widetilde{\Psi}(u)$. Now if there exists a distribution function of a random variable $X$, $F(x)$ such that we may buy this random variable at the ask price for $\Psi$ and sell to $\widetilde{\Psi}$ at the bid for a profit then we have an arbitrage. Let $a$ denote the ask price of $\Psi$ while $b$ is the bid for $\widetilde{\Psi}$. For $a < b$ we have arbitrage.

Now the bid and ask prices for $\widetilde{\Psi}, \Psi$ are respectively (noting that $F_{(-X)}(x) = 1 - F(-x)$)

$$\begin{aligned} b &= \int_{-\infty}^{\infty} x d\widetilde{\Psi}(F(x)) \\ a &= -\int_{-\infty}^{\infty} x d\Psi \left(1 - F(-x)\right) \end{aligned}$$

We may also write that

$$b = \int_0^1 F^{-1}(u)\widetilde{\Psi}'(u)du$$

while

$$\begin{aligned} a &= \int_0^1 F^{-1}(1-u)\Psi'(u)du \\ &= \int_0^1 F^{-1}(u)\Psi'(1-u)du. \end{aligned}$$

Let $G(u) = F^{-1}(u)$ and note that $G$ is an increasing function. We may write the difference between ask and bid prices as

$$\begin{aligned} a - b &= \int_0^1 G(u)\left(\Psi'(1-u) - \widetilde{\Psi}'(u)\right) du \\ &= -\int_0^1 G'(u)\left(1 - \Psi(1-u) - \widetilde{\Psi}(u)\right) du \\ &\geq 0 \end{aligned}$$

The last inequality follows on noting that for all concave distortions $\Psi(u) \geq u$ and so $\Psi(1-u) + \widetilde{\Psi}(u) \geq 1$. Hence the set of measures supporting concave distortions have a nonempty intersection and arbitrage is excluded when modeling different markets using different concave distortions.

Our model for corporate entities in real economies sees the corporations as interacting with a variety of stakeholders. Corporations have a multitude of complementary and competing interests at issue with a variety of stakeholders. We quote from John and Senbet (1998),

"The pay-off structure of the claims of different classes of stakeholders are different. The degree of alignment of interests with those of the agents in the firm who control the major decisions in the firm are also different. This gives rise to potential conflicts among stakeholders, and these incentive conflicts have come to be known as "agency (principal-agent) problems." Left alone, each class of stakeholders pursues its own interest which may be at the expense of other stakeholders. We can classify agency problems on the basis of conflicts among particular parties to the firm, such as conflicts between stockholders (principals) and management (agent) ("managerial agency" or "managerialism"), between stockholders (agents) and bondholders ("debt agency"), between the private sector (agent) and the public sector ("social agency"), and even between the agents of the public sector (e.g. regulators) and the rest of society or taxpayers ("political agency")."

We distinguish in our study and formulation of a corporation four separate stakeholders, the stockholders, the debtholders, the firm itself or its internal managers and decision makers, and the external taxpayer who provides limited liability to the firm even if it was debt free. We take the view that the counterparties of the corporation are not particular stockholders, bondholders, managers, and taxpayers but stock markets, bond markets, markets for managerial talents and markets for the creation of limited liability entities. Each of these markets decides on the residual risks it will hold as appropriate and these are given by potentially different convex cones of random variables that we denote by $\mathcal{A}_S$, $\mathcal{A}_D$, $\mathcal{A}_M$, and $\mathcal{A}_T$.

We take all four cones to be determined by different parameter settings for the distortion $minmaxvar2$. The taxpayers have the most loss aversion and the highest absence of gain enticement and we suppose that for them $\gamma = \lambda = .75$. The managers of a firm we take to be somewhat more tolerant of losses than taxpayers but they are induced by gains and so we suppose $\lambda = .5$ and $\gamma = 0.25$. The debt holders are tolerant of losses but unlike equity holders and firm managers they are not enticed by gains and prefer to see their principal back. They have a cone with $\lambda = 0.1$ but $\gamma = 0.2$. Equity holders however are both tolerant of losses and enticed by gains with $\gamma = \lambda = 0.025$.

We now have a complete specification of the four markets the corporation has to interact with and we can compute the bid and ask prices for cash flows traded in each market. We may write all prices as distorted expectations of cash flows and we abbreviate and write

$$de(X, \lambda, \gamma) = \int_{-\infty}^{\infty} x d\Psi^{\lambda,\gamma}\left(F_X(x)\right).$$

# 3 Determining Required Capital

We have remarked that the classic Mertonian firm has no need for capital reserves as the value of the option to put losses back on the taxpayer or economy is zero by construction. The worst that happens is that assets go to zero wiping out debt and equity holders but there are no adverse consequences for the general economy. Debt holders may worry about there being sufficient equity capital as reserve protecting against their loss exposure. Our concern is not with protecting debt holders and nor should this be the concern of regulatory bodies.

