A dynamic control strategy for multi-item production systems with backlog; part 2

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May 3, 2010

Abstract

This paper is the second of two companion papers dealing with a multi-item production system. Starting from a good fixed cycle policy, using the one-step improvement approach, a dynamic policy is constructed. Numerical results are presented that indicate that the performance is quite good compared to other policies, such as exhaustive base-stock control.

Keywords: Markov decision problem, multi-item production system, one step improvement

1 Introduction

As in part 1, we consider a production system in which one machine produces N different items, but only one at a time. Demand occurs according to independently distributed (compound) Poisson processes. For every item, the machine can make to stock and unsatisfied demand is backlogged.

The system has a resemblance with an intersection controlled by traffic lights. One very often sees a fixed cyclic scheme in which (combinations of) traffic flows receive green, not only in a fixed order but also during a fixed time. In case the intersection is lightly loaded, for instance late in the evening, a dynamic control resembling FCFS is found. For heavily loaded intersections, the dynamic control of a traffic light by a two-step approach is studied in [8]. In the first step an (in some sense) good completely fixed cyclic control scheme is constructed and in the second step a policy is constructed that takes decisions on producing, idling or switching based on an evaluation of the 'relative urgencies' of the different traffic streams in the fixed cycle scheme.

This idea of a two-step approach goes back to Norman [13] and has been used successfully for multi-dimensional problems from different areas, see for example the work of Wijngaard [20] on the control of production and inventory systems and the work of Sassen et al. [17] on telecommunication systems.

In this paper, we use the results from part 1 for a fixed cycle production scheme, including the rules about whether to use a reserved production slot for production or for idling. This fixed cycle policy is used for a two-step approach in multi-item production systems.

The paper is structured as follows. First, we introduce a cost function and, referring to part 1, start with a good stable fixed cycle policy. This fixed cycle scheme is used as a basis for the one step improvement approach. This approach and its resulting dynamic policy are discussed in Sections 3 and 4. In Section 5 a conclusion and some suggestions for further research are given.

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2 Model and notation

The objective function used in this paper is the linear cost function:

$$C(\cdot) = \sum_{i=1}^{N} c_{i,I}\mathbb{E}(I_i) + c_{i,B}\mathbb{E}(B_i), \tag{1}$$

where $\mathbb{E}(I_i)$ and $\mathbb{E}(B_i)$ denote the average number of products on stock and the mean backlog for type i, respectively.
The demand process of every item i is (compound) Poisson with parameter (mean) λ_i. It is assumed that the total load of the system, $\rho := \sum_{i=1}^{N} \lambda_i T_i < 1$. Next we compute a good, stable, fixed cycle according to Algorithm 2, see part 1, with (constant) order-up-to levels $S_i,\ i = 1, \ldots, N$.

This fixed cycle policy reserves a production period of fixed length for every item i, consisting of g_i production times, each with an (average) length of T_i. The order of production is fixed and the decision to produce or not to produce a product is based on the base-stock level S_i. If the stock level of item i reaches this level just before a production slot of type i, the system idles during the next slot. With this assumption the system becomes a combination of N independent queues. Every queue is then analyzed as a single-item production system with periodic vacations, see part 1.

3 One step improvement approach

In this section we show how this fixed cycle can serve as a basis for the one step improvement approach. The improvement step is part of Howard's [9] policy iteration algorithm. To execute it, the relative values (or bias terms) have to be known. The calculation of these values is usually too complex, because of the multi-dimensionality of the system. However, for the fixed cycle policy these relative values can be computed per product type, as the system simplifies to N independent product flows. For each product type one just has a one-dimensional periodic Markov chain. The number of states is still infinite, but this problem can be solved by introducing (large) maximal stock-out levels M_i, $i = 1, \ldots, N$. (These maximal stock out values M_i are chosen such that this value is hardly ever reached. The relative values for stock-out levels can then be approximated, for instance by extrapolation.) Each of the N periodic chains can then be analyzed numerically using successive approximations to compute the $n-$period costs for type i given initial state k_i, with k_i the stock-out.

In the improvement step the decision one looks for is 'what to do next', but this can be seen as looking for the best *time jump* within the fixed cycle, assuming that after this jump the fixed cycle strategy is followed again.

