EURANDOM PREPRINT SERIES
2010-028

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The Pricing of Contingent Capital Notes using Conic Finance

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ISSN 1389-2355

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July 7, 2010

Abstract

In this paper we introduce a fundamental model under which we will price contingent capital notes using conic finance techniques. The model is based on more realistic balance-sheet models recognizing the fact that asset and liabilities are both risky and have been treated differently taking into account bid and ask prices in a prudent fashion. The underlying theory makes use of the concept of acceptability and distorted expectations, which we briefly discuss.

We overview some potential funded and unfunded contingent capital notes. We argue that the traditional core tier one ration is maybe not optimal, certainly when taking into account the presence of risky liabilities; we as an alternative introduce triggers based on capital shortfall. The pricing of 7 variations of funded as well as unfunded notes is overviewed. We further investigate the effect of the dilution factor and the grace factor. In an appendix we show conic balance sheets including contingent capital instruments.

1 Introduction

Contingent Capital or contingent core notes (coconuts) has been put to the fore in the discussion of the aftermath of the credit crunch crises. The main idea is that by means of a financial instrument (derivative), one ensures that in times of heavily distressed markets, corporations in trouble automatically are provide with fresh capital. A contingent capital note is by contract triggered when a certain capital related indicator, like for example the core-tier 1 ratio, is falling below a certain level (say 5%). At that point in time for example new capital comes in, in return for equity according to a rule fixed when the contract is struck; some deals don't bring in equity but cancel some outstanding debt. Many variations are possible. The deal is made upfront and one avoids the need of going to the capital markets in such distressed situations, when it is extremely hard, if not impossible, to raise new funds.

Contingent capital has not come out of the blue. The G20 announcement in September stated specifically that they would be examining contingent capital and comparable instruments. In November 2009, Lloyds Banking Group issued a Lower Tier 2 hybrid capital instrument called Enhanced Capital Notes. These include a contingent capital feature and will reportedly convert into ordinary shares if Lloyds' published consolidated core Tier 1 ratio falls below 5%. This instrument will not be included in the regulatory core Tier 1 ratio until a conversion occurs. Conversion price is based on LBGs stock price at the issue date. Mid 2010 also Rabobank has issued a contingent core note.

The is not much literature about these new kind of products. A pioneering paper proposing such securities is by Flannery [6]. In [9], a form of contingent capital that converts from debt to equity if two conditions are met is proposed. The firm's stock price should be at or below a trigger value and the value of a financial institution index should also be at or below a trigger value. Such a structure gives "protection" during a crisis, when all are perfoming badly, but during normal times permits a badly perfoming bank to go bankrupt.

Acharya et al. [1] have suggested that while contingent capital does restore some market discipline, both contingent capital and equity capital may have incentives to take excessive risks at the expense of taxpayer money. Schoutens [10] has pointed out several potential problems that could arise near the trigger level, were one can anticiapte tha volatility will be high. Triggering one bank, can lead to speculation on other banks. Timing of publishing capital ratios should be synchronized. If one bank reveals it is triggered this increases the triggering probability of other financial players dramatically in a systematic crisis due to high correlation. Further, the most straightworward way coconut holders could hedge against potentially being triggered and baring some heavy losses, is shorting the underlying stock. This could bring stock prices even down further and hence the company actually closer to the trigger level; a kind of death-spiral effect could arise. Furthermore, if coconuts are mainly held by the financial players, the triggering of a few will lead to potential losses on the balance sheets of the others, who held these. This could lead to a domino effect of triggering either new coconuts or defaults. Also some critical analysis is made in [7] on the the mechanics of their operation and the market implications. Bond, Goldstein and Prescott [2] have argued that if agents were to use market prices when taking corrective actions, prices will adjust to reflect such a use and may become potentially less revealing.

In this paper we introduce a fundamental model under which we will price contingent capital notes using conic finance techniques. The model is based on more realistic balance-sheet models recognizing the fact that asset and liabilities are both risky and have been treated differently taking into account bid and ask prices in a prudent fashion. We remark in this context that in a classical Mertonian balance sheet models asset are risky but liabilities are not. In reality however liabilities are risky and can be even unbounded (long-short hedge funds or an insurance company is the basic example). Recently, new balance sheet models have been proposed ([8]) where liabilities are assumed to be risky and correlated to the asset dynamics. Furthermore, in the so-called conic finance balance sheet model, the one-price-market idea is abandoned and one assumes that there are bid-ask spreads (related to the liquidity of the market and the risk appetites of the players in these markets). This theory makes use of the concept of acceptability and distorted expectations, which we briefly discuss in Section 3. In section 2, we overview some potential funded and unfunded contingent capital notes. Section 4, first elaborates on the trigger ratio. We argue that the traditional core tier one ration is maybe not optimal, certainly when taking into account the presence of risky liabilities; we as an alternative introduce triggers based on capital shortfall. The pricing of 7 variations of funded as well as unfunded notes is overviewed in the second part of Section 4. Section 5, investigates the effect of the dilution factor and the grace factor. In an appendix we show conic balance sheets including contingent capital instruments.

2 Contingent Capital

In this section we discuss a variety of different forms of contingent capital. The main idea is to provide fresh capital in times of stress. This can be achieved in different ways. In a funded or unfunded form, either by bringing in new capital, new equity, canceling debt or by taking over assets and/or liabilities. We start with overviewing the funded notes, next we'll move to the unfunded case. Funded notes are hybrid kind of debt instrument where an investor buys the note and receives regular coupons until trigger date or maturity which ever is first. In case the trigger is hit, the investor will get instead of the face value, some equity, some assets or liabilities or just only a fraction (or even zero) of the original face value. In case the note matures, without being triggered the investor gets back the face value.

