Power systems and Queueing theory: Storage and Electric Vehicles

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Plan

Lecture 1: Storage and Arbitrage

Lecture 2: Storage, Buffering and Competition

Lecture 3: Stability and Electric Vehicles

Today:

1. Background
2. The Arbitrage Problem
3. Toy Example - Price taker
4. General Theory - Lagrangian Sufficiency
5. Forecast and Decision Horizon
6. Real Data Example
7. Market Impact

Why do we want electricity storage?

- Need to balance supply and demand at all times.
- Wind power can fluctuate substantially on a short timescale.
- Thermal power plants slow to react.
- Can either use expensive alternatives.
- Alternatively can use electricity storage.

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There are many other uses:

- Arbitrage
- Frequency regulation
- Reactive power support
- Voltage support
- Black start

Dinorwig: capacity: 9 GWh   rate: 1.8 GW   efficiency 0.75–0.80

Storage comes in many forms.

There are many types of storage with different properties:

- Pumped storage
- Battery storage
- Compressed gas storage
- Fuel Cells
- Thermal
- Fly wheels

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As well we can consider dynamic demand as storage:

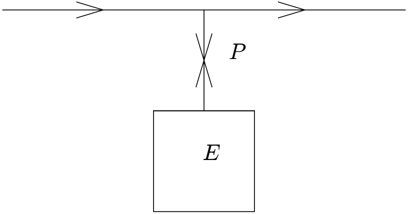
- Control of fridges.
- Thermal inertia of buildings.
- Washing machines.
- Aluminium smelting.

The Problem:

- Storage facilities are expensive, high capital cost.
- To facilitate investment we need to understand the value of storage.
- Today we will only consider money made from arbitrage by an aggregate store.
- Sensible since it is expected most of the profit will have come from this source.
- Need to also be able to compare to alternatives, for example demand side management.
- Also need to understand the effect of multiple competing stores.

Model

We will work in discrete time. (Natural in many electricity markets)

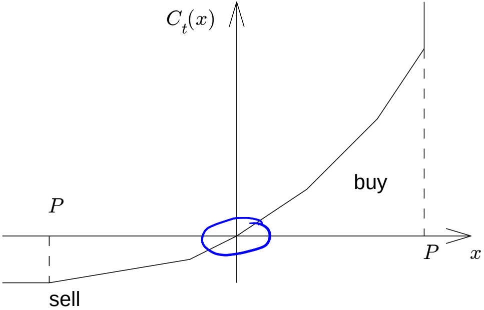


E = size of store — *capacity constraint*

P = max input/output rate — *rate constraint*

Cost function

At any (*discrete*) time t,



$C_t(x) =$ cost of increasing level of store by x (positive or negative)
Assume *convex* (reasonable). This may model

- *market impact*
- *efficiency of store*
- *rate constraints*

Problem

Let S_t = *level* of store at time t, $\quad 0 \leq t \leq T$.

Policy $S = (S_0, \ldots, S_T)$, $\quad S_0 = S_0^*$ (fixed), $\quad S_T = S_T^*$ (fixed).

Define also $x_t(S) = S_t - S_{t-1}$
(*energy* "bought" by store at time t – positive or negative)

Problem: minimise cost

$$\sum_{t=1}^{T} C_t(x_t(S))$$

subject to

$$S_0 = S_0^*, \qquad S_T = S_T^*$$

and

$$0 \leq S_t \leq E, \qquad 1 \leq t \leq T - 1.$$

Small store

This is a store whose activities are not so great as to impact upon the *market*, and which thus has *linear* buy and sell prices. Thus, for all t,

$$C_t(x) = \begin{cases} c_t^{(b)} x & \text{if } 0 \leq x \leq P \\ c_t^{(s)} x & \text{if } -P \leq x < 0 \end{cases}$$

where $0 < c_t^{(s)} \leq c_t^{(b)}$ and P is *rate* constraint.

Optimal control is *bang-bang*. At each time step either:

- *buy* as much as possible,
- *do nothing*,
- or *sell* as much as possible.

