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Option contracts for power system balancing Part 1: Optimal stopping problems

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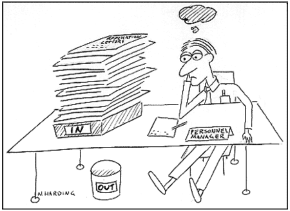
The 37% Rule

At the British Psychological Society's conference in April 1997 Dr Peter Todd, of the Max Planck Institute in Munich, spoke about the best (optimal) strategy for

- finding a partner, or
- finding a suitable new employee from a range of applicants



Source: *Plus magazine*

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He quoted the following rule:

The 37% rule

Once you have seen 37% of the applicants, a coherent picture of the ideal employee is built up and the next person to fulfil these criteria should be given the job.

This is an example of an *optimal stopping problem*: when to stop merely observing, and act.

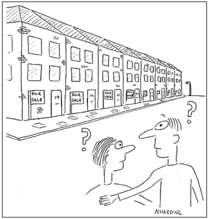


Figure: Another optimal stopping problem (source: *Plus magazine*)

Suppose there is a pool of N candidates and you decide to observe at least M. That is, you

- Observe the first $M-1$ candidates without acting, then
- Select the next candidate which is *better* then all of these $M-1$.

Example: Let $N=4$ and number the candidates from 1 to 4 (4 is best, no ties).

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2413	3412	4312
1432	2431	3421	4321

M=1	**M=2**	**M=3**	**M=4**
4123	1423	1243	1234
4132	1432	1324	1324
4213	2143	1342	2134
4231	2413	2143	2314
4312	2431	2314	3124
4321	3124	2341	3214
	3142	3124	
	3214	3142	
	3241	3214	
	3412	3241	
	3421		

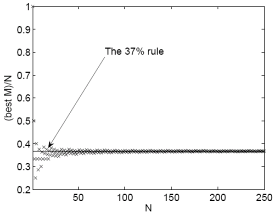


Figure: Left: Permutations when $N=4$. Centre: For each M, permutations yielding best choice: Best $M=2$. Right: The asymptotically optimal proportion for large N is $1/e$ (source: *Plus magazine*)

Optimal stopping has been the subject of wide study:

- First solved in discrete time:
  - Wald (1947): problems of *sequential analysis*
  - Snell (1952): general results for optimal stopping problems
- Then in continuous time, first by Dynkin (1963) using the concepts of excessive and superharmonic functions
- Still very active research area today, underpins Real Options Analysis
- In power systems engineering, applied in context of demand response (see Iwayemi et al. (2011))

Optimal stopping theory aims to tell us the **best possible time** at which to take an action in an **online** fashion.

That is, given the model, there is **no possible better** way to react in real time to the available information.

OS theory is therefore potentially relevant to any time-sensitive situation, provided it can be modelled. Some examples at different timescales in energy systems (together with optimal stopping formulations):

- Providing real-time balancing services (multiple optimal stopping, see next tutorial)
- Charging a battery in a real-time electricity market (singular stochastic control)
- Upgrading (reinforcing) electricity networks (real options analysis)
- Quickest corrective control of power systems (Bayesian quickest detection)

Real Options study of reinforcing electricity networks

In so-called **real options** problems, OS theory is used to value real-world projects having an element of timing flexibility (eg. when to expand or abandon a project) and derive the optimal strategy.

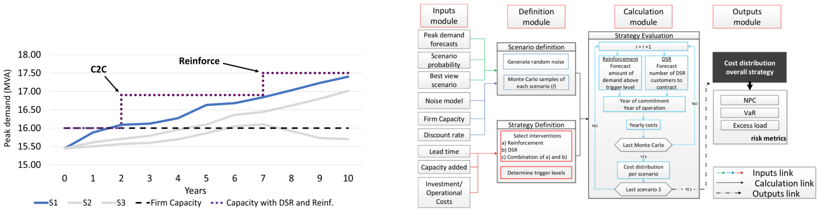


Figure: Left: the (multiple) optimal stopping problem
Right: flowchart of methodology
[from Schachter, Mancarella, M. and Shaw, *Energy Policy* (2016)]

Bayesian quickest detection problem for corrective control of power systems

This was a study of online decision making at the operational timescale:

