A simple SIR-vaccination model: Estimating Parameters.

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Introduction

We model childhood diseases such as measles. Vaccination is key in controlling such diseases. The aim here is to identify which vaccination strategies are optimal given constraint on resources. Pulse vaccination (e.g., Agur, 1993) has been shown to yield better results than constant vaccination, with success stories in Argentina, Brazil, Israel, among others (Chavez and Feng, 1998).

Fitting good parameters are needed to validate any model assumptions or conclusions made, to ensure the model analysis and reporting makes much sense.

Mathematical Analysis

FLOQUET THEORY:

Three independent solutions of the linearization of (1a), with initial conditions (1,0,0), (0,1,0), and (0,0,1) are obtained. The vectors form the columns of the Monodromy manifold - h(s).

The main diagonal defines the spectrum of the matrix, and stability of the disease free periodic orbit is defined if

\[ F = \beta(t)(S + \alpha X - X S) \]

SINGULAR PERTURBATION THEORY:

Both t-Time and I(t)-the Infective Population, are rescaled to obtain the fast system. Then with infinite separation of time, one obtains the limiting fast and slow systems. Below is the slow system, and the slow manifold - h(s).

\[
\begin{align*}
\frac{dX}{dt} &= -\beta S X - g(S) - \frac{\beta X}{\beta(t)} \ln S, \\
\frac{dS}{dt} &= b S - \beta S X, \\
\frac{dI}{dt} &= \frac{\beta S X}{\beta(t)} - \frac{1}{\alpha} S + \ln S - X, \\
\end{align*}
\]

Parameter Estimates

\[ \theta = \{h(t), \mu, \psi(t), \beta(t), \alpha, S(t_0), I(t_0), R(t_0)\} \]

\[
\begin{align*}
\psi(t) &= p \sin(\omega t) \\
\beta(t) &= q \sin(\omega t)
\end{align*}
\]

Parameters: at least 6.

- Birth rate(\(b\))
- Death rate(\(\mu\))
- Vaccination rate(\(\psi\))
- Contact rate(\(\beta\))
- Recovery rate(\(\alpha\))

Model

\[
\begin{align*}
\frac{dS}{dt} &= \alpha S - \mu S - \psi(t) S + \beta(t) S + \alpha I, \\
\frac{dI}{dt} &= -\beta S + \psi(t) S - \alpha I, \\
\frac{dR}{dt} &= \alpha I
\end{align*}
\]

Comment 1:

- Does orbit stability in the sense of Floque't offer a true picture? Figure 1 shows that despite an undetected.

Comment 2:

- Classical methods used to estimate parameters.
- Lack of data (Simulations used) and step changes in inputs, vaccination and contact rates make parameter estimation challenging. Surface plots do not portray true picture. Figure 2 shows that despite an undetected. 

Comment 3:

- Lack of data (Simulations used) and step changes in inputs, vaccination and contact rates. The parameter estimates here are not yet so good, eg, see surface curves or even fig. 3.

Parameter Estimation

- Parameter Estimates: (\(b(t)\)0.53)
- Original Parameters: (\(b(t)\)0.78)
- \(\mu = 2.02\)
- \(\psi\)1.5
- \(\alpha\)2.90
- \(\omega\)2.90
- \(\omega\)3.00
- SSE = 5.37
- SSE = 110.38