Power systems and Queueing theory: Storage and Electric Vehicles

(Joint work with Lisa Flatley, Richard Gibbens, Stan Zachary, Seva Shneer)

James Cruise

Maxwell Institute for Mathematical Sciences
Edinburgh and Heriot-Watt Universities

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Plan

Lecture 1: Storage and Arbitrage

Lecture 2: Storage, Buffering and Competition

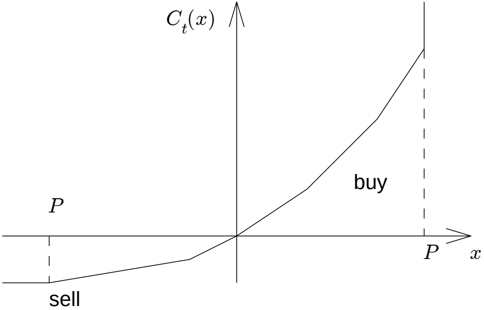
Lecture 3: Stability and Electric Vehicles

Today:

1. Arbitrage Continued
   - Reminders
   - Competition
2. Extending to Buffering
   - Stochastic Dynamic Programming Problem and Reformulation
   - Price Taker Example
   - Price Maker General Theory
3. Directions of Research
   - Multiple Stores and Resource Pooling
   - Demand Side Response
   - Stochastic Prices

Reminder: Cost function

At any (*discrete*) time t,



$C_t(x) =$ cost of increasing level of store by x (positive or negative)
Assume *convex* (reasonable). This may model

- *market impact*
- *efficiency of store*
- *rate constraints*

Reminder: Problem

Let S_t = *level* of store at time t, $\qquad 0 \le t \le T$.

Policy $S = (S_0, \dots, S_T)$, $\qquad S_0 = S_0^*$ (fixed), $\quad S_T = S_T^*$ (fixed).

Define also $x_t(S) = S_t - S_{t-1}$
(*energy* "bought" by store at time t – positive or negative)

Problem: minimise cost

$$\sum_{t=1}^{T} C_t(x_t(S))$$

subject to

$$S_0 = S_0^*, \qquad S_T = S_T^*$$

and

$$0 \le S_t \le E, \qquad 1 \le t \le T-1.$$

N stores in competition

We assume the stores are sufficiently *large* as to have *market impact*: the *activity* of each store *negatively* affects profits which can be made by the others.

Model *costs* at each time t as being derived from a *price function* $p_t(\cdot)$, where $p_t(x)$ is the *price per unit traded* when the total amount traded is x.

Suppose that each store i buys x_{it} at time t. Then *store i* incurs a *total cost* over time of

$$\sum_{t=1}^{T} x_{it} p_t \left(\sum_{j=1}^{n} x_{jt} \right). \qquad (*)$$

What happens now depends of the **RULES OF THE GAME** (the *mechanism* by which the *market is cleared*).

Bertrand competition

Stores bid **"prices"**.

Any single *unconstrained* store able to offer lowest price corners entire market.

If *overcapacity* then typically *little or no profits* to be made.

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CAPITAL COSTS CANNOT BE RECOVERED.

Cournot competition

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Each store *optimises its own profit* given the *activities over time* of all the *other stores*.

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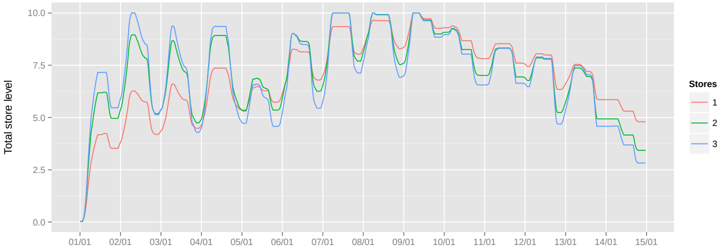
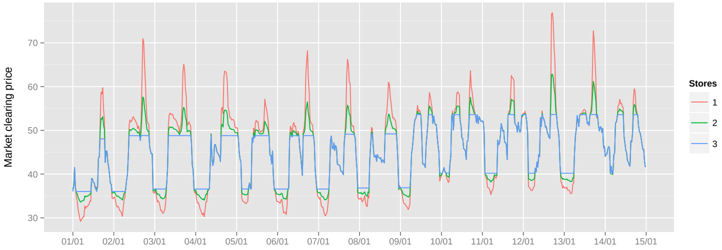
Proof. Given some set of strategies $x = (x_1, \ldots, x_n)$ of all the stores over all time $1, \ldots, T$, suppose that each store i (simultaneously) updates its entire strategy from x_i to x'_i given the activities x_j of all the remaining stores $j \neq i$. This defines a mapping $(x_1, \ldots, x_n) \to (x'_1, \ldots, x'_n)$ which is continuous and defined on a compact convex set.
Hence by the Brouwer fixed point theorem, the above mapping has a *fixed point* $(x_1, \ldots, x_n)$. This (by definition) is a Nash equilibrium.