Later on, we elaborate on the pricing of some of these notes in detail. Here we describe the payoff of this notes in a simplified capital structure. We assume that the firm holds some capital and has invested in risky assets and moreover also has some risky liabilities. Denote the firm value by $V = \{V_t, t \geq 0\}$. V_0 is then the initial value of the firm. We have

$$V_t = (M \exp(rt) + A_t - L_t)^+,$$

where M is the capital put aside at time zero; $A = \{A_t, t \geq 0\}$ is the risky asset price process and $L = \{L_t, t \geq 0\}$ is the risky liability price process. The firm typically will have issued some ordinary debt and some equity. In case of contingent or junior debt, we assume that ordinary debt holders have a higher seniority than contingent debt holders, which are more senior than equity holders.

2.1 Funded Coconuts

2.1.1 Funded Equity for Debt Coconut

In case of being triggered, the holder of the *funded equity for debt coconut* gets new equity, but will not receive back the face value of the note (nor any further coupons). Here there is a clear dilution effect, because new equity is created. The Lloyd's deal is a clear example of such structure.

We analyze this case in a zero-coupon-bond (ZCB) structure. We assume ordinary ZCB debt's face value is given by F and contingent capital's (ZCB) face value is F_C. Assume, for the sake of the illustration, that the maturity for both debt forms is T. In case of triggering, we assume that at time zero we have N stocks and in case of being triggered $\tilde{N}$ new stocks are issued.

The cash flow at time T of ordinary bond holders is $\min(V_T, F)$. The ordinary bond holder has the highest seniority. Next comes, the coconutholder and hence the payoff of coconut holder is : $(\min(V_T - F, F_C))^+$ if no conversion took place and equals in case of the conversion $\frac{\tilde{N}}{N+\tilde{N}}(V_T - F)^+$. This is the typical equity payoff with a factor $\frac{\tilde{N}}{N+\tilde{N}}$ taking into account the dilution effect. An equity holder has per stock either the payoff $\frac{1}{N}(V_T - F - F_C)^+$ in case of no conversion of the coconut and $\frac{1}{N+\tilde{N}}(V_T - F)^+$ in case a conversion took place before maturity time T. We give a survey of these payoffs in Table 1. An overview of the main ingredients at time zero and at maturity in case of trigger and no trigger are given in Table 2; here the outstanding debt (with or without being triggered) indicates the maturing face value.

Investor	payoff in case of no trigger	payoff in case of trigger
Debt	$\min(V_T, F)$	$\min(V_T, F)$
Convertible	$(\min(V_T - F, F_C))^+$	$\frac{\tilde{N}}{N+\tilde{N}}(V_T - F)^+$
Equity	$\frac{1}{N}(V_T - F - F_C)^+$	$\frac{1}{N+\tilde{N}}(V_T - F)^+$

Table 1: Payoff - Funded Equity for Debt Coconut

2.1.2 Funded Debt Reduction Coconut

In case of being triggered, the holder of the *funded debt reduction coconut* just looses all or part of its face value, but doesn't receive any equity (nor

	at time zero	at time T in case of no trigger	at time T in case of trigger
Number of stock	N	N	$N + \tilde{N}$
Capital	M	$\exp(rT)M$	$\exp(rT)M$
Risky Assets	A_0	A_T	A_T
Risky Liabilities	L_0	L_T	L_T
Ordinary debt	F	F	F
Contingent debt	F_C	F_C	0

Table 2: Overview - Funded Equity for Debt Coconut

any further coupons). Here there is no dilution effect, because no new equity is created. The Rabobank deal is an example of such structure.

The cash flow at time T of ordinary bond holders is $\min(V_T, F)$. The cash flow of coconut holder is : $(\min(V_T - F, F_C))^+$ if no conversion took place and in case of being triggered the holder receives $(\min(V_T - F, \kappa F_C))^+$, with $0 \leq \kappa < 1$. An equity holder has per stock either the payoff $\frac{1}{N}(V_T - F - F_C)^+$ in case of no conversion of the coconut and $\frac{1}{N}(V_T - F - \kappa F_C)^+$ in case the coconut has been triggered before maturity T.

Investor	payoff in case of no trigger	payoff in case of trigger
Debt	$\min(V_T, F)$	$\min(V_T, F)$
Convertible	$(\min(V_T - F, F_C))^+$	$(\min(V_T - F, \kappa F_C))^+$
Equity	$\frac{1}{N}(V_T - F - F_C)^+$	$\frac{1}{N}(V_T - F - \kappa F_C)^+$

Table 3: Payoff - Funded Debt Reduction Coconut

	at time zero	at time T in case of no trigger	at time T in case of trigger
Number of stock	N	N	N
Capital	M	$\exp(rT)M$	$\exp(rT)M$
Risky Assets	A_0	A_T	A_T
Risky Liabilities	L_0	L_T	L_T
Ordinary debt	F	F	F
Continent debt	F_C	F_C	κF_C

Table 4: Overview - Funded Debt Reduction Coconut

2.1.3 Funded Troubled Asset Reduction Coconut

In case of being triggered, the holder of the *funded troubled asset reduction coconut* gets instead of his face value, a fraction $0 < \kappa \leq 1$ of the assets, (which are assumed to have low value). κ could be for example F_C/A_0. Here there is no dilution effect, because no new equity is created. The coconut outstanding debt is canceled. This note is very similar to the TARP or bad bank construction, where troubled assets are transferred, here to the coconut holder in return for the cancelation of the outstanding debt.

We now have that the firm's value $V_T = (M\exp(rT)+A_T-L_T)^+$, if no conversion took place and equals $V_T = (M\exp(rT)+(1-\kappa)A_T-L_T)^+$ in case of being triggered. The cash flow at time T of ordinary bond holders is $\min(V_T, F)$, with $V_T = M\exp(rT)+A_T-L_T$ in case of no trigger and $V_T = M\exp(rT)+(1-\kappa)A_T-L_T$ if the coconut has been triggered; The cash flow of coconut holder is : $(\min(V_T - F, F_C))^+$ in case of no trigger ($V_T = M\exp(rT)+A_T-L_T$) and κA_T in case of triggering. An equity holder has per stock the payoff $\frac{1}{N}(M\exp(rT)+A_T-L_T-F-F_C)^+$ in case of no trigger and $\frac{1}{N}(M\exp(rT)+(1-\kappa)A_T-L_T-F)^+$. We see here that the trigger affects ordinary debt holders as well as the contingent debt holders and the equity holders.