Extreme cases

Considering extreme cases provides insight into problem structure. Two natural extreme cases to consider:

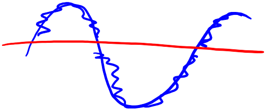
- No capacity constraint, $E = \infty$.
- No rate constraint, $P = \infty$.

- To add to the simplicity we assume equal buy and sell prices,

$$c_t = c_t^{(b)} = c_t^{(s)}.$$

- This is equivalent to assuming 100% efficient.
- In both cases we can give a clean representation of optimal control.

$E = \infty$



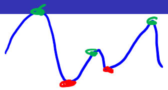
Solution:

- We find a single global reference price π.
- If $c_t > \pi$ we sell maximum amount.
- If $c_t < \pi$ we buy maximum amount.
- π is selected such that buy and sell for an equal number of time periods.

Comments:

- Time horizon for making decisions is long
- Need all data to decide on value of π
- Optimal solution is global in nature.

$P = \infty$



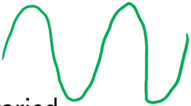
Solution:

- Only buy and sell at local maximums and minimums.
- Fill store completely at minimums.
- Empty store completely at maximums.

Comments:

- Time horizon for making decisions is short.
- Only need to look one time step ahead to decide if local maximum or minimum.
- Optimal solution is local in nature.

Example: Periodic Cost functions

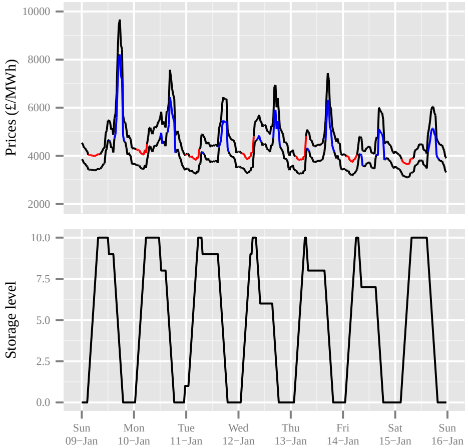


- Consider sinusoidal prices.
- Interested in what happens as frequency is varied.
- Assume $P = 1$.
- Then for a given value of E, there exists a pair $\mu_b < \mu_s$ such that we buy if $c_t < \mu_b$ and sell if $c_t > \mu_s$.
- Further μ_b is increasing in E and μ_s is decreasing in E.
- Also μ_b is increasing and μ_s is decreasing in frequency.
- These are bounded by μ_b^* and μ_s^*, the parameters obtained for $E = \infty$ which does not depend on frequency.
- For a given E, as frequency increases profit increases upto the unconstrained case, and there is a frequency beyond which you obtain no further benefit.

Example: Real prices with Dinorwig parameters

$E/P = 5$ hrs $\quad$ *Efficiency* $= 0.85$ (ratio of sell to buy price).

Solution is *bang-bang*: red points buy, blue points sell



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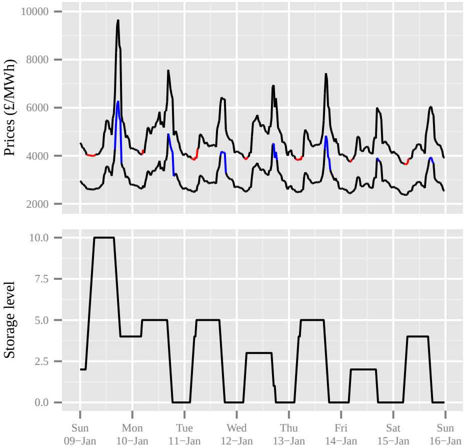


General Result: Lagrangian sufficiency

Suppose there exists a *vector* $\mu^* = (\mu_1^*, \ldots, \mu_T^*)$ and a *value* $S^* = (S_0^*, \ldots, S_T^*)$ of S such that

- S^* is *feasible*,
- $x_t(S^*)$ *minimises* $C_t(x) - \mu_t^* x$ over all x, $1 \leq t \leq T$,
- the pair (S^*, μ^*) satisfies the *complementary slackness conditions*, for $1 \leq t \leq T-1$,

$$\begin{cases} \mu_{t+1}^* = \mu_t^* & \text{if } 0 < S_t^* < E, \\ \mu_{t+1}^* \leq \mu_t^* & \text{if } S_t^* = 0, \\ \mu_{t+1}^* \geq \mu_t^* & \text{if } S_t^* = E. \end{cases} \tag{1}$$

Then S^* *solves* the stated problem.

