Extending Dynamic Convex Risk Measures From Discrete Time to Continuous Time: a Convergence Approach*

Mitja Stadje†
Eurandom, Eindhoven Technical University
P.O. Box 513, 5600 MB Eindhoven, The Netherlands

Abstract

We present an approach for the transition from convex risk measures in discrete time to their counterparts in continuous time. The aim of this paper is to show that a large class of convex risk measures in continuous time can be obtained as limits of discrete time-consistent convex risk measures. The discrete-time risk measures are constructed from properly rescaled (‘tilted’) one-period convex risk measures, using a \( d \)-dimensional random walk converging to a Brownian motion. Under suitable conditions (covering many standard one-period risk measures) we obtain convergence of the discrete risk measures to the solution of a BSDE, defining a convex risk measure in continuous time, whose driver can then be viewed as the continuous-time analogue of the discrete ‘driver’ characterizing the one-period risk. We derive the limiting drivers for the semi-deviation risk measure, Value at Risk, Average Value at Risk, and the Gini risk measure in closed form.

Key words: Dynamic convex risk measures; time-consistency; \( g \)-expectation; discretization; convergence; special drivers

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†Corresponding Adress: Eurandom, Eindhoven Technical University, P.O. Box 513, 5600 MB Eindhoven, The Netherlands, Tel.: +31 40 2478109, Fax: +31 40 2478190, Email: Stadje@tue.nl