Behaviour of stores

At a *Nash equilibrium* each *store* tend to *"overtrade"* (compared to an optimal *cooperative* solution): it thereby increases its *revenue*, the *excess costs* (in terms of price impact) being *borne by the other stores*.

Example: Competition example





Linearised price functions

Assume that, for each t,

$$p_t(x) = a_t + b_t x$$

(a reasonable first approximation).

Result. There is a unique *Nash equilibrium*, given by the *minimiser* $(x_1, \ldots, x_n)$ of the *quadratic* function

$$\sum_{t=1}^{T} \left[a_t \sum_{i=1}^{n} x_{it} + \frac{1}{2} b_t \left(\sum_{i=1}^{n} x_{it}^2 + \left(\sum_{i=1}^{n} x_{it} \right)^2 \right) \right] \qquad (**)$$

subject to the given *rate* and *capacity constraints* on each store.

Proof. For each i, minimisation in x_i of the above function is equivalent to minimisation of the earlier function (*).

Unconstrained stores in competition

Assume *linear prices* and n stores subject to *neither capacity nor rate constraints*, and each of which has the same starting and finishing level.

Result. At the (unique and necessarily symmetric) *Nash equilibrium*, the *quantity traded per store* is proportional to $1/(n+1)$ and the *profit per store* is proportional to $1/(n+1)^2$.

Proof. This follows easily from the observed symmetry of the solution and the use of strong Lagrangian theory to minimise the function (**) subject to the constraint $\sum_{i=1}^{n} x_{it} = 0$ for all i.

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Consequence. In comparison to the optimal *cooperative* solution, the *n unconstrained stores* in *Cournot competition*

- *overtrade* by a factor $2n/(n+1)$
- make a *total profit* proportional to $4n/(n+1)^2$.

Buffer shocks

- Major role of storage is to provide very fast response:
  - TV pick-up
  - Buffering renewable generation (forecast errors)
  - Generation failure
- Leads to cost dependent on storage level.
- Random fluctuations in storage due to unexpected demands.

Stochastic Dynamic Programme

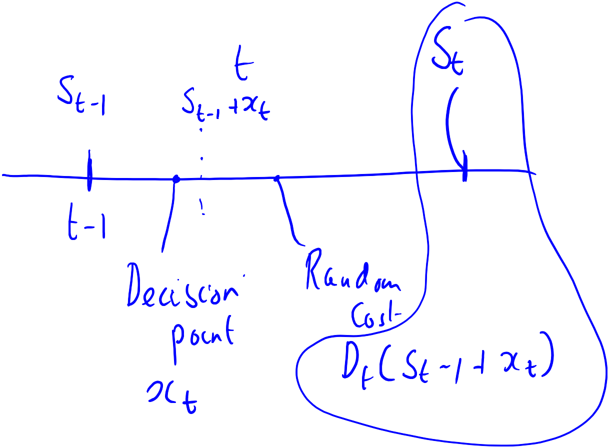
Decision at time t. As before:

- Current store level s_{t-1}
- Cost functions C_t
- Purchase amount x_t

Additionally:

- Random cost $D(s_{t-1} + x_t)$
- Random storage level at next time period, s_t.

Problem (Diagram)



Solution Strategy

Instead of writing down full SDP consider non-standard version:

$$V_{t-1}(s_{t-1}) = \min_{\substack{x_t \in X_t \\ s_{t-1}+x_t \in \cap [0, E_t]}} \left[C_t(x_t) + A_t(s_{t-1}+x_t) + V_t(s_{t-1}+x_t) \right], \tag{1}$$

- C_t costs as before,
- $A_t(s_{t-1}+x_t)$ expected cost of the shock (see later)
- V_t the expected future cost under optimal strategy.