Investor	payoff in case of no trigger	payoff in case of trigger
Debt	$\min(M\exp(rT)+A_T-L_T, F)$	$\min(M\exp(rT)+(1-\kappa)A_T-L_T, F)$
Convertible	$\min(M\exp(rT)+A_T-L_T-F, F_C)$	κA_T
Equity	$\frac{1}{N}(M\exp(rT)+A_T-L_T-F-F_C)^+$	$\frac{1}{N}(M\exp(rT)+(1-\kappa)A_T-L_T-F)^+$

Table 5: Payoff - Funded Troubled Asset Reduction Coconut

	at time zero	at time T in case of no trigger	at time T in case of trigger
Number of stock	N	N	N
Capital	M	$\exp(rT)M$	$\exp(rT)M$
Risky Assets	A_0	A_T	$(1-\kappa)A_T$
Risky Liabilities	L_0	L_T	L_T
Ordinary debt	F	F	F
Contingent debt	F_C	F_C	0

Table 6: Overview - Funded Troubled Asset Reduction Coconut

2.2 Unfunded Coconuts

2.2.1 Unfunded Equity for Cash Coconut

In case of being triggered, the holder of the *unfunded equity for cash coconut* gets new equity in return for cash. The deal is similar to a CDS, but in case of being triggered new equity is transferred for cash. This cash amount can come along with a periodic fee payment during the life time of the deal.

The cash flow of coconut holder is : $\exp(rT)C$, if no conversion took place and where C is the present value of the periodically paid fees. In case of the conversion the coconut holder gets $\frac{\tilde{N}}{N+\tilde{N}}((M+\tilde{M})\exp(rT)+A_T-L_T-F)^+-\exp(rT)\tilde{M}$, where $\tilde{N}$ is the number of new stocks and $\tilde{M}$ is the present value of agreed cash injection net the fee payments.

The payoff at time T of ordinary bond holders is $\min(V_T, F)$, with $V_T = (M-C)\exp(rT)+A_T-L_T$ in case of no triggering and $V_T = (M+\tilde{M})\exp(rT)+A_T-L_T$ in case of triggering.

An equity holder has per stock either the payoff $\frac{1}{N}((M-C)\exp(rT)+A_T-L_T-F)^+$ in case of no conversion of the coconut and $\frac{1}{N+\tilde{N}}((M+\tilde{M})\exp(rT)+A_T-L_T-F)^+$ in case a conversion took place before maturity time T. We note that the coconut holder can have a negative cash-flow at maturity.

Investor	payoff in case of no trigger	payoff in case of trigger
Debt	$\min(((M-C)\exp(rT)+A_T-L_T)^+, F)$	$\min(((M+\tilde{M})\exp(rT)+A_T-L_T)^+, F)$
Convertible	$\exp(rT)C$	$\frac{\tilde{N}}{N+\tilde{N}}((M+\tilde{M})\exp(rT)+A_T-L_T-F)^+$ $-\exp(rT)\tilde{M}$
Equity	$\frac{1}{N}((M-C)\exp(rT)+A_T-L_T-F)^+$	$\frac{1}{N+\tilde{N}}((M+\tilde{M})\exp(rT)+A_T-L_T-F)^+$

Table 7: Payoff - Unfunded Equity for Cash Coconut

	at time zero	at time T in case of no trigger	at time T in case of trigger
Number of stock	N	N	$N+\tilde{N}$
Capital	M	$\exp(rT)(M-C)$	$\exp(rT)(M+\tilde{M})$
Risky Assets	A_0	A_T	A_T
Risky Liabilities	L_0	L_T	L_T
Ordinary debt	F	F	F

Table 8: Overview - Unfunded Equity for Cash Coconut

2.2.2 Unfunded Debt Coconut

In case of being triggered, the holder of the *unfunded debt coconut* gets new junior debt in return for cash. The deal is similar to a CDS, but in case of being triggered new junior debt is transferred for cash. This cash amount can come along with a periodic fee payment during the life time of the deal.

The payoff at time T of ordinary bond holders is $\min(V_T, F)$, with $V_T = (M-C)\exp(rT)+A_T-L_T$ in case of no triggering and $V_T = (M+\tilde{M})\exp(rT)+A_T-L_T$ in case of triggering and where C and $\tilde{M}$ are the present value of the periodically paid fees and the present value of agreed cash injection respectively net the fee payments. The payoff of coconut holder is : $\exp(rT)C$, if no conversion took place. In case of the conversion the coconut holder gets $(\min((M+\tilde{M})\exp(rT)+A_T-L_T-F,F_J))^+-\exp(rT)M$, where F_J is the amount of junior debt and $\tilde{M}$ is the present value of agreed cash injection net the fee payments. An equity holder has per stock either the payoff $\frac{1}{N}((M-C)\exp(rT)+A_T-L_T-F)^+$ in case of no conversion of the coconut and $\frac{1}{N}((M+\tilde{M})\exp(rT)+A_T-L_T-F-F_J)^+$ in case a conversion took place before maturity time T.

