Exact FCFS matching rates for two infinite multi-type sequences

Ivo Adan∗ Gideon Weiss†

June 1, 2010

Abstract

We consider an infinite sequence of items of types \( C = \{c_1, \ldots, c_I\} \), and another infinite sequence of items of types \( S = \{s_1, \ldots, s_J\} \), and a bipartite graph \( G \) of allowable matches between the types. Matching the two sequences on a first come first served basis defines a unique infinite matching between the sequences. For \((c_i, s_j) \in G\) we define the matching rate \( r_{c_i, s_j} \) as the long term fraction of \((c_i, s_j)\) matches in the infinite matching, if it exists. We assume that the types of items in the two sequences are i.i.d. with given probability vectors \( \alpha, \beta \). We describe this system by a Markov chain, obtain conditions for ergodicity, and derive its stationary distribution which is of product form. We show that if the chain is ergodic, then the matching rates exist almost surely, and give a closed form formula to calculate them.

Keywords: Service system; first come first served policy; multi type customers and servers; infinite bipartite matching; infinite bipartite matching rates; Markov chains; product form solution.

2000 Mathematics Subject Classification: Primary 60J10; Secondary 90B22; 68M20.

∗Department of Mathematics and Computer Science, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, the Netherlands, and Department of Quantitative Economics, University of Amsterdam, P.O.Box 19268, 1000 GG Amsterdam, the Netherlands; email iadan@win.tue.nl Research supported in part by the Netherlands Organization for Scientific Research (NWO).

†Department of Statistics, The University of Haifa, Mount Carmel 31905, Israel; email gweiss@stat.haifa.ac.il Research supported in part by Israel Science Foundation Grants 454/05 and 711/09, hospitality of the Newton Institute on Mathematics is gratefully acknowledged.