Model and calibration risks for the Heston model

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Abstract

Parameters of equity pricing models, such as the Heston’s stochastic volatility model, have to be calibrated every day to new market data of European vanilla options by minimizing a particular functional. Hence, the optimal parameter set might turn out to vary significantly on a daily basis, depending on the quality of the initial guess and therefore on the local minima which is reached by the local optimizer method. However, thanks to the emergence of market data for volatility derivatives, practitioners might resort to time series or market quotes to determine some of the model parameters beforehand and perform therefore a calibration on a reduced parameter set. In particular, the spot variance $v_0$ of the Heston model can be inferred beforehand from the spot value of the volatility index whereas the long run variance $\eta$ can be estimated either from the time series of the volatility index or from the VIX option surface.

This paper provides a market-implied estimate of $\eta$ which is inferred from the Put-Call parity for long maturity options on the VIX. We then compare the such obtained mark-to-market estimate with the $\eta$ parameter obtained from the calibration and with the estimate inferred from the VIX time series by using either a moving window or the exponentially weighted moving average technique. Moreover, this paper features a detailed calibration performance study of the Heston model for the two calibration procedures, i.e. the standard calibration on the whole parameter set and the different reduced calibrations on the parameter set $\{\kappa, \lambda, \rho\}$. We also investigate the calibration risk which arises by considering different objective functions and/or different calibration methodologies and price a wide range of exotic options for the different calibration settings. For the numerical study, we consider daily S&P500 and VIX market quotes for a period extending from the 24th of February 2006 until the 31st of October 2009, including therefore the credit crunch.

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