Recast as a deterministic optimisation which we solve with using Lagrangian methods.

Price taker example

Consider special case of cost function:

$$C_t(x) = \begin{cases} c_t^{(b)} x, & \text{if } x \geq 0 \\ c_t^{(s)} x, & \text{if } x < 0. \end{cases} \quad (2)$$

Then:

Proposition

Suppose that, for each t, we have $c_t^{(b)} = c_t^{(s)} = c_t$ say; define

$$\hat{s}_t = argmin_{s \in [0, E_t]} [c_t s + A_t(s) + V_t(s)]. \quad (3)$$

Then, for each t and for each s_{t-1}, we have $\hat{x}_t(s_{t-1}) = \hat{s}_t - s_{t-1}$ provided only that this quantity belongs to the set X_t.

Price taker example

If the store is not total efficient we need A_t is convex.
Define:

$$s_t^{(b)} = \operatorname{argmin}_{s \in [0, E_t]} [c_t^{(b)} s + A_t(s) + V_t(s)] \tag{4}$$

and similarly define

$$s_t^{(s)} = \operatorname{argmin}_{s \in [0, E_t]} [c_t^{(s)} s + A_t(s) + V_t(s)]. \tag{5}$$

Proposition

Suppose that the cost functions C_t are as given by (2) and that the functions A_t are convex. Then the optimal policy is given by: for each t and given s_{t-1}, take

$$x_t = \begin{cases} \min(s_t^{(b)} - s_{t-1},\ P_{It}) & \text{if } s_{t-1} < s_t^{(b)}, \\ 0 & \text{if } s_t^{(b)} \le s_{t-1} \le s_t^{(s)}, \\ \max(s_t^{(s)} - s_{t-1},\ -P_{Ot}) & \text{if } s_{t-1} > s_t^{(s)}. \end{cases} \tag{6}$$

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Relation to previous work

Interested in buffering against wind forecast errors, minimising excess conventional generation.

- Bejan, Kelly, Gibbens, "Statistical aspects of storage systems modelling in energy networks"

- Gast,Tomozei,Le Boudec, "Optimal storage policies with wind forecast uncertainties"

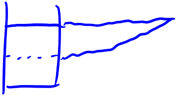
Function A_t

Reminder: $A_t(s_{t-1} + x_t)$ average cost of the shock

Made of two parts:

1. Cost of due to the shock, e.g. energy unserved

2. Cost due to random fluctuation in store level.



Estimating A_t

- $\bar{A}_t(s_{t-1} + x_t)$ the expected additional cost to *immediately* returning the level of the store to its planned level $s_{t-1} + x_t$ by the end of time period.
- The cost of the energy which will be purchased to rectify the situation as well as penalty costs.
- Here $\bar{A}_t$ is readily determinable, since it does not depend on how the store is controlled outside the time period t.

Estimating A_t

Then $\bar{A}_t$ is a good approximation of A_t if one of the following is true:

- Linear cost functions, $C_t(x) = c_t x$.
  - since the optimal level for the store is unchanged.
- Shocks are rare but expensive.
  - since the major contribution to A_t is the cost due to the shock not the readjustment.

Approximation can be improved by allowing longer time periods for the coupling.

In many applications the value of A_t may need to be determined by observation.

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Optimal control

Define also the following (deterministic) optimisation problem:
P:Choose $s = (s_0, \dots, s_T)$ with $s_0 = s_0^*$ so as to minimise

$$\sum_{t=1}^{T} [C_t(x_t(s)) + A_t(s_t)] \tag{7}$$

subject to the capacity constraints

$$0 \leq s_t \leq E_t, \quad 1 \leq t \leq T, \tag{8}$$

and the rate constraints

$$x_t(s) \in X_t, \qquad 1 \leq t \leq T. \tag{9}$$

It can be shown that the solution to this problem solves the SDP up to times of shocks.