Therefore, the relative values are calculated for every time slot within the fixed cycle. For time slot n, the relative value for state $(k_1, \ldots, k_N)$ is denoted by $v(n, k_1, \ldots, k_N)$ and it is just the sum of the N relative values, one for each item:

$$v(n, k_1, \ldots, k_N) = \sum_{i=1}^{N} v_i(n, k_i).$$

In order to compute $v_i(n, k_i)$, we need to be aware of the fact that the fixed cycle strategy is periodic. Also note that these relative values $v_i(n, k_i)$ only have to tell the difference in expected costs between starting in one slot or starting in another slot.

Determination of the relative values

In the calculation of the relative values, it is assumed that arriving demand that finds a stock-out of M_i is lost.

For a non-periodic Markov chain the $t-$period costs w_t satisfy

$$w_t = tg + v + o(1) \quad (t \to \infty) \ ,$$

with t the number of periods, g the *gain* (in our case average costs) and v the relative value vector. In the policy improvement step one does not need the exact value of v, any vector $v + \alpha$ with α an arbitrary constant vector will do. For a periodic Markov chain with cycle time c one can use as relative value for state k,

$$\frac{1}{c} \sum_{m=tc+1}^{(t+1)c} w_m(k),$$

provided t is sufficiently large.

For product type i, denote the state of the system as (n, k_i), with n the slot within the fixed cycle and k_i the number of items short compared to S_i. For the fixed cycle strategy, and noting that the slots have different durations, we get the following (approximate) relative value for item i and state (n, k_i):

$$\tilde{v}_i(n, k_i) = \frac{1}{\sum_{m=1}^{c} T_m} \sum_{m=tc+1}^{(t+1)c} T_{m-tc} \ w_{i,m}(n, k_i),$$

with t sufficiently large. The overall relative value $v(n, k_1, \ldots, k_N)$ for time slot n and state $(k_1, \ldots, k_N)$ is then approximated by the sum of the approximate relative values for the N products and pairs (n, k_j), $j = 1, \ldots, N$:

$$\tilde{v}(n, k_1, \ldots, k_N) = \sum_{i=1}^{N} \tilde{v}_i(n, k_i).$$

4 Results

In this section, we give an overview of the results of the fixed cycle policy and the one step improvement approach and compare them with the results for a number of other policies. For a policy Γ, the following notation is used for the optimal decision levels and expected costs:

$\boldsymbol{S_\Gamma}$ **:**	Optimal decision levels if policy Γ is used.
$\boldsymbol{c_\Gamma}$ **:**	Expected costs per time unit if policy Γ is used.

The different policies will be denoted by:

FC:	Fixed cycle policy,
1SI:	One step improvement policy,
EXH:	Exhaustive policy,
G:	Gated policy,
EXH*:	Exhaustive policy adjusted: If, just before a switch, the stock level of the next item is equal to the decision level, this item is skipped.

In the examples studied in this section, the holding costs per time unit are all equal to 1 and backlogging costs equal 50. Furthermore, all production and set-up times are deterministic and equal to 1. The arrival processes at the different stock points are either Poisson with intensity λ

or compound Poisson. In the case of a compound Poisson arrival process, batches with customers arrive with intensity $\lambda/2$ and each batch is of size 4 with probability 1/3 and of size 1 with probability 2/3. The average batch size is then equal to 2, so the average number of arriving customers per time unit equals λ.

The dynamic policy is compared with a gated, exhaustive and an adjusted exhaustive base-stock policy. The exhaustive and gated base-stock policies set base-stock levels for all items and produce the different items in a fixed, cyclic order. The server switches if the base-stock level of the item currently set up is reached or if the shortfall seen upon arrival is produced, otherwise it produces another unit of this type. In this way, the server never idles and decisions only depend on the stock level of the item currently set-up. The adjusted exhaustive base-stock policy applies the same rules as the exhaustive base-stock policy, but skips an item if, just before the switch to this item, the stock level of this type is equal to the base-stock level of this type. If all stock levels equal their base-stock levels, the server switches to the next item.

The gated, exhaustive and adjusted exhaustive base-stock policies are analysed as follows. First, all base-stock levels are set to zero, then a simulation is performed to find the limiting distribution of the shortfall levels. These distributions are not influenced by any of the N base-stock levels, so a newsvendor equation can be used to obtain the optimal base-stock levels.

In realistic settings for multi-item production systems, about 80 percent of the demand is for only 20 percent of the product types. This is also the case in the paper of Winands et al. [21], where 12.5% of the product types is responsible for 67% of the demand. Therefore, the settings for Tables 1 and 2 are similar: A low percentage (20%) of the product types is responsible for a high percentage ($80 - 90\%$) of the demand.