We recognize that corporations may access random and potentially unbounded liabilities with the possibility that come year end, or whatever suitable horizon one works with, we find that the random liabilities exceed assets in place and losses either fall on counterparties in the economy who are not made whole, or there is an explicit bail out with counterparties being made whole by the general taxpayer and then the losses fall uniformly across the community of taxpayers. Every limited liability entity accessing random and potentially unbounded liabilities should ensure that there is a sufficient stake up front from the organizers of the enterprise or else the enterprise should not be allowed to exist and get the limited liability status.

The upfront stake is reserve cash capital whose purpose is to cover for losses if and when they should arise. The regulatory body granting the limited liability has to consider the total cash flow that is being accessed and this should be acceptable to taxpayers. For random assets $A(T)$ and random liabilities $L(T)$ with a capital stake of $M^*$ we have to ensure that

$$M^* e^{rT} + A(T) - L(T) \in \mathcal{A}_T.$$

A simple computation reveals (see Madan (2009)) that

$$M^* = -e^{-rT} de\left(A(T) - L(T), \lambda_T, \gamma_T\right). \tag{1}$$

Given a model for the evolution of the random or stochastic components of the balance sheet or the joint law for $A(T)$, $L(T)$ and the parameters of the taxpayer cone one may determine the capital $M^*$ that must be placed at stake to get the limited liability license from the taxpayers making the proposed business generally economically acceptable. From the viewpoint of the general economy the base law for $A(T), L(T)$ to be distorted is a risk neutral law for one has to ensure at a minimum a positive expectation after risk compensation. Hence we shall define acceptability by distorting risk neutral laws. The actual funds the firm has to raise in the capital markets is

$$M^* + A(0) - L(0).$$

By way of a simple example helpful in illustrating the issues involved consider the model of geometric Brownian motion for the random assets and liabilities with correlated Brownian motions. Risk neutrally we suppose that

$$\begin{aligned} A(t) &= A(0)e^{rt+\sigma_A W_A(t)-\frac{\sigma_A^2}{2}t} \\ L(t) &= L(0)e^{rt+\sigma_L W_L(t)-\frac{\sigma_L^2}{2}t} \\ dW_A dW_L &= \rho dt \end{aligned}$$

For an annual horizon ($T = 1$) with equal volatilities of 25% , a 50% correlation, an interest rate of 3% and starting values of 100, 90 for assets and liabilities we determine $M^*$ for the taxpayer cone parameters $\lambda = \gamma = 0.75$ at 17.2399 dollars as per equation (1). The capital to be raised in the markets for this firm to exist or start operations is then 27.2399.

# 4 Firm Existence and the Value of the Taxpayer Put

We first consider the possibility of an all equity firm approaching the equity markets for funding. The equity market has high tolerance for losses and may be easily enticed by gains. For this reason we model the equity market with a wide cone and $\lambda_S = \gamma_S = 0.025$. We could take zero values and then we have a single risk neutral measure and no bid ask spread. We allow for a small spread and relate this spread to the cost of entering financial markets. In a liquid market there are no costs to entering the market as the round trip cost is zero given that we trade in both directions at the same price. For markets with two prices, bid and ask, defined by cones of acceptable cash flows you sell to market at bid but you have to buy back at ask and the difference is a cost that must be funded upfront. The difference may be held as cash reserves in recognition of the expected cost of reversing the trade.

These costs address the separate treatment of assets and liabilities when transacting in two price markets. On the asset side one unwinds by selling to market and one receives the bid price. On the liability side however one unwinds by buying from market and this takes place at the ask price. Hence all assets are to be marked at their bid prices while all liabilities are to be marked at their ask prices. When we issue liabilities like stocks or bonds we receive for them the bid price, but we mark then up to the ask price we expect to pay for an unwind. The difference is a cost we incur and hold as reserve.