Lagrangian Theory

Theorem

Let s^* denote the solution to the problem **P**. *Then there exists a vector $\lambda^* = (\lambda_1^*, \ldots, \lambda_T^*)$ such that*

1. *for all vectors s such that $s_0 = s_0^*$ and $x_t(s) \in X_t$ for all t (s is not otherwise constrained),*

$$\sum_{t=1}^{T} \left[C_t(x_t(s)) + A_t(s_t) - \lambda_t^* s_t \right] \geq \sum_{t=1}^{T} \left[C_t(x_t(s^*)) + A_t(s_t^*) - \lambda_t^* s_t^* \right]. \tag{10}$$

Lagrangian Theory

Theorem

2. *the pair* (s^*, λ^*) *satisfies the complementary slackness conditions, for* $1 \leq t \leq T$,

$$\begin{cases} \lambda_t^* = 0 & \text{if } 0 < s_t^* < E_t, \\ \lambda_t^* \geq 0 & \text{if } s_t^* = 0, \\ \lambda_t^* \leq 0 & \text{if } s_t^* = E_t. \end{cases} \tag{11}$$

Conversely, suppose that there exists a pair of vectors (s^*, λ^*), *with* $s_0 = s_0^*$, *satisfying the conditions* (1) *and* (2) *and such that* s^* *is additionally feasible for the problem* $\mathbf{P}$. *Then* s^* *solves the problem* $\mathbf{P}$.

Finding (s^*, λ^*)

Proposition

Suppose that the functions A_t are differentiable, and that the pair (s^*, λ^*) is such that s^* is feasible for the problem **P**, *while (s^*, λ^*) satisfies the conditions of previous Theorem. For each t define*

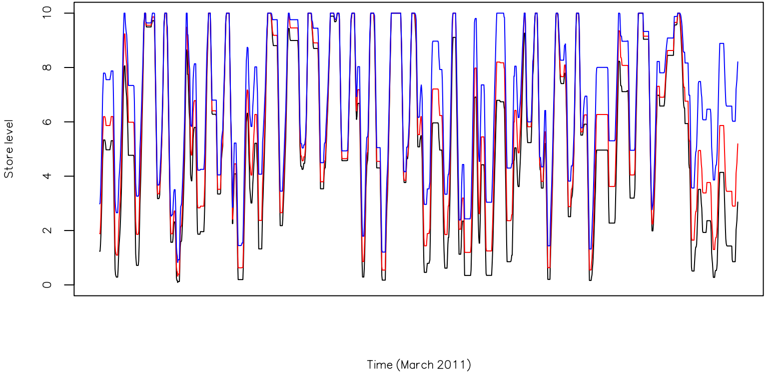
$$\nu_t^* = \sum_{u=t}^{T} [\lambda_u^* - A_u'(s_u^*)]. \tag{12}$$

Then the condition that (s^*, λ^*) satisfies the condition (1) of previous Theorem is equivalent to the condition that

$$x_t(s^*) \text{ minimises } C_t(x) - \nu_t^* x \text{ in } x \in X_t, \qquad 1 \le t \le T. \tag{13}$$

UK Market Example

$E/P = 5$ hrs $\quad$ *Efficiency* $= 0.85$ (ratio of sell to buy price).
$A_t(S) = \nu/S$ (Black:$\nu = 0.02$,Red:$\nu = 0.2$, Blue: $\nu = 1$)



Two Cooperating Stores

- Consider two stores working in co-operation
- Already looked at case of two identical stores
- Interesting question if two stores are very different:
  - A fast small store (demand side response)
  - A small large store (pumped storage)

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- Consider two stores working in co-operation
- Already looked at case of two identical stores
- Interesting question if two stores are very different:
  - A fast small store (demand side response)
  - A small large store (pumped storage)
- Best outcome is complete resource pooling, i.e. can treat as a single store with parameters as sum of individuals.
- But when will this occur?
- How far away from this are we?

Demand Side Response

- Demand side response can be viewed as also moving energy through time.
- We now have an energy debt S, such that $-E \leq S \leq 0$.
- So we have to sell before we can buy, but the problem formulation is the same.
- Often demand response has further binding constraints:
  - Frequency at which actions can be taken.
  - Length of time energy debt can be maintained.
- But for a first approximation this work provides some insight.

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Stochastic Prices

- So far nearly everything we have done has assumed deterministic price functions.
- In reality this problem is stochastic in nature.
- We have side stepped that issue by talking about a re-optimisation framework.
- Under a specific stochastic model the optimal behaviour does not changes, multiplicative errors.
- But if these are not true how sub-optimal is re-optimization?
- E.G. if prices follow a mean reverting process?