Limit theorems for 2D invasion percolation

Michael Damron†
Artēm Sapozhnikov‡

September 2010

Abstract: We prove limit theorems and variance estimates for quantities related to ponds and outlets for 2D invasion percolation. We first exhibit several properties of a sequence \( (O(n)) \) of outlet variables, the \( n \)-th of which gives the number of outlets in the box centered at the origin of side length \( 2n \). The most important of these properties describes the sequences renewal structure and exponentially fast mixing behavior. We use these to prove a central limit theorem and strong law of large numbers for \( (O(n)) \). We then show consequences of these limit theorems for the pond radii and outlet weights.

Keywords: Invasion percolation; invasion ponds; critical percolation; near critical percolation; correlation length; scaling relations, central limit theorem.

†Mathematics Department, Princeton University, Fine Hall, Washington Rd., Princeton, NJ 08544. Email: mdamron@math.princeton.edu; Research funded by an NSF Postdoctoral Fellowship.
‡ETH Zürich, Department of Mathematics, Rämistrasse 101, 8092 Zürich. Email: artem.sapozhnikov@math.ethz.ch; Research partially supported by Excellence Fund Grant of TU/e of Remco van der Hofstad.