Option contracts for power system balancing Part 3: Power system balancing and multiple optimal stopping

John Moriarty (Queen Mary University of London)
Joint work with Jan Palczewski (Leeds)

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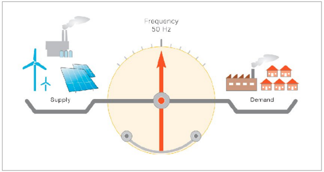
1. Power system balancing
   - Imbalance markets
   - Option contracts for batteries
   - Controlling the battery

2. Modelling via multiple optimal stopping

The main task of an **electric power system operator** is to continuously match (balance) electricity generation with demand.

- If balance is lost, the system frequency deviates from 50Hz and control actions are taken to compensate
- Too little generation: system is ‘short’, **incremental** reserve needed
- Too much generation: system is ‘long’, **decremental** reserve needed



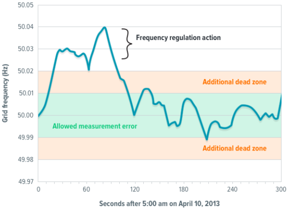


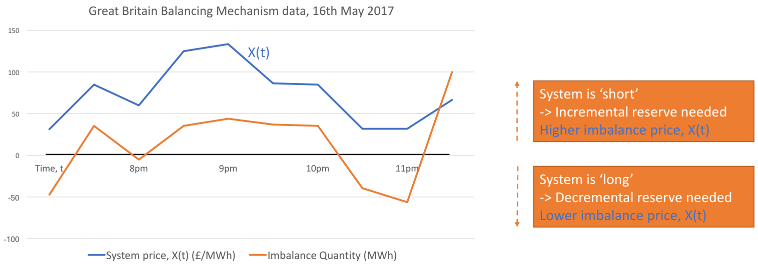
Figure: Left: Generation - demand balance (source: esc.ethz.ch)
Right: Actual frequency variations in TENNET (source: smartpowergeneration.com)

Imbalance markets associate a financial value to a unit of power for use in system balancing.

- System is short: insufficient power, higher imbalance prices
- System is long: excess power, lower imbalance prices

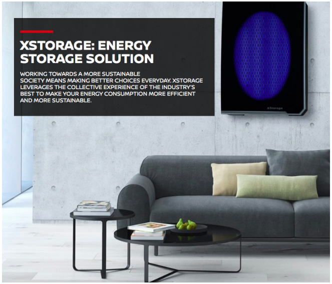
Today, in the UK we have the *system price* which is not usable for real-time control (determined *ex post*).

In a future market setup, we assume a real-time imbalance price usable as a control signal.



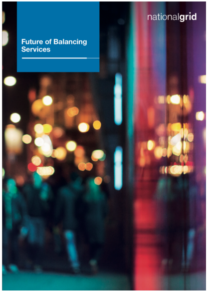
Balancing services are of multiple kinds (eg frequency response, spinning/non-spinning, replacement. . .) and are provided by multiple technologies. Here we focus on **batteries**, which can provide

- decremental reserve when system is long (by charging)
- incremental reserve when system is short (by discharging)



Left: Nissan xStorage Home, which could "provide Grid Services"
(source: nissan.co.uk).
Right: The world's largest battery at Hornsdale wind farm, Australia
(source: hornsdalepowerreserve.com.au)

- National Grid (UK) is consulting on future balancing services
- including greater use of shorter-term contracts
- In this talk we will consider the potential use of **American-style option contracts**



(source: nationalgrid.com)

In financial markets,

- an **American option** is a contract sold by one party (the option writer) to another party (the option holder)
- contract offers the buyer the right, but not the obligation, to buy (call) or sell (put) an asset
- at an agreed-upon price (the strike price) during a certain period of time.

Question: Could the asset be one unit of power for balancing a power system?

We will consider an **American call option on one unit of power** for balancing

- This is one possible new, short term contract for incremental reserve
- Devices eg. home batteries could participate

Some natural questions:

- How much should the option cost? (the *premium*)
- What should the pre-agreed strike price be?
- How would devices optimally engage with the contract?
- Would it stabilise or destabilise the system?
- Would it lower or raise costs overall?

Comparison with reserve contracts today

Present-day reserve contracts:

- Provide an option on power which can be exercised eg. several times within a pre-specified window of time
- Specify payments for both *availability* (£/ MW / h) and *utilisation* (£/ MWh) [like a financial option's premium and strike]
- But usually don't specify total energy to be provided

In contrast, our proposed American-style contract provides one unit of power at one time. So it can:

- Match the capacity of the battery, avoiding under-delivery
- Allow casual / opportunistic participation

Under the call option:

- System operator (SO) directly controls battery's discharging
- Battery operator (BO) chooses when to recharge. If this is from
  - local generation then there's no issue
  - the grid, this could create/worsen imbalance

  One solution would be to **expose the BO to imbalance pricing when charging**.