Investor	payoff in case of no trigger	payoff in case of trigger
Debt	$\min((M-C)\exp(rT)+A_T-L_T,F)$	$\min((M+\tilde{M})\exp(rT)+A_T-L_T,F)$
Convertible	$\exp(rT)C$	$(\min((M+\tilde{M})\exp(rT)+A_T-L_T-F,F_J))^+$
Equity	$\frac{1}{N}((M-C)\exp(rT)+A_T-L_T-F)^+$	$\frac{1}{N}((M+\tilde{M})\exp(rT)+A_T-L_T-F-F_J)^+$

Table 9: Unfunded Debt Coconut

	at time zero	at time T in case of no trigger	at time T in case of trigger
Number of stock	N	N	N
Capital	M	$\exp(rT)(M-C)$	$\exp(rT)(M+\tilde{M})$
Risky Assets	A_0	A_T	A_T
Risky Liabilities	L_0	L_T	L_T
Ordinary debt	F	F	F
Junior debt	0	0	F_J

Table 10: Overview - Unfunded Debt Reduction Coconut

2.2.3 Unfunded Troubled Asset Reduction Coconut

In case of being triggered, the holder of the *unfunded troubled asset reduction coconut* gets in return for cash, a fraction $0<\kappa\leq 1$ of the assets, (which are assumed to have low value). The deal is similar to a CDS, but in case of being triggered, troubled assets are transferred for cash. This cash amount can come along with a periodic fee payment during the life time of the deal. Here there is no dilution effect, because no new equity is created. This note is a kind of TARP swap.

We now have that the firm's value $V_T=((M-C)\exp(rT)+A_T-L_T)^+$, in case of no trigger and $V_T=((M+\tilde{M})\exp(rT)+(1-\kappa)A_T-L_T)^+$ in case of being trigger, where C and $\tilde{M}$ are the present value of the periodically paid fees and the present value of agreed cash injection respectively net the fee payments.

The cash flow at time T of ordinary bond holders is $\min(V_T,F)$, with V_T as given above in case of trigger and no trigger. The cash flow of coconut holder is : $\exp(rT)C$ in case of no trigger and $\kappa A_T-\exp(rT)M$ in case of triggering. An equity holder has per stock the payoff $\frac{1}{N}(V_T-F)^+$, with V_T again as given above in case of trigger and no trigger.

Investor	payoff in case of no trigger	payoff in case of trigger
Debt	$\min((M-C)\exp(rT)+A_T-L_T,F)$	$\min((M+\tilde{M})\exp(rT)+(1-\kappa)A_T-L_T,F)$
Convertible	$\exp(rT)C$	$\kappa A_T-\exp(rT)\tilde{M}$
Equity	$\frac{1}{N}((M-C)\exp(rT)+A_T-L_T-F)^+$	$\frac{1}{N}((M+\tilde{M})\exp(rT)+(1-\kappa)A_T-L_T-F)^+$

Table 11: Payoff - Unfunded Troubled Asset Reduction Coconut

	at time zero	at time T in case of no trigger	at time T in case of trigger
Number of stock	N	N	N
Capital	M	$\exp(rT)(M-C)$	$\exp(rT)(M+\tilde{M})$
Risky Assets	A_0	A_T	$(1-\kappa)A_T$
Risky Liabilities	L_0	L_T	L_T
Ordinary debt	F	F	F

Table 12: Overview - Unfunded Troubled Asset Reduction Coconut

2.2.4 Unfunded Liabilities Reduction Coconut

In case of being triggered, the holder of the *unfunded liabilities reduction coconut* gets, a fraction $0<\kappa\leq 1$ of the liabilities; in return he gets a (periodic) fee. Here there is no dilution effect, because no new equity is created. This note is a kind of TARP swap, but now on the liabilities side.

We now have that the firm's value $V_T=((M-C)\exp(rT)+A_T-L_T)^+$, in case of no trigger and $V_T=((M-C)\exp(rT)+A_T-(1-\kappa)L_T)^+$ in case of being trigger, where C is the present value of the periodically paid fees.

The cash flow at time T of ordinary bond holders is $\min(V_T,F)$, with V_T as given above in case of trigger and no trigger. The cash flow of coconut holder is : $\exp(rT)C$ in case of no trigger and $\exp(rT)C-\kappa L_T$ in case of triggering. An equity holder has per stock the payoff $\frac{1}{N}(V_T-F)^+$, with V_T again as given above in case of trigger and no trigger.

Investor	payoff in case of no trigger	payoff in case of trigger
Debt	$\min((M-C)\exp(rT)+A_T-L_T,F)$	$\min((M-C)\exp(rT)+A_T-(1-\kappa)L_T,F)$
Convertible	$\exp(rT)C$	$\exp(rT)C-\kappa L_T$
Equity	$\frac{1}{N}((M-C)\exp(rT)+A_T-L_T-F)^+$	$\frac{1}{N}((M-C)\exp(rT)+A_T-(1-\kappa)L_T-F)^+$

Table 13: Payoff - Unfunded Liabilities Reduction Coconut

	at time zero	at time T in case of no trigger	at time T in case of trigger
Number of stock	N	N	N
Capital	M	$\exp(rT)(M-C)$	$\exp(rT)(M-C)$
Risky Assets	A_0	A_T	A_T
Risky Liabilities	L_0	L_T	$(1-\kappa)L_T$
Ordinary debt	F	F	F

Table 14: Overview - Unfunded Liabilities Reduction Coconut

Market	λ	γ
Equity	2.5 %	2.5 %
Debt	10 %	20 %
Convertible	6.25%	11.25 %
Taxpayer	75%	75 %

Table 15: Absence of gain enticement and loss aversion for different markets

3 Conic Finance

In this section, we summarize the basic techniques used. We will discuss non-linear distorted expectation, acceptability and bid-ask pricing.

In this paper, we will make use of a distortion function from the minmaxvar family parameterized as given in Equation 1 by two parameters lambda λ and gamma γ.

$$\Phi(u;\lambda,\gamma) = 1 - \left(1 - u^{\frac{1}{1+\lambda}}\right)^{1+\gamma} \tag{1}$$

Lambda determines the rate of loss aversion of the investor; gamma determines the absence of gain enticement. Securities are traded in their own markets and we model different markets using different levels of lambda and gamma to reflect the different preferences of investors in these market.