Comment

- The above result (essentially an application of the Lagrangian Sufficiency Principle) does *not* require *convexity* of the functions C_t.

- However, *convexity* the functions C_t is sufficient to *guarantee* the *existence* of a pair (S^*, μ^*) as above.

  The latter result is a standard application of *Strong Lagrangian* theory (i.e. the *Supporting Hyperplane Theorem*).

Algorithm

- We need to identify the relevant value of μ_t^* at each time t.
- It is important to note that the value of μ_t^* only changes at times when the store is full or empty.
- Further μ_t^* acts as a reference level since the optimal action at time t is given by the x which minimizes:

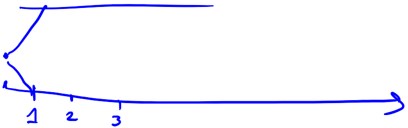
$$C_t(x) - \mu_t^* x.$$

- This is generally equivalent to finding the amount to sell such that μ_t^* is the marginal price.
- We can use this to carry at a search for μ^*.
- Further this method is local in time, as we can ignore times after filling or emptying the store.

Computational Cost

Careful consideration of how we find μ_t^* allows to bound computational cost of this process.

- Can show we need to carry out at most $2T$ linear searches in general case.
- Work forward from time 0 considering unconstrained problem (Energy).
- Find
  - μ_t^u, value need to fill the store at time t
  - μ_t^l, value need to empty the store at time t
- Find time when $\min_{0,t}(\mu_t^u)$ and $\max_{0,t}(\mu_t^l)$ cross.



Definition of Forecast and Decision Horizon

Consider the problem of finding the optimal decision at time 0. In some problems we can find times:

- τ, a Decision Horizon
- $\bar{\tau}$, a Forecast Horizon
- Have $\tau \leq \bar{\tau}$

Such that:

- We can make all optimal decisions up to time τ
- With no need for any information after $\bar{\tau}$

So in this example if we can find a pair τ and $\bar{\tau}$, changing the cost functions C_t for $t > \bar{\tau}$ will not change the optimal decisions upto time τ.

Importance of such Horizons

Why do we care?

- Often we are making decisions based on forecasts with increasing uncertainity.
- If the forecast horizon is short, do not need much future knowledge to make decisions.
- Allows decomposition of long/infinite horizon problems.

Methodology:

- Initially only solve the problem up to the first decision horizon
- Extend the solution when this time is reached.

Horizons and Coupling

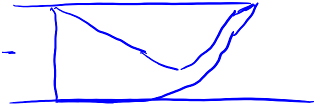
Forecast horizons often occur because of path coupling.

Examples include:

- Trunking
- Holding costs
- Constrained state space

Existence of Horizons

- The constraint on the store level (between 0 and E) leads to a forecast horizon.
- Paths couple as squeezed against boundaries
- Similar to ideas from coupling from the past.
- Means that the forecast horizon length is on the order the length of the filling/emptying cycle.
- In many examples this will be on the order of a 1 or 2 days in reality.



