

## Lajos Takács

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Professor emeritus Lajos Takács passed away on December 4, 2015, in Cleveland Heights, Ohio. He is survived by his wife Dalma, their two daughters Judith and Susan, and their families. Lajos Takács will be remembered as a brilliant mathematician, a groundbreaking contributor to the theory of stochastic processes, a world-leading queueing theorist, and a very kind person. We provide here a short biography and a discussion of his main contributions to queueing and fluctuation theory.

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Lajos Takács was born on August 21, 1924, in Maglód, Hungary. As a young person in highschool he had shown an interest in mathematics, and with the guidance of his teachers, who supplied him textbooks, he taught himself subjects like calculus, linear algebra, set theory, number theory, and complex function theory. In 1943 he entered the Technical University of Budapest, where he studied mathematics and physics. In particular, he studied probability and statistics under Professor Charles Jordan, and also became student–assistant of Professor Zoltan Bay in the Tungram Research Laboratory of Atomic Physics. His work in the project of detecting microwave echoes from the moon led him to the study of stochastic phenomena.

In 1948 Takács obtained his Ph.D writing the thesis ‘On a probability–theoretical investigation of Brownian motion’, and in 1957 he obtained his Habilitation for his thesis ‘Stochastic processes arising in the theory of particle counters’. Until 1955 Takács held a joint appointment at the Telecommunications Research Institute and the Research Institute of Mathematics of the Hungarian Academy of Sciences. In that period he published a large number of papers on queueing, inventories, dams, and insurance risk.

As he wrote in the autobiographical essay [14], the year 1958 was a turning point in his life. He made the decision to leave Hungary permanently and continue his work in the United States. After spending some time in Imperial College, London, on a visiting appointment, he accepted an offer from Columbia University in New York. He worked in the Statistics Department until 1966, teaching probability theory, stochastic processes and queueing theory. He also served as a consultant for Bell Labs and IBM Research. He was subsequently appointed as Professor of Mathematics at Case Western Reserve University (CWRU) in Cleveland, Ohio, where he worked until his retirement in 1987. At CWRU, he wrote over 100 research papers, fluctuation theory, occupation times and random graphs being his main areas of interest.

## 1 Takács’ contributions to queueing theory

The influence of Takács is felt in several areas of applied probability—he made many contributions to point processes, binomial moments, occupation time problems, random walks, ballot theorems, order statistics, fluctuation theory, combinatorial problems, branching processes, and random graphs. He also introduced, in [7], what later would be called semi-Markov processes. From his 225 publications (1–211 are listed in [3] and 212–225 in [5]), only about 65 are directly devoted to queueing problems. Still, some of his most influential publications are on queueing theoretic problems, and as he wrote in [14], p. 148: “The area that I have found most exciting and versatile is queueing theory. Apart from its practical importance, queueing theory is an excellent proving ground for new mathematical ideas. It is hard to find a probabilistic method which has not had its impact on queueing theory.”

In his extremely productive years 1954–1958, Takács published 55 papers, which included [8], one of the classics in queueing theory. In this paper, he considered the FIFO  $M/G/1$  single server queue. He studied, a.o., a stochastic process which has later been called the Takács process or the *virtual waiting time process*, the virtual waiting time at time  $t$  denoting the amount of time required to serve all the work that

is in the system at time  $t$ . He observed that this process is Markovian, and indicated how the transient behavior of the virtual waiting time in the  $M/G/1$  queue can be studied using what was later called the integro-differential equation of Takács. He even allowed the arrival rate of the Poisson process to be time-dependent. Takács also showed that the steady-state distribution of the virtual waiting time equals that of the waiting time. The paper [8] also contains a detailed and beautiful analysis of the distribution of the busy period length in the  $M/G/1$  queue, exploiting a relation to branching processes. Such a relation was already sketched by Kendall [6]; see also the comments on this paper by I.J. Good (p. 182–183 of [6]). Takács' landmark paper [8] ends with a discussion of the  $GI/G/1$  queue; in particular, he derives a relation between the steady-state distributions of virtual waiting time and waiting time. In later publications on fluctuations of partial sums of sequences of random variables [12, 13], Takács has also obtained the transient distribution of the virtual waiting time process for  $GI/G/1$ , by solving operator recurrence equations. In this respect, it is interesting to cite from an email which Takács sent to Haghghi on May 23, 2015 (cf. p. 657–658 of [5]): “An additional note: in July 1973 I completed my book *Theory of Random Fluctuations*, which unfortunately is still in manuscript form. While I was working on the book, John Wiley was so interested that they offered me a contract before the book was finished. I did not like to sign a contract until my work was ready for the press. The finished manuscript turned out to be 1600 pages which Wiley considered too big for a book. They offered to publish it in lecture note form, which was unacceptable to me. I was also unwilling to shorten the MS, so it is still unpublished.” Presently the option is being considered to make this manuscript available as Eurandom report, via <http://eurandom.tue.nl>.

## 2 Takács and his books

Lajos Takács published six books; three in Hungarian and three in English [9–11]. His best-known books are [10] and [11]. *Introduction to the Theory of Queues* [10] is a gem, and in our view still an absolute must for any scholar in queueing theory. The analytic and probabilistic skills demonstrated by the author are dazzling, most of the results were new at the time or at least derived by Takács in the preceding six years, and still the writing is clear and quite accessible. The 135-page chapter on *Single Server Queueing Processes*, and the beautiful short chapter on *Queues with Infinitely Many Servers* are still most worthy of a careful study. We cite the first sentences of the Preface: “The aim of this book is to give an introduction to the probabilistic treatment of mass servicing. We deal with different models which can be applied successfully to the theory of telephone traffic, airplane traffic, road traffic, storage, operation of dams, serving of customers, operation of particle counters, and others. Our interest is chiefly in the time-dependent or transient behavior of these processes.” The last part of the Preface is also interesting: “This book is not a complete monograph of the theory of queues, but the methods which we consider most important are presented here. By using these methods one can approach several types of queueing processes not discussed in this book. In several cases, the solution of queueing problems results in an apparently complicated formula; but fortunately we are living in an age of electronic computers,

when complicated formulas in the old-fashioned sense can easily be handled.” This last statement, written in 1962 when he and contemporary leading queueing theorists like Cohen and Prabhu were wrestling with the “Laplacian curtain” (complicated transform expressions which in those days were hard to invert numerically), 50 years later has become even more true.

His book [11] demonstrates his incredible proficiency in combinatorial methods. It starts from the classical ballot theorem of De Moivre and Bertrand, generalizes that theorem, and uses that generalization to obtain explicit results for the distribution of the maximum of random variables, or the supremum of stochastic processes—results with an immediate bearing on queueing theory. Most of the results in this book, too, were obtained by the author in the 6 years preceding its publication.

Takács had 9 Ph.D students at Columbia University, the first one being Paul J. Burke and the last one Peter Welch; he had 14 Ph.D students in CWRU, including Aliakbar Montazer-Haghighi who wrote a moving personal remembrance and obituary in [5].

Lajos Takács was an applied probabilist and queueing theorist who was way ahead of his time. He received several distinctions. In 1993 he was elected a foreign member of the Hungarian Academy of Sciences. In 1994, the Institute for Operations Research and Management Science bestowed upon him the prestigious Von Neumann Prize. In addition, in that year two special issues were devoted to Takács and his work [2,4]. His books, papers and students form a long-lasting legacy; he will not be forgotten.

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