We will assume, that the firm operates and raises capital in four different markets:

- Equity market (S) : tolerant to losses and enticed by gains;
- Debt market (D) : quite tolerant to losses but not really enticed by gains;
- Convertible Bond Market (C) : moderately tolerate to losses and moderately induced by gains;
- Taxpayers (T) : most loss aversion and highest absence of gain enticement.

In the numerical examples presented, the parameters of Table 15 are assumed.

In each market we use a different non-linear expectation to calculate (bid and ask) prices. The prices arise form the theory of acceptability.

We say that a risk X is acceptable ($X \in \mathcal{A}$) if

$$E_Q[X] \geq 0 \text{ for all measures } Q \text{ in a convex set } \mathcal{M}.$$

The convex set is called a cone of measures; operational cones were defined by Cherney and Madan [3] and depend solely on the distribution function $G(x)$ of X and a distortion function Φ. $X \in \mathcal{A}$, if the distorted expectation is non-negative. More precisely, the distorted expectation of a random variable X with distribution function $G(x)$ relative to the distortion function Φ (we use the one given in Equation (1), but other distortion function are also possible), is defined as

$$de(X; \lambda, \gamma) = \mathrm{E}^{\lambda,\gamma}[X] = \int_{-\infty}^{+\infty} x \mathrm{d}\Phi(G(x); \lambda, \gamma). \tag{2}$$

Note that if $\lambda = \gamma = 0$, $\Phi(u; 0, 0) = u$ and hence $de(X; 0, 0) = \mathrm{E}[X]$ is the ordinary linear expectation.

The ask price of payoff X is determined as

$$ask(X) = -\exp(-rT)\mathrm{E}^{\lambda,\gamma}[-X].$$

This formula is derived by noting that the cash-flow of selling X at its ask price is acceptable in the relevant market: $ask(X) - X \in \mathcal{A}$

Similarly, the bid price of payoff X is determined as

$$bid(X) = \exp(-rT)\mathrm{E}^{\lambda,\gamma}[X].$$

Here the cash-flow of buying X at its bid price is acceptable in the relevant market : $X - bid(X) \in \mathcal{A}$.

One can prove that the bid and ask prices of a positive contingent claim X with distribution function $G(x)$ can be calculated as:

$$\begin{aligned} bid(X) &= \int_0^{+\infty} (1 - \Phi(G(x); \lambda, \gamma))\mathrm{d}x \\ ask(X) &= \int_0^{+\infty} \Phi(1 - G(x); \lambda, \gamma))\mathrm{d}x \end{aligned}$$

4 Contingent Capital Pricing

4.1 Trigger Ratio

The Lloyds coconut and some others use the core tier-1 ratio as the trigger. Essentially this level is based on the ratio of equity to (risk weighted) assets. For us risk weighted assets are just assets marked at bid and the risk weighting is equivalent to the shaving of value done by bid prices relative to prices. For an all equity firm with risky assets risk weighted at A and equity at J with zero debt we have a balance sheet showing risk weighted liabilities marked at ask and cash reserves M. Later on we will introduce and elaborate more on some more "advanced" balance sheets, but for the sake of the next argument we will use the simple balance sheet as in Table 16. We have $M + A = L + J$.

ASSETS		LIABILITIES	
Risky Assets A	10	Risky Liabilities L	15
Capital M	7	Equity J	2
TOTAL	17	TOTAL	17

Table 16: Simple Balance Sheet

The core tier-1 ratio could be $J/(M+A) = 2/17 = 11.76\%$ and this could be perceived as totally fine but the firm could be in trouble as $A - L = -5$, and default occurs when $A - L = -7$ (because $M = 7$). Hence the "distance to default" is $2(= J)$.

The volatility under zero correlation is proportional to $A+L = 25$ and $J/(A+L) = 8\%$ and with an 8% volatility we are hence only one standard deviation from default.

We therefore argue that the trigger of a contingent capital note in order to be effective should better be set on the basis of the required capital. We hence introduce the notion of require capital and capital shortfall. The former is the capital needed to set up limited liability at a given time, the later is the positive part of the difference of this number with the actual capital held. If this deviates too much (below governed by the grace parameter β) the coconut is triggered. In the light of this discussion, we note that balance sheets should therefore also report on the asset side the capital shortfall. On the liability side this equivalent amount could resort under the heading *grace equity*.

4.2 Conic Finance Pricing

We now analyze how limited liability firms should be set up and how their capital structure is formed. We will at the end have a firm that has issued normal debt, contingent capital debt and equity. We assume that assets are risky as usual but now also that we have risky liabilities. Key in the argument is that we abandon the one-price-market idea and assume that there are bid-ask spreads (related to the liquidity of the market) as explained in the above section. We hence will make extensive use of the theory of acceptability and distorted expectations.

First we determine the capital required for existence. Remark that the classical Mertonian firm has no need for capital reserves. Throughout the paper an example will be used based on correlated geometrical Brownian motions, but the theory can be readily applied for fat-tailed models as well. Assume we want to set-up a firm with risky assets A and risky liabilities L. We will denote with A_t the value of the assets at time t and with L_t the value of the liabilities at time t. $A = \{A_t, t \geq 0\}$ and $L = \{L_t, t \geq 0\}$ are (dependent) stochastic processes. For the example, we take $A_0 = 100; L_0 = 90$ and

$$A_t = A_0 \exp\left((r - \sigma_A^2/2)t + \sigma_A W_t\right) \text{ and } L_t = L_0 \exp\left((r - \sigma_L^2/2)t + \sigma_L \tilde{W}_t\right),$$

where $\sigma_A = \sigma_L = 25\%$, $r = 3\%$ and $W = \{W_t, t \geq 0\}$ and $\tilde{W} = \{\tilde{W}_t, t \geq$

$0\}$ are two standard Brownian motions with correlation parameter equal to $\rho = 50\%$.