The firm we approach the equity market with has final limited liability cash flow for maturity $T = 5$ of

$$V(T) = (M^* + A(T) - L(T))^+ .$$

The bid price from the equity market is given by

$$bJ = de\,(V(T), \lambda_S, \gamma_S)$$

while the ask price will be

$$aJ = -de\,(-V(T), \lambda_S, \gamma_S)\,.$$

We receive $bJ$ but mark the liability at $aJ$ and the funds raised must provide us with

$$M^* + A(0) - L(0) + aJ - bJ.$$

The all equity firm can come into existence provided

$$M^* + A(0) - L(0) + aJ - bJ \leq bJ. \tag{2}$$

For our example, we have $bJ = 35.4621$ and $aJ = 39.0824$ and the left hand side of the above inequality is 30.8602. As a result the all equity firm can come into existence. On the other hand if equity markets were more averse to losses and less responsive to gains with $\lambda_S = .05$, $\gamma_S = .05$ then $bJ = 33.8428$ and the left hand side is 34.5178 and the firm has insufficient funds.

The option to put losses back to the taxpayer has the payoff

$$(-M^* e^{rT} - A(T) + L(T))^+,$$

that is an asset of the firm to be valued at the bid price for the market to which it could be sold. If we temporarily suppose that this is the equity market, the bid price is 9.8789. We later relax this assumption in Section 7 where we construct the reported balance sheet.

# 5 Determining Debt Levels

If a firm cannot exist as an all equity firm, we show later under conservative assumptions that it cannot exist by issuing debt as well. Assuming we have a firm that can exist as an all equity firm, we consider debt issues of pure discount bonds with a face value $F$ and maturity of $T = 5$ years. On a debt issue of face value $F$ the cash flow to bond holders is

$$CFD = \min(V(T), F)$$

while equity holders receive

$$JD = (V(T) - F)^+.$$

The bid prices for these securities in the bond and equity markets are

$$\begin{aligned} bD &= de(CFD, .1, .2) \\ bJD &= de(JD, .025, .025) \end{aligned}$$

while the ask prices are

$$\begin{aligned} aD &= -de(-CFD, .1, .2) \\ aJD &= -de(-JD, .025, .025) \end{aligned}$$

For a debt issue at level $F$ the funds raised must cover the cost of such an issue and we require that

$$M^* + A(0) - L(0) + aJD - bJD + aD - bD \leq bD + bJD. \tag{3}$$

We first note that if the equity and debt markets had the same cone of acceptable risks then the bid and ask prices for debt $bD, aD$ and equity with debt, $bJD, aJD$ add up to the corresponding all equity prices $bJ, aJ$. There are no advantages to issuing debt when evaluated at the bid or the ask. In general as the bid prices are computed as an infimum over expectations taken with respect to a set of measures and ask prices are supremums we have that

$$\begin{aligned} bD + bJD &\leq bJ \\ aD + aJD &\geq aJ \end{aligned}$$

When the cones are the same in both the debt and equity markets we may observe the required equalities as follows. Let $X = CFD$ and $Y = JD$. The distribution function of $J = X + Y$ for values $a < F$ is just $F_Y(a)$. Hence we note that for $a < F$,

$$F_J(a) = F_Y(a).$$

For values $a > F$ the probability that

$$\begin{aligned} P(V &= X + Y < a) = P(X < a - F) \\ &= F_X(a - F) \end{aligned}$$

Hence we have that

$$\begin{aligned} bJ &= \int_0^F ad\Psi(F_Y(a)) + \int_F^\infty ad\Psi\left(F_X(a - F)\right) \\ &= \int_0^F ad\Psi(F_Y(a)) + \int_0^\infty (F + x)d\Psi\left(F_X(x)\right) \\ &= \int_0^F ad\Psi(F_Y(a)) + F\left(1 - \Psi(F_Y(F\_))\right) + \int_0^\infty xd\Psi(F_X(x)) \\ &= bD + bJD. \end{aligned}$$

A similar argument supports the additivity of ask prices under the same cone.

When the cone of the debt market is smaller and the set of supporting measures larger than for the equity market the sum of the bid prices for debt and equity with debt will be lower than the all equity bid price and similarly the sum of the ask prices will be larger than $aJ$. Now the value of a firm is the cost of acquiring the firm or a comparable firm. This value is given by the ask prices and we value all our liabilities, debt and equity at the ask prices. From this perspective there is an advantage to issuing debt and appealing to the higher pricing of liabilities in the debt market with a view to raising the value of the firm. The amount of debt one can issue is however limited by the constraint (3) as eventually the cost of debt rises and one hits the boundary of the constraint with insufficient funds available to cover the reserve costs of debt issue.