Table 1 shows results for a 5-item production system with Poisson arrivals, where the first item is responsible for $80-90\%$ of the total demand. The total load on the system is 0.7, which is the sum of the arrival intensities of the demand processes. Table 2 shows results for a 5-item production system with compound Poisson arrivals, all other settings are the same as in Table 1.

It is seen that in 4 out of 6 cases $1SI$ performs better than $EXH*$. Particularly if the demand variation is larger (Table 2) $1SI$ clearly outperforms $EXH*$. We want to note that it is very important to adjust the order-up-to levels when moving from the fixed cycle system to the dynamic system: the base-stock levels are considerably decreased and the difference in costs is huge (around 70 percent).

λ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
$(0.56, 0.03, 0.05, 0.02, 004)$	65.79	21.85	17.79	15.93	15.16
$(0.60, 0.02, 0.04, 0.01, 003)$	29.69	29.48	16.08	13.90	15.36
$(0.63, 0.02, 0.015, 0.025, 0.01)$	43.24	20.22	14.85	12.72	12.76

Table 1: Poisson demand

λ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
$(0.56, 0.03, 0.05, 0.02, 004)$	85.24	45.53	41.16	38.18	35.92
$(0.60, 0.02, 0.04, 0.01, 003)$	84.39	45.64	38.68	36.26	33.51
$(0.63, 0.02, 0.015, 0.025, 0.01)$	79.26	44.92	36.53	34.51	32.07

Table 2: Compound Poisson demand

N	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
5	47.51	25.69	24.10	23.84	28.26
6	58.03	29.84	28.27	27.98	32.32
7	70.04	33.78	32.45	32.14	35.67

Table 3: Poisson demand, $\lambda = (0.7/N, \ldots, 0.7/N)$

N	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
5	128.85	53.62	52.02	50.00	47.73
6	161.36	61.66	59.34	57.08	53.88
7	199.78	69.02	67.00	63.37	59.75

Table 4: Compound Poisson demand, $\lambda = (0.7/N, \ldots, 0.7/N)$

Number of product types

Table 4 shows results for completely symmetric production systems. The load in the system is equal to 0.7, with $N = 5, 6, 7$ so the average number of arriving customers is $0.7/N$ per time unit for each product flow. Although these settings are not realistic, the results are illustrative for the performance of the (one step and) two step improvement policy. It is seen that even if the system is symmetric, in all cases $1SI$ outperforms EXH^*. Moreover, the performance of $1SI$ is better if N is larger. It further seems that for systems with a large demand variation, the dynamic strategy outperforms the adjusted exhaustive strategy.

Production order

The relative value function of the fixed cycle policy depends on the order of production. This effect is illustrated in Figure 1 for an empty 3-item production system with Poisson demand processes and demand intensities $\lambda = (0.25, 0.15, 0.10)$. Holding costs are all equal to 1 and backlogging costs are 50 for all items. The production periods of the fixed cycle are equal to $g = (6, 4, 3)$ and the base-stock levels are $S = (6, 5, 4)$. In Tables 5 up to 8, results are shown for production systems with Poisson demand (Tables 5 and 6) and compound Poisson demand (Tables 7 and 8). The relative value function of the equivalent system with $\lambda = (0.25, 0.10, 0.15)$ is shown in Figure 2.

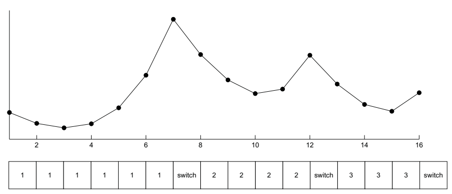

Figure 1: Relative value function for an empty 3-item production system, $\lambda = (0.25, 0.15, 0.10)$

For 6-item productions systems, results are shown in Tables 5 up to 12. The order of production does have an effect on the performance of the one step improvement policy, but it seems quite random which of the two studied production orders is best. Therefore, it is difficult to give a good intuition for the best order of production.

ρ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
0.70	45.67	23.48	22.43	21.83	26.83
0.75	60.40	28.11	26.88	26.10	28.27
0.80	93.90	34.35	32.78	32.60	32.34

Table 5: Poisson demand, $\lambda = (0.25\rho, 0.15\rho 0.10\rho, 0.25\rho, 0.15\rho, 0.10\rho), b = 20$

ρ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
0.70	45.67	23.49	22.43	21.83	26.86
0.75	60.40	28.11	26.88	26.08	28.27
0.80	93.90	34.35	32.78	32.59	32.38

Table 6: Poisson demand, $\lambda = (0.25\rho, 0.25\rho 0.15\rho, 0.15\rho, 0.10\rho, 0.10\rho), b = 20$