The regulatory body granting limited liability has to set the capital M^* such that the firms total cash-flow is acceptable to the tax-payer (with e.g. a one-year horizon : $T = 1$). Therefore M^* is set such that

$$\exp(rT)M^* + A_T - L_T \in \mathcal{A}_T.$$

M is calculated by distorted expectation:

$$M^* = -\exp(-rT)de(A_T - L_T; \lambda_T, \gamma_T) = \mathrm{E}^{\lambda_T,\gamma_T}[A_T - L_T].$$

This value M^* is dependent ofcourse on the stochastic processes chosen for the risky assets and liabilities. Under the chosen model (here the geometrical Brownian motions) it can be seen as a function of the model parameters (interest rate r, the volatilities of the risky assets and liabilities σ_A and σ_L and the correlation parameterρ), the preset maturity ($T = 1$) and the start ($t = 0$) values of the risky assets and liabilities, resp. given by A_0 and L_0. To stress the dependency on the later, we will later on therefore write $M(A_0, L_0)$. Actually, we will be recalculating the value for the newly revealed A_t and L_t's at certain points t in time (using the same parameter and maturity assumptions). If we just write M^*, we always refer to $M(A_0, L_0)$.

The firm will approach the equity market which has final limited liability cash flow

$$V_T = (M^* \exp(rT) + A_T - L_T)^+;$$

Using distorted expectation we calculate the bid and ask price in the equity market of this cash flow. We receive the bid (bJ) and mark the liability at ask (aJ). The all-equity firm comes into existence provided

$$M^* + A_0 - L_0 + aJ - bJ \leq bJ;$$

The option to put losses back to the taxpayer has payoff

$$(-M^* \exp(rT) - A_T + L_T)^+.$$

This is an asset of the firm to be valued at the bid price and is called the Taxpayer put option (see [5]).

Assume we have a firm that can exist as an all equity firm. Next, we consider the debt issuing. We assume two kinds of debt will be issued. First classical debt, a pure zero coupon bond with face value F maturing at T and secondly, contingent equity for debt note with face value F_C (no coupons), which in case the conversion trigger is hit, converts into equity. Assume there are N numbers of stock issued for the moment; at the time there is a conversion αN new stocks are issued and we have a dilution effect. We assume there are a number of trigger dates, $t_i, i = 1, \ldots, n$ (in the example quarterly), on these dates one checks if the firm needs extra capital. The conversion happens the first time

$$(1 + \beta)M^* \exp(rt_i) < M(A_{t_i}, L_{t_i}).$$

This means, that if we would re-calculate at the monitor date t_i the required capital needed to be granted limited liability, it can not deviate too

	Funded	Equity for Debt	Funded	Debt R. $\kappa = 0$	Funded	TAR $\kappa = 5\%$
	bid	ask	bid	ask	bid	ask
Equity	26.29	29.45	26.31	29.47	26.26	29.42
Debt	51.37 %	66.50 %	51.37 %	66.50 %	51.23 %	66.39 %
Convertible	48.95 %	58.76 %	48.52 %	58.22 %	58.22 %	68.28 %

Table 17: Bid and ask prices funded notes

much (governed by the grace parameter $\beta > 0$) from the original capital held. To be precise $(1 + \beta)$ times the original capital M^* should suffice otherwise a conversion of the debt into equity occurs.

The cash flow at time T of ordinary bond holders is $\min(V_T, F)$. The cash flow of coconut holder is : $(\min(V_T - F, F_C))^+$ if no conversion took place and equals in case of the conversion $\frac{\alpha}{1+\alpha}(V_T - F)^+$ This is the typical equity payoff with a factor $\alpha/1 + \alpha$ taking into account the dilution effect (governed by the parameter α, the conversion factor). Equity holders have either the payoff $(V_T - F - F_C)^+$ in case of no conversion of the coconut and $\frac{1}{1+\alpha}(V_T - F)^+$ in case a conversion took place before maturity time T.

These cash flows have bid prices (bD, bC, bE) and ask prices (aD, aC, aE), which are calculated as in Section 3. We incorporate the fact that the market to which the cash-flow belongs are different by using different distortion functions reflecting the difference of loss aversions and absence of gain enticements among the three markets.

For these issues the funds raised must cover the cost and we require that

$$M^* + A_0 - L_0 + (aD - bD) + (aC - bC) + (aE - bE) \leq bD + bC + bE$$

A similar pricing can be done for the other variations of the coconuts described in Section 2, by replacing the above payoff formulas with the corresponding ones.

As illustration, we show the pricing of the funded notes in Table 17 and the unfunded ones in Table 18. We assume $F = 10$, $F_C = 5$, $T = 5$ years, $\alpha = 10\%$, $\beta = 6$, $C = 2$ and $\tilde{M} = 10$. The prices of the debt instruments are given as percentage of the face value. For the convertible this is also the case for funded notes, but for unfunded notes is just given in currency units. The probability the coconuts are converted, with the given grace parameter, is around 10%.

5 Analysis

Next, we investigate the sensitivity of the dilution factor α and the grace factor β to the pricing of a equity for debt funded note.

In Figure 1 we have plotted the conversion probability for a range of β values. We clearly see that increasing β decreases the probability that

	Unfunded	Equity for Cash	Unfunded	TAR $\kappa = 5\%$	Unfunded	TLR $\kappa = 5\%$
	bid	ask	bid	ask	bid	ask
Equity	27.47	30.98	28.79	32.14	28.79	32.14
Debt	50.09 %	65.47 %	50.24 %	65.60 %	50.17 %	65.54 %
Convertible	0.32	1.02	0.89	1.37	0.85	1.37

Table 18: Bid and ask prices unfunded notes

a conversion will take place. We note that the conversion probability is independent from the dilution parameter α.