We demonstrate that the when the debt market is more conservative than the equity market as represented by a debt market distortion $\widetilde{\Psi}(u)$ that everywhere dominates the equity market distortion $\Psi(u)$ then the firm value measured by the sum of $aD$ plus $aJD$ is increasing in the face value of debt. Hence the optimal debt is defined by equality

$$M^* + A(0) - L(0) + aJD - bJD + aD - bD = bD + bJD. \tag{4}$$

We shall now use here the fact that for positive random variables $X$ with distribution function $G(x)$ the bid and ask prices, $b(X)$, $a(X)$ respectively are

$$\begin{aligned} b(X) &= \int_0^\infty (1 - \Psi(G(x)))\, dx \\ a(X) &= \int_0^\infty \Psi\, (1 - G(x))\, dx. \end{aligned}$$

To observe that firm value increases with the face value of debt offered we note that the ask price of debt for a face value $F$ and a firm value distribution function $H(v)$ and density $h(v)$ is given by

$$aD(F) = \int_0^F \widetilde{\Psi}(1 - H(v))dv$$

while the ask price of equity given debt is

$$aJD(F) = \int_0^\infty \Psi\, (1 - H(F + v))\, dv$$

The derivative of firm value with respect to the face value of debt is

$$\begin{aligned} aD'(F) + aJD'(F) &= \widetilde{\Psi}(1 - H(F)) + \int_0^\infty \Psi'(1 - H(F+v))h(F+v)dv \\ &= \widetilde{\Psi}(1 - H(F)) - \Psi(1 - H(F)) \\ &\geq 0 \end{aligned}$$

where the last inequality follows from the assumption of debt markets being more conservative.

We define by $g(F)$ the funding gap for debt at face value $F$ as

$$g(F) = M^* + A(0) - L(0) + aD(F) + aJD(F) - 2\,(bD(F) + bJD(F))\,.$$

For an all equity firm that is allowed to exist we have that

$$g(0) < 0.$$

We now observe that the derivative of this funding gap is positive as

$$\begin{aligned} g'(F) &= aJD'(F) + aD'(F) - 2\,(bD'(F) + bJD'(F)) \\ &= \widetilde{\Psi}(1 - H(F)) - \Psi(1 - H(F)) - 2\left(1 - \widetilde{\Psi}(H(F)) - (1 - \Psi(H(F))\right) \\ &= \widetilde{\Psi}(1 - H(F)) - \Psi(1 - H(F)) + \widetilde{\Psi}(H(F)) - \Psi(H(F)) \\ &\geq 0. \end{aligned}$$

It follows that if the funding gap is initially positive at zero debt it will grow as debt is introduced and so if a firm cannot exist as an all equity firm, it cannot exist with positive debt. Furthermore, for a firm that is allowed to exist with a negative initial funding gap at zero debt, there will be at most one solution to the equation for a zero funding gap as the derivative is universally positive. We also observe that as $F$ tends to infinity the funding gap tends to

$$g(\infty) = M^* + A(0) - L(0) + \int_0^\infty \widetilde{\Psi}(1 - H(v))dv - 2\int_0^\infty \left(1 - \widetilde{\Psi}\left(H(v)\right)\right) dv.$$

This is the funding gap for financing as an all equity firm in the debt market and we suppose this funding gap is positive for a sufficiently conservative debt distortion $\widetilde{\Psi}$. In such a case there is exactly one solution to the equation for a zero funding gap and we we have a formula for the optimal debt $F^*$ in terms of the distribution function of firm value given by

$$\begin{aligned} & M^* + A(0) - L(0) + \int_0^{F^*} \widetilde{\Psi}(1 - H(v))dv + \int_0^\infty \Psi\left(1 - H(F^* + v)\right) dv \\ = \quad & 2\left[\int_0^{F^*} \left(1 - \widetilde{\Psi}\left(H(v)\right)\right) dv + \int_0^\infty \left(1 - \Psi\left(H(F^* + v)\right)\right) dv\right]. \end{aligned}$$

In our example the highest level of debt that can be funded is a face value of 23. At this level the ask price of the debt security is 14.3752 and the value of equity with debt at this level is 26.2107. The increase in the value of the firm is 1.5035.

# 6 Balance Sheet Construction

We now present the construction of the balance sheet to be reported. We begin with the complete markets balance sheet with all items valued under the single risk neutral measure and the value of the tax payer put recognized as an asset of the firm. In this case we have for the assets, $M + A_0 + P$ where $P$ is the value of the put option. On the liability side we have $L_0$ and the value of the all equity firm $J$ or equivalently in the presence of debt we write the value of debt $D$ and the value of equity with debt $JD$. The balance sheet reflects the equality

$$M + A_0 + P = L_0 + D + JD. \tag{5}$$

We next recognize that in our incomplete markets the debt and equity are carried on the books at the ask price for their respective markets and their cones. These are the cones $\mathcal{A}_D$, $\mathcal{A}_S$. We have to raise the extra funds in reserves and this is accomplished by adding the difference between the mark and the complete market value to both sides of equation (5) to get the equality

$$M + aD - D + aJD - JD + A_0 + P = L_0 + aD + aJD. \tag{6}$$

However, both the debt and equity markets deliver their bid prices instead of the complete markets value. We thus add the difference to cash reserves for the assets and to equity for the liabilities. This yields

$$M + aD - bD + aJD - bJD + A_0 + P = L_0 + aD + aJD + D - bD + JD - bJD.$$

We have already ensured that the funds raised are sufficient as per equation (3).