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Example

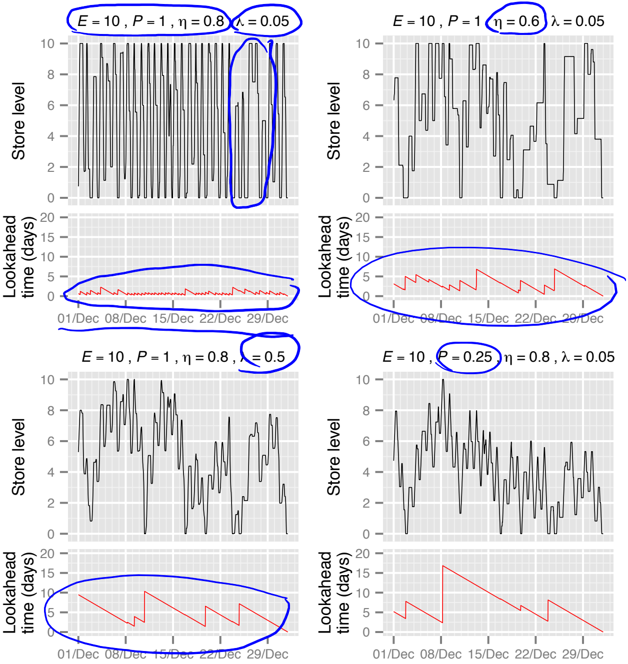
- Again consider real price data but incorporate market impact by the store.
- Use a quadratic cost function:

$$C_t(x) = \begin{cases} p_t x(1+\lambda x) & \text{if } 0 \leq x \leq P \\ -p_t \nu x(1+\lambda \nu x) & \text{if } -P \leq x < 0 \end{cases}$$

- p_t is the historic price.
- λ is a measure of market impact by the store.
- ν is the round trip efficiency of the store.
- P is the power constraint.

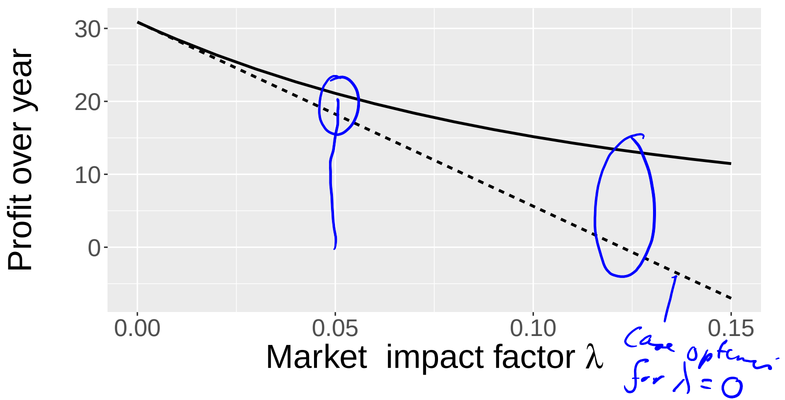
Case Study

- Cost series $(p_1, \ldots, p_T)$ corresponding to the real half-hourly spot market wholesale electricity prices in Great Britain for the year 2011.
- Provided by National Grid plc
- Prices show a strong daily cyclical behaviour
- Initially consider parameter choices $E = 10$, $P = 1$.
- Relates to Dinorwig pumped storage facility in Snowdonia.



E = 10 , P = 1 , η = 0.8 , λ = 0.05
E = 10 , P = 1 , η = 0.6 , λ = 0.05
E = 10 , P = 1 , η = 0.8 , λ = 0.5
E = 10 , P = 0.25 , η = 0.8 , λ = 0.05
Store level
Lookahead time (days)
01/Dec 08/Dec 15/Dec 22/Dec 29/Dec

2011



Profit over year
30
20
10
0
0.00
0.05
0.10
0.15
Market impact factor λ

Market impact of storage

A *large store* may impact *costs* (be a *price-maker*), and hence the rest of *society*.

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Impact of **storage** on **consumer surplus** is in general *beneficial*, but not *necessarily*.

Example ($T = 2$): Buy x at time 1 (increasing price by p_1) and sell at time 2 (decreasing price by p_2).

Increase in consumer surplus is

$$p_2 d_2 - p_1 d_1$$

where d_1 and d_2 are the respective demands at times 1 and 2.

In general expect $d_2 > d_1$ and p_2 to be comparable to p_1, so effect on consumer surplus is *beneficial*.

However, the latter need not be the case.