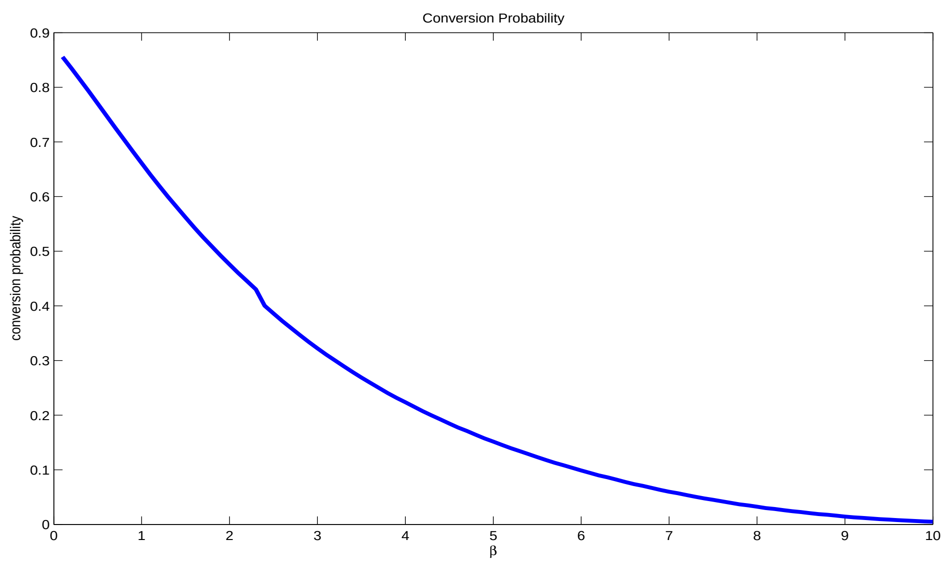

Figure 1: Conversion Probabilities for a range of β values

In Figure 2 we show the effect of β (which is controlling the conversion probability) on the equity prices. We plot both bid and ask prices. We observe that the more unlikely conversion is, the higher the equity price. This is very natural because the probability of being diluted as equity holder then becomes smaller.

In Figure 3 we see the dilution effect (α) on the price of the convertible. We observe that the price increase the more equity one obtains in case of a conversion. We note that as can be expected we don't see any effect on the ordinary bond price. For low and moderate α, the convertible price is lower than the ordinary bond price reflexing a high risk and hence a higher yield. However, for extreme high α, we see that the bid price of

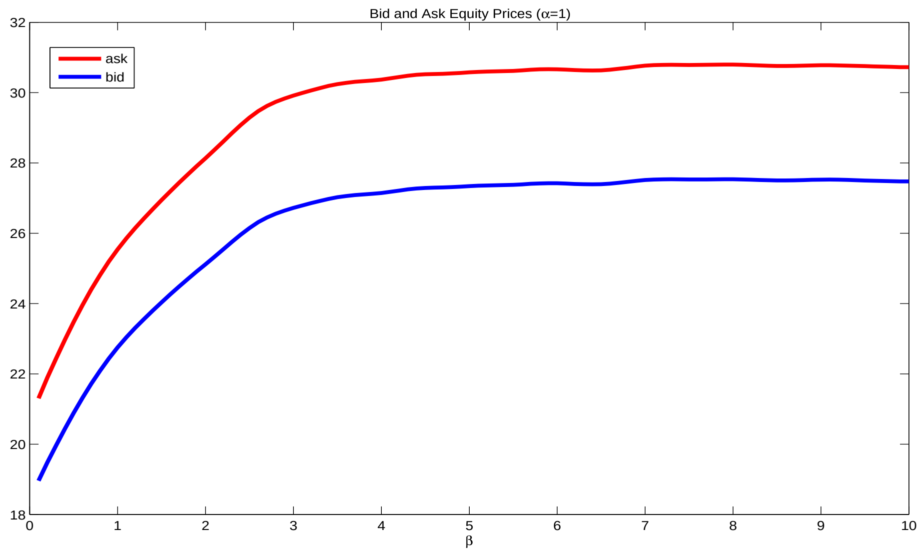

Figure 2: Effect of β on equity price.

the convertible is actually higher than the bond's bid, because the high amount of stock the convertible holder gets in case of conversion.

In Figure 4 we see the dilution effect (α) on the price of the equity. We observe that the price decrease since as equity holder one gets more diluted.

Appendix : Conic Corporate Balance Sheets

In this appendix, we show how balance sheets as developed in [8] are now extended with contingent capital instruments. As discussed above, we also argue that balance sheets should report on the asset side the capital shortfall and on the liability side an equivalent amount as grace equity. In below's balance sheets both numbers are equal to zero; we report balance sheets at initiation when there is by construction no capital shortfall. However, if time is progressing capital shortfall can be non zero.

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ASSETS		LIABILITIES	
Risky Assets A_0	100	Risky Liabilities L_0	90
Capital M	16.63		
Taxpayer Put bP	9.63		
Cost of Equity $aE - bE$	3.28	Equity aE	30.65
Cost of Debt $aD - bD$	1.51	Debt aD	6.66
Cost of Coco $aC - bC$	0.48	Coco aD	2.96
		Synthetic Equity	1.27
TOTAL	131.53	TOTAL	131.53

Table 19: Funded Equity for Debt Coconut Balance Sheet

ASSETS		LIABILITIES	
Risky Assets A_0	100	Risky Liabilities L_0	90
Capital M	16.63		
Taxpayer Put bP	9.63		
Cost of Equity $aE - bE$	3.28	Equity aE	30.66
Cost of Debt $aD - bD$	1.51	Debt aD	6.66
Cost of Coco $aC - bC$	0.48	Coco aD	2.94
		Synthetic Equity	1.26
TOTAL	131.53	TOTAL	131.53