This balance sheet reports the equity value, $JR$ at

$$JR = aJD + D - bD + JD - bJD$$

that includes the liability valued at the ask plus the shortfall associated with raising funds in a conic market. The value of equity plus debt equals net assets plus cash reserves made up of externally required capital plus reserves associated with the cost of financing, and the value of the tax payer put.

We now make a final adjustment to the value of the taxpayer put. Currently it stands as valued at the base risk neutral measure. We illustrated earlier in Section 4, a value taken at the bid price for the equity market cone $\mathcal{A}_S$. However, this put option is not an explicitly traded contract in the equity market. It comes into existence when the firm is created by its owner managers who then organize the financing structure. The value one may realize for it may then have to come from the managerial market where the cones are narrower and the bid price in this market could be lower. If we mark this put option value down to its managerial bid price then we reduce the reported equity down by the mark down in this put option value. This value is given in our example by 3.8490. The complete markets value was 10.7166. The reported equity is now

$$aJD + D - bD + JD - bJD - (P - bP)$$

where $P$ is the complete markets put value and $bP$ is the bid price in the managerial market. On the assets side we have

$$M + aD - bD + aJD - bJD + A_0 + bP.$$

The rest of the liability side includes

$$L_0 + aD.$$

In our example the final balance sheet is as presented in Table 1 with zero coupon debt issued at a face value of 23 for a five year maturity.

TABLE 1

| Assets | | Liabilities | |
|---|---|---|---|
| Required Capital | 17.2400 | Risky Liabilities | 90 |
| Cost of Debt | 3.6541 | Debt | 14.3752 |
| Cost of Equity | 2.9811 | Equity | 23.3490 |
| Risky Assets | 100 | | |
| Taxpayer Put | 3.8490 | | |
| Total | 127.7242 | | 127.7242 |

# 7 Gaussian Balance Sheet Models

We investigate the relationships between the risk characteristics of the balance sheet and seven outputs of interest in the corporate balance sheet, for firms that are allowed to exist on account of being able to raise enough capital in the equity markets. These outputs are the level of required capital ($RC$) as determined by the taxpayer cone. The cost of debt in the debt market ($CD$), the cost of equity post debt issue in the equity market ($CJD$), the value of the taxpayer put as valued in the managerial market ($bP$), the face value of debt issued ($F$) and the level of debt ($aD$) and reported equity ($JR$) as they appear in the final balance sheet.

We investigate these matters here for firms with a Gaussian model underlying the randomness in assets and liabilities. The initial asset level is set at 100. For the initial level of liabilities we take 5 settings of 10, 25, 50, 75, and 90. We allow for three level of the interest rate at 2.5%, 5%, and 10%. The maturity of the debt has three levels of 5, 10 and 15 years. For the asset and liability volatilities we take three level each of .15, .3, and .6. The correlations are set 7 levels of $-.75$, $-.5$, $-.25$, 0, .25, .5, and .75. In all we have $5 * 3 * 3 * 3 * 3 * 7 = 2835$ potential Gaussian balance sheets.

Apart from the firm's risk characteristics we have to specify the structure of the taxpayer, debt market, equity market and managerial market cones of market acceptable risks. These are given by the level of $\lambda$, and $\gamma$, the coefficients for loss aversion and absence of gain enticement in the respective markets. For our first cone the values of $\lambda$ in the four markets are .75, .1, .025, and .5 respectively for the taxpayer, debt, equity and managerial markets. The corresponding values for $\gamma$ are .75, .2, .025, and .25.

Our second set of cones just opens up the taxpayer cone to levels of $\lambda, \gamma$ set at .5 instead of .75. Finally in our third cone we set the taxpayer cone back to the level of .75 for $\lambda, \gamma$ and narrow the debt market cone to $\lambda = .15$ and $\gamma = .3$.