To stabilise the grid, the operational outcome we seek is like this:

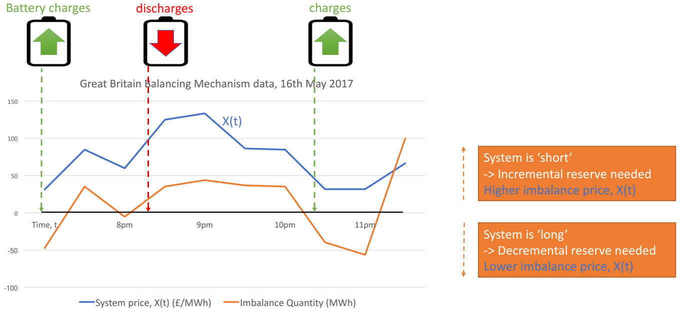


Figure: Grid-stabilising battery operation

...rather than this:

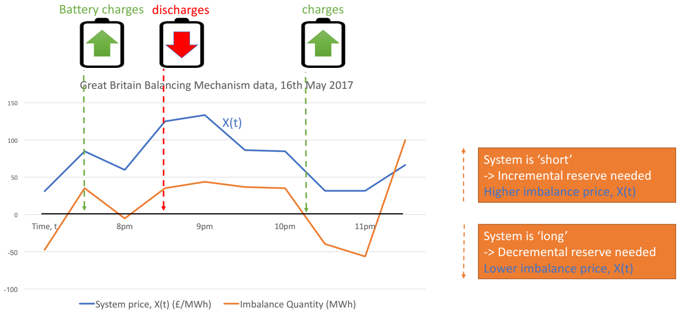
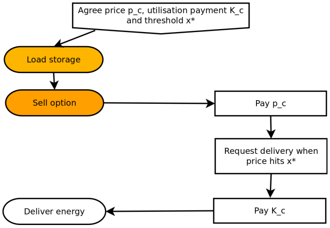


Figure: Grid-destabilising battery operation

If the BO pays the imbalance price $(X_t)_{t \geq 0}$ (and if recharging is instant) then recharging can be considered an **optimal stopping problem for the BO**.

- The BO and SO both respond to the imbalance price signal $(X_t)_{t\geq 0}$, which we model as a regular diffusion process
- The BO can sell a call option to the SO at any time, with a fixed premium (p, also called the 'utilisation payment') and strike price (K, also called the 'availability payment')
- The SO exercises its option when $X \geq x^*$ (ie. when the system is too short)



Battery operator System operator

This cycle can be repeated indefinitely, meaning a **multiple optimal stopping problem** for the BO – the **'lifetime problem'**.

We would like to know:

1. Can we predict when the battery would be charged?
2. Do the premia (p and K) give the battery operator sufficient profit to participate?

Let's begin with a lemma whose proof is trivial: whatever the imbalance process X, if the battery is full then

the BO never waits to sell the call option

since immediate sale means:

- the option premium p is received immediately, and
- the strike price is received at the earliest opportunity.

Predicting the BO's charging times is a non-trivial multiple optimal stopping problem driven by the imbalance price process X.

Problem setup

Imbalance process. We model the imbalance process X as an Ornstein-Uhlenbeck process (Recall: no explicit expression for ϕ or ψ)

Objective function. We consider two different optimisations: Discounted net present value of

- One option contract
- An infinite (or 'lifetime') series of option contracts

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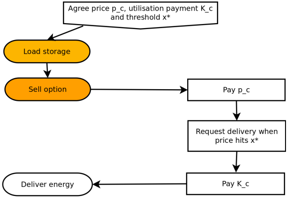
What is the optimal stopping gain function? (Single option, BM)



Suppose we charge & sell an option **when the spot price is y.**

- Exercise occurs at $\tau_e := \inf\{t \geq 0 : X_t \geq x^*\}$.
- Expected NPV of strike price:

$$h_c(y) = E^y\{e^{-r\tau_e} K\} = \begin{cases} K, & y > x^*, \\ Ke^{-a(y-x^*)}, & y \leq x^*. \end{cases} \quad (1)$$

So the payoff is $-X_\tau + p + h_c(X_\tau)$ (non-smooth).

Brownian motion case (seen yesterday)

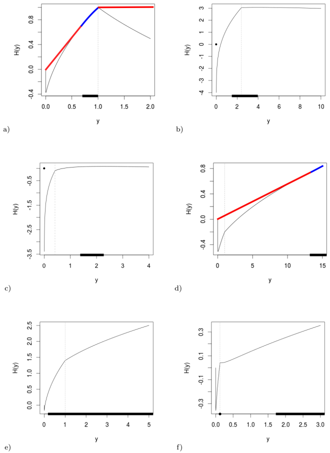


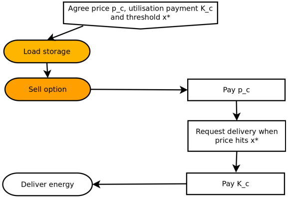
Figure: Six qualitatively different solutions in the Brownian case

The case of general diffusions (OU,. . .)

