Regular $g$-measures are not always Gibbsian

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Abstract
Regular $g$-measures are discrete-time processes determined by conditional expectations with respect to the past. One-dimensional Gibbs measures, on the other hand, are fields determined by simultaneous conditioning on past and future. For the Markovian and exponentially continuous cases both theories are known to be equivalent. Its equivalence for more general cases was an open problem. We present a simple example settling this issue in a negative way: there exist $g$-measures that are continuous and non-null but are not Gibbsian. Our example belongs, in fact, to a well-studied family of processes with rather nice attributes: It is a chain with variable-length memory, characterized by the absence of phase coexistence and the existence of a visible renewal scheme.

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