For our first set of cones, and for each of our 2835 potential balance sheets we first simulate 100000 scenarios for the random assets and liability levels at year end. The required capital is then determined in accordance with equation (1). We then evaluate whether the firm can raise sufficient capital to actually come into existence as defined by equation (2). Unlike the Mertonian world, not all firms are allowed to exist. For our first cone we found that 1536 out of 2835 firms are allowed to exist. For a more generous taxpayer cone with $\lambda, \gamma$ at .5, the number of firms allowed to exist goes up marginally by an additional 36 firms to 1572 firms. We therefore concentrate attention on the first and third of our sets of market cones, that differ in the size of the debt market cone with the third being narrower. We find that the presence of risky liabilities helps the existence of firms as it reduces the need for funding from equity markets and helps meet the required constraint (2). For firms with the level risky liabilites at 90 only 9 failed to meet the criterion for existence. For the lower levels of risky liabilities at 75, 50, 25 and 10 the numbers that failed to exist were 18, 207, 498, and 567 respectively. Our further analysis is restricted to the 1536 firms that come into existence and we report on the results using the first and third set of market cones that we shall refer to as the base debt cone and the narrow debt cone.

For these existing firms we determine the face value of debt ($F$) issued at the maturity in question in accordance with equation (3). We may then determine the cost of debt ($CD$), the cost of equity post debt ($CJD$), the managerial bid price for the taxpayer put option ($bP$), the level at which debt is marked ($aD$), and the level of reported equity ($JR$). For each of these six items plus the required capital ($RC$) we analyse by regression the effects of liability levels ($L0$), the effects of interest rates ($r$), the maturity ($T$), the asset volatility ($\sigma_A$), the liability volatility ($\sigma_L$) and the level of correlation ($\rho$). The levels of $L0$, $r$, $T$, $\sigma_A$, $\sigma_L$ and $\rho$ are proxied by dummy variables to give six sets of explanatory variables with respectively $5, 3, 3, 3, 3$ and 7 variables. There were not enough surviving firms at $L0 = 10$ and so we used 4 dummy variables in the regressions on the liability levels for the levels 25, 50, 75 and 90. In each case we regressed all 7 variables of interest $RC$, $CD$, $CJD$, $bP$, $F$, $aD$ and $JR$ on dummy variables for the levels of $L0$, $r$, $T$, $\sigma_A$, $\sigma_L$ and $\rho$. The result of these regressions consists of 6 matrices of coefficients of dimension $4, 3, 3, 3, 3$ and 7 by 7. The six matrices are in duplicate, one for the base debt cone, and the other for the narrow debt cone. The columns for $RC$ and $bP$ are the same for the base debt cone and the narrow debt cone as these values are independent of the debt market cone. The results are presented in Tables 2 and 3 for the base debt cone and the narrow debt cone respectively. We comment on the relationship of each of the seven independent variables on the set of six regressors in turn.

The required capital ($RC$) rises with the level of risky liabilities, and is independent of rates and maturity. Furthermore, the level of capital required rises with asset and liability volatility and decreases with an increase in the correlation.

The cost of debt rises with the level of risky liabilities, is independent of rates, and rises with maturity. The cost of debt decreases with asset volatility and rises with the volatility of the liabilities. It is invariant to correlation.

The cost of equity post debt also rises with the level of risky liabilities, is insensitive to rates and rises with maturity. It rises with asset volatility and declines somewhat with the volatility of liabilities. It also falls somewhat with correlation.

The value of the taxpayer put rises with the level of risky liabilities and maturity and is insensitive to rates. It rises with asset volatilities, falls with liability volatility and correlation.

The face value of debt issued rises with the level of risky liabilities, and is positively related to rates and maturity. It falls with asset volatilities and rises with liability volatility and correlation.

The marked value of debt however, rises with the level of risky liabilities, is insensitive to rates, and rises with maturity. It falls with asset volatility, rises with liability volatility and falls with correlation.

The level of reported equity falls with the level of risky liabilities, is insensitive to rates, and falls with maturity. It is positively related to asset volatility, negatively related to liability volatility and decreases with correlation.

The face values of debt issued are uniformly substantially reduced when the debt cone is narrowed.

## 8 Conclusion

Markets are modeled as passively accepting a convex cone of cash flows. These cones contain the set of nonnegative cash flows as they are acceptable to all. Different markets are conceptualized as accepting different cones and conditions are established to exclude the possibility of arbitrage between markets. Operationally these cones are defined by positive expactation under a concave distortion of the distribution function of the cash flow delivered to market. Different cones are then constructed using different distortions. A two parameter family of distortions is introduced that calibrates the level of loss aversion in markets and the level of the absence of gain enticement.