Table 20: Funded Debt Reduction Coconut Balance Sheet

ASSETS		LIABILITIES	
Risky Assets A_0	100	Risky Liabilities L_0	90
Capital M	16.63		
Taxpayer Put bP	10.03		
Cost of Equity $aE - bE$	3.28	Equity aE	30.62
Cost of Debt $aD - bD$	1.51	Debt aD	6.65
Cost of Coco $aC - bC$	0.49	Coco aD	3.43
		Synthetic Equity	1.24
TOTAL	131.94	TOTAL	131.94

Table 21: Funded Asset Reduction Coconut Balance Sheet

ASSETS		LIABILITIES	
Risky Assets A_0	100	Risky Liabilities L_0	90
Capital M	16.63		
Taxpayer Put bP	9.04		
Cost of Equity $aE - bE$	3.35	Equity aE	32.17
Cost of Debt $aD - bD$	1.53	Debt aD	6.57
Cost of Coco $aC - bC$	0.70	Coco aD	1.02
		Synthetic Equity	1.47
TOTAL	131.25	TOTAL	131.25

Table 22: Unfunded Cash for Equity Coconut Balance Sheet

ASSETS		LIABILITIES	
Risky Assets A_0	100	Risky Liabilities L_0	90
Capital M	16.63		
Taxpayer Put bP	9.42		
Cost of Equity $aE - bE$	3.25	Equity aE	32.14
Cost of Debt $aD - bD$	1.54	Debt aD	6.56
Cost of Coco $aC - bC$	0.48	Coco aD	1.37
		Synthetic Equity	1.34
TOTAL	131.42	TOTAL	131.42

Table 23: Unfunded TAR Coconut Balance Sheet

ASSETS		LIABILITIES	
Risky Assets A_0	100	Risky Liabilities L_0	90
Capital M	16.63		
Taxpayer Put bP	9.39		
Cost of Equity $aE - bE$	3.36	Equity aE	32.14
Cost of Debt $aD - bD$	1.54	Debt aD	6.55
Cost of Coco $aC - bC$	0.52	Coco aD	1.37
		Synthetic Equity	1.37
TOTAL	131.43	TOTAL	131.43

Table 24: Unfunded TLR Coconut Balance Sheet

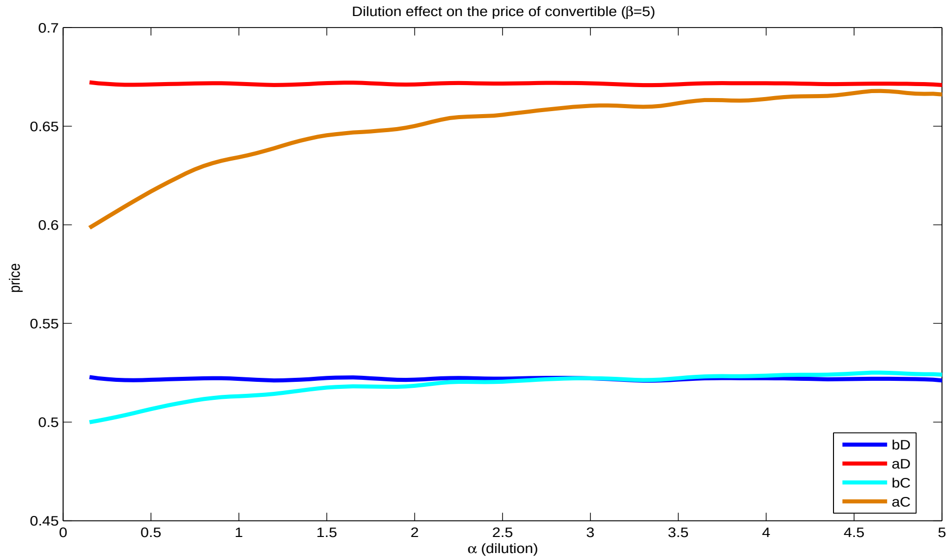

Figure 3: Dilution Effect on bond prices. ($\beta = 5$)

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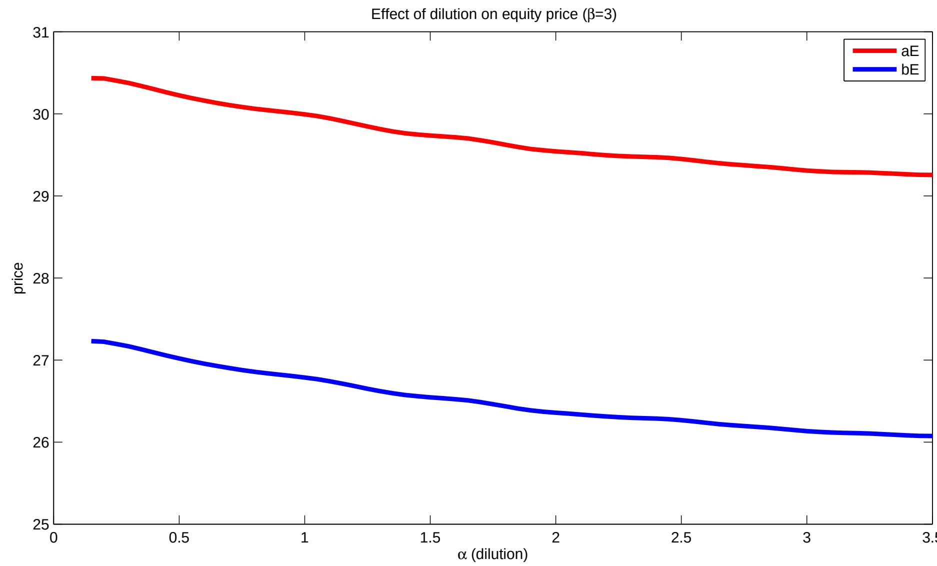

Figure 4: Dilution Effect on equity price. ($\beta = 3$)

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