Since ϕ and ψ are not explicit we need some help!

In general, let $x \mapsto V(x)$ be the value function. Then:

1. From the general theory, stopping occurs when X first hits the optimal stopping set (when gain = value) – say, at $\check{x}$
2. We pay $\check{x}$ for the power and receive the option premium p
3. SO exercises option when price rises to x^*
4. We receive K+(expected value of all remaining options)



We impose the following fair conditions:

S1. *The BO has a positive expected profit from the offer and exercise of the option.*

S2. *The option cannot lead to a certain financial loss for the SO.*

Lemma

When taken together, the sustainability conditions **S1** *and* **S2** *are equal to the following quantitative conditions:*

S1*: $\sup_{x\in(a,b)} h(x) > 0$, *and*

S2*: $p + K < x^*$.

Idea: We don't need to know the entire value function!

- Since $p + K < x^*$, it cannot be optimal to buy power at the price x^* or greater
  $\Rightarrow$ don't care about value function above x^*
- Once we buy power at $\check{x}$, forced to wait until price rises to x^* before buying again
  $\Rightarrow$ don't care about value function below $\check{x}$

So *we may not need to know the full geometry of the gain function to solve the problem!*

Define $L := \limsup_{x \to a} \frac{\vartheta(x)^+}{\phi_r(x)}$.

Theorem

(Single option problem) Assume that conditions **S1*** *and* **S2*** *hold. There are three exclusive cases:*

(A) $L \le \frac{h(x)}{\phi_r(x)}$ *for some* $x \implies$ *there is* $\hat{x} < x^*$ *that maximises* $\frac{h(x)}{\phi_r(x)}$*, and then, for* $x \ge \hat{x}$, $\tau_{\hat{x}}$ *is* **optimal***, and*

$$V(x) = \phi_r(x)\frac{h(\hat{x})}{\phi_r(\hat{x})}, \qquad x \ge \hat{x}. \tag{2}$$

(B) $\infty > L > \frac{h(x)}{\phi_r(x)}$ *for all* $x \implies V(x) = L\,\phi_r(x)$ *and there is* **no optimal** *stopping time.*

(C) $L = \infty \implies V(x) = \infty$ *and there is* **no optimal** *stopping time.*

Moreover, in cases A and B the value function V is continuous.

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Let $x \mapsto \hat{\mathscr{T}}^n \mathbf{0}(x)$ be the value function of n options.

Lemma (existence)

If the single option value function is finite then

1. *The functions $\hat{\mathscr{T}}^n \mathbf{0}$ are strictly positive and uniformly bounded in n*
2. *The limit $\hat{\zeta} = \lim_{n \to \infty} \hat{\mathscr{T}}^n \mathbf{0}$ exists and is a strictly positive bounded function. Moreover, the lifetime value function $\hat{V}$ coincides with $\hat{\zeta}$*
3. *The lifetime value function $\hat{V}$ is a fixed point of $\hat{\mathscr{T}}$*

Finally, we can calculate the lifetime value function numerically:

Lemma

The lifetime value function evaluated at x^* satisfies

$$\hat{V}(x^*) = \max_{z \in (a, x^*)} y(z),$$

where

$$y(z) := \frac{-z + p + \frac{\psi_r(z)}{\psi_r(x^*)} K}{\frac{\phi_r(z)}{\phi_r(x^*)} - \frac{\psi_r(z)}{\psi_r(x^*)} A}.$$

Summary

- Proposed an energy limited balancing services contract designed for battery storage
- Based on the American call option in mathematical finance
- Fixed revenues paid to battery operator (availability and utilisation payments) rather than potentially low market prices
- Mathematically we have derived the optimal charging strategy (when it exists!) and contract value
- Studied single option for opportunistic use, lifetime problem for investment analysis
- Explicit results available for OU imbalance prices; numerical results for any regular diffusion

Presentation based on:

- Moriarty J. and J. Palczewski (2017). Real option valuation for reserve capacity. EJOR 257 (1), 2017, 251–260
- Moriarty J. and J. Palczewski (2016). Energy imbalance market call options and the valuation of storage. arXiv:1610.05325

Conclusions

- Optimal stopping problems pop up everywhere, including energy applications! (Real options, corrective control, demand response, charging, trading . . .)
- Geometric viewpoint is easy for 1d Brownian motion, and guides intuition for other 1d regular diffusions
- Rich and practically useful theory, much known, much still to prove

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Thanks for your attention!