Firms or corporations are seen as accessing risky assets and risky liabilities where the latter may dominate the former and capital requirements are set by taxpayers via their regulatory bodies to ensure that sufficient capital is put at stake by the creators of a firm to make the residual risk of excess loss acceptable to the taxpayer cone that reflects the highest level of loss aversion and the highest level of absence of gain enticement. Firms approach equity markets that have lowest level of risk aversion and the highest level of gain enticement and can come into existence if they can raise sufficient equity capital.

Firms that are allowed to exist approach debt markets using securities particularly attractive to such markets to generate favorable funding opportunities. Debt is however costly as it is marked at the ask price of the debt market though the securities issued raise just the bid price. The difference is the cost of debt and this rises as the debt level is increased and sets a limit to the amount of debt a firm may issue. This constraint on covering the cost of debt coupled with the clientele effects of debt markets determines the optimal level of debt.

Firms with lognormally distributed and correlated assets and liabilities are analysed for their required capital, their optimal debt levels, the value of the option to put losses back to the taxpayer, the costs of debt and equity, and the level of finally reported equity in the balance sheet. The relationship between these entities and the risk characteristics of a firm are analysed and reported in detail.

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TABLE 2

| | | Base Debt Cone | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | RC | CD | CJD | bP | F | aD | JR |
| | 25 | 6.14E-14 | 3.030026 | 5.952812 | 2.576113 | 74.85895 | 29.75127 | 56.80768 |
| L0 | 50 | 7.130793 | 6.24139 | 9.091613 | 6.941819 | 84.24166 | 28.24098 | 51.16464 |
| | 75 | 29.37849 | 10.16274 | 8.46755 | 8.643333 | 168.5517 | 42.84339 | 38.80873 |
| | 90 | 49.64672 | 12.79738 | 8.158569 | 9.38603 | 251.7783 | 54.97642 | 35.01227 |
| | | | | | | | | |
| | | RC | CD | CJD | bP | F | aD | JR |
| | 0.025 | 30.20751 | 9.880376 | 8.388601 | 8.241798 | 105.5077 | 43.24053 | 41.13401 |
| rates | 0.05 | 30.20751 | 9.880376 | 8.388601 | 8.241798 | 145.7898 | 43.24053 | 41.13401 |
| | 0.1 | 30.20751 | 9.880368 | 8.388602 | 8.241798 | 273.1544 | 43.24052 | 41.13401 |
| | | | | | | | | |
| | | RC | CD | CJD | bP | F | aD | JR |
| | 5 | 32.93585 | 5.877256 | 6.363468 | 5.883728 | 51.74522 | 31.21331 | 45.17166 |
| maturity | 10 | 28.99678 | 10.16005 | 8.528502 | 8.647565 | 133.7709 | 44.41079 | 40.69988 |
| | 15 | 29.07139 | 13.06092 | 9.99921 | 9.871599 | 322.8031 | 52.46268 | 38.07976 |
| | | | | | | | | |
| | | RC | CD | CJD | bP | F | aD | JR |
| | 0.15 | 21.35833 | 12.86909 | 2.484566 | 6.035251 | 290.2499 | 58.84556 | 13.67822 |
| Asset Volatility | 0.3 | 26.93009 | 11.67511 | 6.25841 | 7.897233 | 156.2029 | 46.39542 | 33.96543 |
| | 0.6 | 43.86292 | 4.506576 | 17.43674 | 11.12326 | 64.65967 | 22.06708 | 80.17887 |
| | | | | | | | | |
| | | RC | CD | CJD | bP | F | aD | JR |
| | 0.15 | 15.66517 | 4.382674 | 9.458356 | 9.460592 | 51.80309 | 12.60448 | 48.98469 |
| Liability Volatility | 0.3 | 22.55852 | 7.57368 | 8.426037 | 8.243515 | 90.8424 | 27.18165 | 44.89788 |
| | 0.6 | 46.23984 | 15.48354 | 7.620295 | 7.398485 | 325.5177 | 76.97238 | 32.76495 |
| | | | | | | | | |
| | | RC | CD | CJD | bP | F | aD | JR |
| | -0.75 | 37.73271 | 10.77034 | 9.432175 | 9.417795 | 145.4134 | 46.6149 | 51.09956 |
| | -0.5 | 35.1163 | 10.43043 | 9.314972 | 9.139587 | 144.135 | 45.04822 | 49.07502 |
| | -0.25 | 32.55553 | 10.10414 | 9.173598 | 8.85425 | 154.7413 | 43.52417 | 46.47585 |
| correlation | 0 | 29.96487 | 9.974582 | 8.785176 | 8.418177 | 175.4633 | 43.00403 | 42.32059 |
| | 0.25 | 27.64379 | 9.510025 | 8.183466 | 7.794504 | 179.862 | 41.41846 | 37.69924 |
| | 0.5 | 24.37293 | 9.519097 | 6.89682 | 7.01605 | 238.3385 | 42.25277 | 29.50374 |
| | 0.75 | 18.76015 | 8.206099 | 5.777023 | 6.0304 | 213.7006 | 38.99181 | 23.02747 |