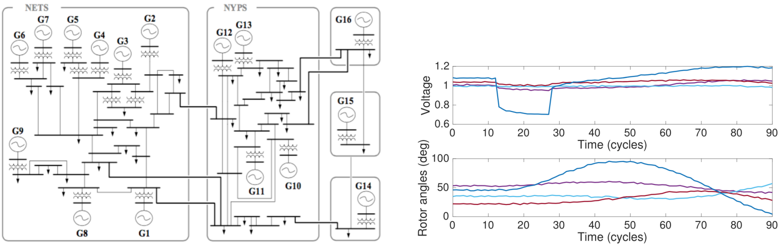


Figure: Left: IEEE 68 bus test network. Right: Simulated post-fault responses of voltages (top panel) and rotor angles (bottom panel) [from Gonzalez, Kitapbayev, Guo, Milanovic, Peskir & M., *CDC 2016*]

OS theory suggests the structure of the solution, which can then be calibrated using machine learning:

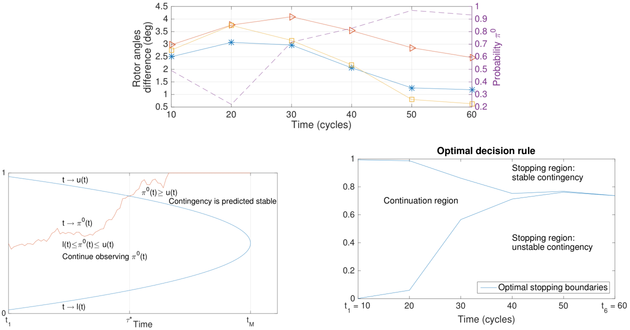


Figure: Top: Training data and probabilistic neural network output
Bottom left: Stylised illustration of the optimal boundaries
Bottom right: Computed optimal boundaries

Singular stochastic control problem of charging a battery in a real-time electricity market with $\pm$ prices

OS theory also connects to several other stochastic optimisation problems. An example is **singular stochastic control**:

- Optimal stopping problem is the 'derivative' of the singular stochastic control problem 'in the direction of the control'
- Connection first used heuristically by Bather and Chernoff (1963) but not proved rigorously until work of Karatzas and Shreve in 1984
- τ represents the first time at which any control is exercised

In energy trading context, possibly negative prices make this relationship more complex. . .

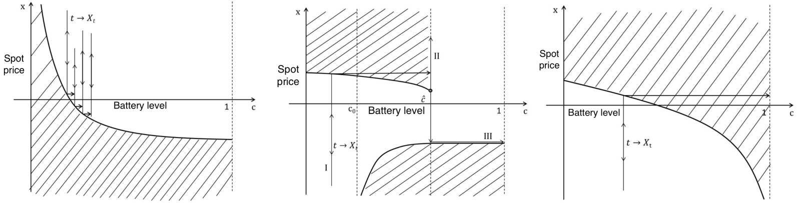
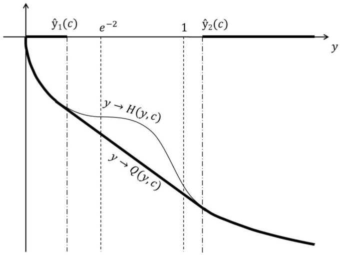


Figure: Stylised illustrations of the optimal singular stochastic control boundaries in three different parameter regimes. [From De Angelis, Ferrari & M. *SICON* (2015)]



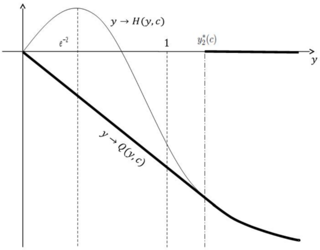


Figure: Example members of the underlying family of optimal stopping problems

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Negative prices can also lead to strange, non-smooth optimal stopping boundaries:

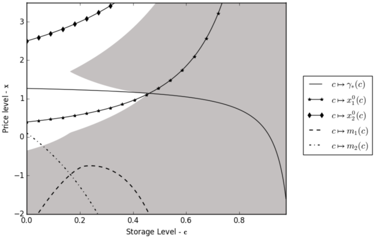
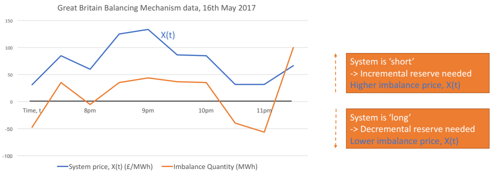


Figure: 'Kinked' optimal boundary for a problem of optimal entry into a charging contract. [From De Angelis, Ferrari, Martyr & M. *MAFE* (2017)]

- Note the hiring problem was
  - purely combinatorial (no statistical model)
  - binary (win / lose)
- In contrast, in energy applications we often
  - have relevant historic datasets
  - seek to minimise costs (subject to operational constraints)
- Data series can often be modelled as diffusion processes, eg:



- Can then perform **model-based** optimisations.