ρ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
0.70	57.01	29.30	28.23	27.20	30.74
0.75	76.74	34.57	33.23	32.74	32.92
0.80	123.37	42.49	40.69	40.49	38.78

Table 7: Poisson demand, $\lambda = (0.25\rho, 0.15\rho 0.10\rho, 0.25\rho, 0.15\rho, 0.10\rho), b = 50$

ρ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
0.70	57.01	29.30	28.23	27.21	30.90
0.75	76.74	34.58	33.25	32.73	32.96
0.80	123.37	42.51	40.65	40.47	38.96

Table 8: Poisson demand, $\lambda = (0.25\rho, 0.25\rho 0.15\rho, 0.15\rho, 0.10\rho, 0.10\rho), b = 50$

ρ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
0.70	124.23	47.80	45.72	42.78	42.80
0.75	173.30	55.84	53.74	51.20	50.61
0.80	284.24	67.14	65.12	63.36	64.42

Table 9: Compound Poisson demand, $\lambda = (0.25\rho, 0.15\rho 0.10\rho, 0.25\rho, 0.15\rho, 0.10\rho), b = 20$

ρ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
0.70	124.23	47.79	45.73	42.78	42.85
0.75	173.30	55.83	53.75	51.23	50.56
0.80	284.24	67.11	65.10	63.39	67.37

Table 10: Compound Poisson demand, $\lambda = (0.25\rho, 0.25\rho 0.15\rho, 0.15\rho, 0.10\rho, 0.10\rho), b = 20$

ρ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
0.70	159.74	61.11	59.37	55.90	52.61
0.75	224.64	70.91	68.81	66.38	62.29
0.80	441.63	84.86	83.28	81.43	79.61

Table 11: Compound Poisson demand, $\lambda = (0.25\rho, 0.15\rho 0.10\rho, 0.25\rho, 0.15\rho, 0.10\rho), b = 50$

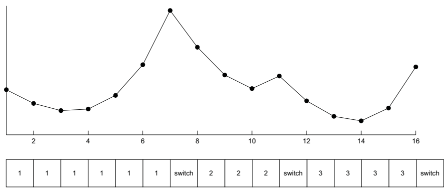

Figure 2: Relative value function for an empty 3-item production system, $\lambda = (0.25, 0.10, 0.15)$

ρ	c_{FC}	c_G	c_{EXH}	c_{EXH*}	c_{1SI}
0.70	159.74	61.12	59.38	55.89	52.54
0.75	224.64	70.87	68.79	66.33	62.09
0.80	441.63	84.84	83.18	81.44	78.76

Table 12: Compound Poisson demand, $\lambda = (0.25\rho, 0.25\rho 0.15\rho, 0.15\rho, 0.10\rho, 0.10\rho), b = 50$

Load, backlogging costs and stochasticity

Although we were not able to get a good intuition for a good production order in the fixed cycle policy, the obtained results in Tables 5 up to 12 give us a lot of information on the performance of the one step improvement policy. The results show that the one step improvement policy performs better if the load on the system is higher, which was also seen from the results in Table 4. Further, an increase of the backlogging costs has a positive effect on the performance of the one step improvement policy. The fact that the new strategy is able to react to sudden changes in demand also leads to good results, because the one step improvement policy performs better if demand is more stochastic, as in Tables 9 up to 12.

5 Conclusion

A multi-item production system is analysed in which demand is backlogged if it can not be satisfied from stock. For every type, holding and backlogging costs are considered and in order to minimize the total expected costs per time unit, we analysed a one step improvement policy. This one step improvement policy is constructed by starting with a good fixed cycle control and then performing one policy iteration of Howard's policy iteration algorithm [9]. After this policy iteration, the base-stock levels are adjusted by estimating the limiting distribution of the shortfall for the new policy and using a newsvendor type equation.

Numerical and simulation results are given to compare the fixed cycle policy, the gated and exhaustive base-stock control policy and the new one step improvement policy. An adjusted exhaustive base-stock control policy is constructed by changing the switching rule in the exhaustive base-stock control policy. The machines still switches to another product type if the stock level of the current product type equals its base-stock level, but skips the next item if at that moment, i.e. just before the switch, the stock level of the next item equals the base-stock level of that item. (In the case that no item has a shortfall, the machine switches to the next item.)

It is shown that the one-step improvement approach leads to a very good dynamic control of the production system, particularly for systems with a large number of product types, systems with a high load, systems with high backlogging costs and systems with stochastic demand.

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