TABLE 3

Narrow Debt Cone

| | | RC | CD | CJD | bP | F | aD | JR |
|---|---|---|---|---|---|---|---|---|
| | 25 | 6.14E-14 | 2.753306 | 6.153449 | 2.576113 | 54.84408 | 22.3922 | 64.09066 |
| L0 | 50 | 7.130793 | 5.773739 | 9.488208 | 6.941819 | 56.53911 | 21.92556 | 57.409 |
| | 75 | 29.37849 | 9.473637 | 9.107141 | 8.643333 | 91.94996 | 34.29289 | 47.30971 |
| | 90 | 49.64672 | 12.01175 | 8.955683 | 9.38603 | 119.7148 | 44.66665 | 45.33353 |

| | | RC | CD | CJD | bP | F | aD | JR |
|---|---|---|---|---|---|---|---|---|
| | 0.025 | 30.20751 | 9.226633 | 9.008747 | 8.241798 | 57.02694 | 34.62829 | 49.71266 |
| rates | 0.05 | 30.20751 | 9.226633 | 9.008747 | 8.241798 | 76.77493 | 34.62829 | 49.71266 |
| | 0.1 | 30.20751 | 9.226633 | 9.008747 | 8.241798 | 142.4085 | 34.62829 | 49.71266 |

| | | RC | CD | CJD | bP | F | aD | JR |
|---|---|---|---|---|---|---|---|---|
| | 5 | 32.93585 | 5.415004 | 6.739304 | 5.883728 | 35.46411 | 22.96871 | 53.32985 |
| maturity | 10 | 28.99678 | 9.447175 | 9.169014 | 8.647565 | 81.23796 | 35.62115 | 49.41716 |
| | 15 | 29.07139 | 12.30131 | 10.81013 | 9.871599 | 151.9978 | 43.71176 | 46.882 |

| | | RC | CD | CJD | bP | F | aD | JR |
|---|---|---|---|---|---|---|---|---|
| | 0.15 | 21.35833 | 12.12034 | 3.28216 | 6.035251 | 125.9105 | 47.96192 | 24.6107 |
| Asset Volat | 0.3 | 26.93009 | 10.84216 | 6.996816 | 7.897233 | 101.3787 | 36.71268 | 43.55361 |
| | 0.6 | 43.86292 | 4.158972 | 17.72487 | 11.12326 | 43.42186 | 17.2138 | 84.97267 |

| | | RC | CD | CJD | bP | F | aD | JR |
|---|---|---|---|---|---|---|---|---|
| | 0.15 | 15.66517 | 4.112951 | 9.730034 | 9.460592 | 32.76709 | 9.289306 | 52.30182 |
| Liability Vo | 0.3 | 22.55852 | 7.048937 | 8.901856 | 8.243515 | 58.54062 | 20.37884 | 51.65177 |
| | 0.6 | 46.23984 | 14.46356 | 8.594119 | 7.398485 | 159.2784 | 63.28473 | 46.40644 |

| | | RC | CD | CJD | bP | F | aD | JR |
|---|---|---|---|---|---|---|---|---|
| | -0.75 | 37.73271 | 9.971102 | 10.11633 | 9.417795 | 94.72321 | 36.24493 | 61.35445 |
| | -0.5 | 35.1163 | 9.655462 | 9.978202 | 9.139587 | 92.45607 | 35.27866 | 58.73284 |
| | -0.25 | 32.55553 | 9.351659 | 9.817105 | 8.85425 | 90.51236 | 34.39299 | 55.49806 |
| correlation | 0 | 29.96487 | 9.264121 | 9.417428 | 8.418177 | 91.23544 | 34.44038 | 50.80603 |
| | 0.25 | 27.64379 | 8.851562 | 8.784941 | 7.794504 | 89.09065 | 33.48031 | 45.5804 |
| | 0.5 | 24.37293 | 8.970907 | 7.487021 | 7.01605 | 94.01355 | 34.82043 | 36.97809 |
| | 0.75 | 18.76015 | 8.0449 | 6.248658 | 6.0304 | 92.56294 | 33.14361 | 29.1861 |